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REVIEW-ESSAY

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In [Anellis & Houser 1991] it was noted that, through much of the
twentieth century, major works on the history of logic, such as Kneale
and Kneale's [Kneale & Kneale 1962] *Development of Logic* and Bo-
cheński's [Bocheński 1970] *Formal Logic*, gave scant attention to the
work in the algebraic logic or algebra of logic of the second half of the
nineteenth century. The proportion of pages devoted to the work of
George Boole (1815–1864), Augustus De Morgan (1806–1871), Charles
Peirce (1839–1914), William Stanley Jevons (1835–1882), John Venn
(1834–1923), and Ernst Schröder (1841–1902), not to mention such
less well known of their colleagues as Evgenii Bunitskii (1874–1952),
Platon Poretskii (1846–1907), or even Louis Couturat (1868–1914),
amount to less than that given to those of their contemporaries whose
contributions to logic itself were scant, but whose influence on what
we might call philosophy of logic was large, Immanuel Kant (1724–
1804), Georg Wilhelm Friedrich Hegel (1770–1831), and John Stuart
Mill (1806–1873), for example. Thus, for example [Bocheński 1970]
devotes a mere ten pages to "the Boolean calculus" and only a dozen
more to the logic of relations, which focuses attention primarily on the
work of Russell, rather than that of De Morgan, Peirce, and Schröder,
while [Kneale & Kneale 1962] devotes a mere thirty pages to Boolean
algebra and the logic of relations. [Anellis & Houser 1991] sought an
explanation, and suggested there, and in more detail in [Anellis 1995b],
what might be the reason for this relative neglect. One clue was
found in the comments of van Heijenoort [Van Heijenoort 1967b, vi],
in which the algebraic logic is in effect regarded as a minor sidelight

in the history of the development of mathematical logic. Van Heijenoort traced the birth of mathematical logic to Frege's *Begriffsschrift* of 1879 (see, e.g., [Van Heijenoort 1967a], [Van Heijenoort 1967b, vi], [Van Heijenoort 1987], [Van Heijenoort 1992, 242]). Van Heijenoort (in a work written in 1974 and published in 1992) plainly, unequivocally, and uncompromisingly stated [Van Heijenoort 1992, 242]: "Modern logic began in 1879, the year in which Gottlob Frege (1848–1925) published his *Begriffsschrift.*" He immediately added [Van Heijenoort 1992, 242] that "Frege's contribution marks one of the sharpest breaks that ever occurred in the development of a science." It should come as no surprise that [Kneale & Kneale 1962] devoted so little space to the algebraic tradition of Boole-Peirce-Schröder after noting that William Kneale, although he also wrote two articles [Kneale 1948, Kneale 1956] on Boole and his work, likewise took Frege to be the founder of mathematical logic [Kneale 1957]. But this hardly accounts for the paucity of their attention to the algebraic tradition as compared with the attention lavished upon Kant and Mill, not to mention the extravagant amount of attention devoted to the medievals, if the purpose of their work, as they averred, was to trace the developments that led to modern mathematical logic as conceived and configured by Frege and those who followed his example.

In fact, both Augustus De Morgan and Alfred North Whitehead (1861–1947) provided clues to the mystery of the lack of attention to *algebraic* logic, which, in the second half of the nineteenth century, was the mathematical logic of that day, as Edward Huntington (1874–1952) and Christine Ladd-Franklin (1847–1930) wrote in using the terms synonymously [Huntington & Ladd-Franklin 1905, 1]: "Symbol Logic, or Mathematical Logic, or the Calculus of Logic, —called also the Algebra of Logic (Peirce), Exact Logic (Schröder), and Algorithmic Logic or Logistic (Couturat), —covers exactly the same field as Formal Logic in general . . . ." De Morgan [De Morgan 1868, 71] wrote of the mathematical sect wanting to put out the logical eye, while the logical sect wanted to put out the mathematical eye. Whitehead, somewhat more prosaically in his *Treatise of Universal Algebra* [Whitehead 1898, vi], similarly complained that Symbolic Logic has been "disowned" by mathematicians for being too logical, and by some logicians as being too mathematical. He did so in pointing to the kinship between the Boole's Symbolical Logic, William Rowan Hamilton's quaternions [Hamilton 1844], and Hermann Grassmann's calculus of extension (*Ausdehnungslehre*) as the "chief examples" of "the various systems of Symbolic Reasoning allied to ordinary Algebra."

But this is less than half the explanation. Another crucial aspect was the absorption of algebraic logic and the calculus of relatives into the logistic system, inspired by Frege, that was begun by Whitehead in the "Memoir on the Algebra of Symbolic" [Whitehead 1901] and "On Cardinal Numbers" [Whitehead 1902], in which Bertrand Russell contributed an important section, and Russell's own "Sur la logique des relations . . . " [Russell 1901] and *Principles of Mathematics* [Russell 1903a], where the first steps were to render the algebraic logic of Boole, Peirce, and Schröder into the notation of Giuseppe Peano, far more elegant and economical than Frege's notation, in preparation for ultimately incorporating it within the grand system of the *Principia Mathematica.*

The remainder of the explanation seems to reside in the philosophical debates which raged in the second half of the nineteenth century and into the first decade or two of the twentieth century. In Germany and Britain, philosophical journals took up the question, in its various aspects: "*Was ist Logik?*" It will suffice us to note the series of articles by Hermann Ulrici (1806–1884) under the title "Zur logischen Frage" [Ulrici 1869–70] in *Zeitschrift für Philosophie und philosophische Kritik*, and the essay *Ueber die Begriff der Logik* [Löwe 1849] by Johann Heinrich Löwe (or Loewe; 1808–1892), to appreciate the longevity of the problem of the nature and scope of logic raised in the middle of the nineteenth century by the appearance of competing developments in logic in those years. The compilation of an exhaustive bibliography of these and related works in German and English into the 1920s would serve only to show that there was serious concern and extensive discussion on the nature and scope of logic arising from new developments in logic since the middle of the nineteenth century. It would show that the competing systems and interpretations that vied for the attention of logicians, philosophers, and mathematicians, as well as linguistics and psychologists. All this even after Kant pronounced the view in the second edition of his *Critik der reinen Vernunft* [Kant 1787, vii] and in his lecture notes on logic [Kant 1800], edited and published by his student Gottlob Benjmain Jäsche (1762–1842) at the end of the eighteenth century, that logic had been essentially completed and perfected by its founder, Aristotle.

Aristotelian logic, or at least that version of Aristotelian logic handed down by the medieval scholastic philosophers, first came into serious and continuous question in the Renaissance, and René Descartes (1596–1650), by no means the first to challenge the fecundity or value of

this scholastic Aristotelian logic, nor the first to propose an alternative *mathesis*, was among the more outspoken to call for applying the example of mathematics as such a *mathesis*.

The first and most prominent to undertake the development of a mathematical logic was Leibniz, and the present volume of the *Handbook* opens with the extensive essay by Wolfgang Lenzen on "Leibniz's Logic" (pp. 1–83). Meanwhile, however, reformers of the stripe of Petrus Ramus (Pierre de la Ramée; 1515–1572) and Antoine Arnauld (1612–1694) and Pierre Nicole (1625–1695) had taken up the challenge of the Renaissance humanists, of Francis Bacon (1561–1626), and especially of Descartes, to devise logics free of the scholastic baggage which they held served as little more than futile exercises of ingenuity and, more crucially, served as a hindrance to the organization and advancement of science. The Ramist logic and the Cartesian or Port-Royalist logics were each designed to serve as an *ars inveniendi*, rather than merely, as had traditional—scholastic—logic, as solely an *ars cogitandi*. In her essay on "Kant: From General to Transcendental Logic" (pp. 85–130), Mary Tiles (p. 85) saw in Kant the "architect who provides conceptual design sketches for the new edifice that was to be built on the site once occupied by Aristotelian, syllogistic logic but which in the eighteenth century was covered by rubble left by Ramist and Cartesian demolition gangs." Yet the logics introduced by these "demolition gangs" had a large following, especially among Protestant scholars, through the seventeenth and early eighteenth centuries. If Kant was the architect for much of the logic of the nineteenth century, then the Ramists and Port-Royalists can be considered the precursors of another goodly portion of the logic of the nineteenth and early twentieth centuries, from the empirical and inductive logic of John Stuart Mill and his followers to the experimental logic of John Dewey (1859–1952) except for occasional brief mentions, such as that by Tiles. However, the "demolition gangs" of the Renaissance through the seventeenth century are omitted from consideration in the present volume. (We may perhaps anticipate that they will receive due attention in the second, not yet published, volume of the series.) The British logicians of the nineteenth century, among them no doubt Mill, and covering the period from Richard Whately (1787–1863) and George Bentham (1800–1884) to Francis Herbert Bradley (1846–1924) and Bernard Bosanquet (1884–1923), await the publication of another volume, as do the twentieth century logicians from Russell to Gödel, which will presumably also include *fin de siècle* logicians such as Peano and members of his school, spanning the last years of the nineteenth and first years of the twentieth centuries, in their own separate volume (see p. vii).

The Ramist and Port-Royalist Cartesian logics of the seventeenth century gave way to the empiricist-experimentalist logics of Mill and Dewey, among others, in the nineteenth century. Kant's *Critik* and his lectures on logic, prepared and published by his student Jäsche in 1800, sought to define the limitations of pure reason and its logic [Kant 1781, Kant 1787, Kant 1800]. This led to the development of three schools of logic in the nineteenth century. Bolzano's great contribution was to separate the theory of propositions, considering the *Satz-an-sich*, from the theory of judgment, which properly belongs to psychology. Bolzano's conception of logic is dealt with by Paul Rusnock and Rolf George in their essay "Bolzano as Logician" (pp. 177–205). Others took Kant's consideration of the limits of pure reason in other directions. The leader of the school which sought to identify logic and metaphysics was Hegel, while some, such as the early Husserl, took a psychologistic approach, and sought to identify logic and epistemology or psychology. Wilhelm Wundt (1832–1920) and Christoph Sigwart (1830–1905), who contributed to experimental psychology at least as much as to philosophy, can be seen as precursors of Dewey in the effort to make logic into a branch of psychology.[1] In his little booklet *Ueber den Begriff der Logik* [Löwe 1849, 5], writing just after the appearance of Boole's *Mathematical Analysis of Logic* [Boole 1847] and just before the appearance of his *Investigation of the Laws of Thought* [Boole 1854], Löwe began by asserting: "Eine gangbare und gemeinsaßliche Erklärung nennt die Logik die Wissenschaft von den Gesetzen des Denkens."

Despite the efforts of a few such hardy souls as Alois Riehl (1844–1924) who in articles such as "Die Englische Logik der Gegenwart" [Riehl 1877] sought to familiarize their fellow philosophers with the new "English" logic of the Booleans, "formal logic," whether rejected or embraced by philosophers, continued predominantly to mean Aristotelian logic until the early twentieth century. Rather what was at stake in the debates concerning the question of "*Was ist Logik?*" was in the first place, whether logic was "the *art* of correct reasoning" or "the *science* of the laws of thought." For the most part, the debates that occurred between philosophers and others who had anything to say on the question devolved into a turf battle: did logic belong to philosophy, to psychology, to linguistics, or, *horribile dictu*, to mathematics? And if it belonged to philosophy, to metaphysics, or to epistemology? This aspect of the debate is chronicled, especially from the

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[1]For an attempt to fill in the gap in the history of logic between Kant and Hegel, see [Holzheimer 1936].

perspective of German-speaking philosophers, in [Pulkkinen 1994], albeit with inadequate treatment of the developments of formal logic in the contributions, among others of the Grassmann brothers (Hermann Günther Grassmann [1809–1877] and Robert Grassmann [1815–1901]) and Schröder. With the appearance of Frege's *Begriffsschrift*, a new dimension to this debate was added, mainly between Frege and Schröder. A more rounded survey of this little-known aspect of the history of logic, from Kant to the appearance of Frege's *Begriffsschrift*, is given by [Vilkko 2002]. The philosophical discussions of the crucial question, *Was ist Logik?*, is barely touched upon in the current volume, although it exercised a goodly portion of the attention of those who, from the mid-nineteenth century until the mathematical logic of Frege, Peano, Hilbert, Whitehead and Russell, secured dominance over the formal traditional logic of Aristotle and the algebraic logic of the "Booleans," from Boole through Peirce to Schröder in the 1920s and early 1930s. In 1918, Morris Raphael Cohen (1880–1947) still found it incumbent upon himself to urge fellow philosophers to surrender the hodge-podge of linguistics, psychology, metaphysics and syllogistics that they taught under the rubric of "logic" and take up the new mathematical logic [Cohen 1918], while others, such as Harry Todd Costello (1885–1963), continued to dismiss the mathematical logic as the pouring of old wines into new bottles [Costello 1918]. The decisive break came in the late 1920s and early 1930s, when analytic philosophers, led by Russell and followed by the logical positivists, most notably Rudolf Carnap (1891–1970), were able to demonstrate the applicability of the new logic to old philosophical problems. Carnap's [1932] "Überwindung der Metaphysik durch logische Analyse der Sprache" may be viewed as the manifesto of this new attitude, while Russell's work, in particular his development in "On Denoting" [Russell 1905] of the theory of descriptions and his use of that theory to dispatch the chimeras of Meinong's ontology, filled with golden mountains, round squares, and "the present King of France," bald or hirsute, was sufficient to lead Gilbert Ryle (1900–1976) to suggest that Russell's use of the logistic persuaded philosophers of its value. In particular, he declared [Ryle 1957, 9–10] that, once the idea of relation was "made respectable" by De Morgan and the resulting relational inferences were codified by Russell in *The Principles of Mathematics*, then "the potentialities of the $x \ R \ y$ relational pattern, as against those of the overworked s-p pattern, were soon highly esteemed by philosophers, who hoped by means of it to bring to order all sorts of recalcitrancies into the notions of knowing, believing, ..." [Collins 1998, 710–1] holds

that the "three efforts in basic logic—Frege's, Peirce's and Russell's[—]" led to "developing a new, highly generalized logic," but that "[o]nly after 1900, when the mathematical foundations controversy attracted general attention in philosophy did interest in a new logic crystallize in attention space." The Rylean interpretation accommodates into this historical development the "logischen Frage" of the second half of the nineteenth century and earliest years of the twentieth, whereas Collins's clearly refers only to the problems raised by the set-theoretical and logical paradoxes originating in Cantor's set theory and Frege's Basic Law V regarding quantification over higher-order predicates, as formulated in the Russell Paradox and as dealt with by logicists, formalists, and intuitionists. The Rylean interpretation therefore would seem to have greater application than does Collins', since it more specifically accounts for the shift from what Volker Peckhaus has called the "omnibus logic" that was subject of the "logischen Frage" towards mathematical logic, whereas the foundational debates arose, so far as I understand the history, as a result of the attendance by philosophers to mathematical logic, rather than serving as the lure that enticed philosophers to shift from "omnibus" logic to mathematical logic.

Wolfgang Lenzen, in his essay "Leibniz's Logic," quite naturally does not deal with the nineteenth century issues of the "Was ist Logik?" debates, but does take note of the fact that the definitions of "logic" have "changed quite a lot during the development of logic from ancient to present times" (p. 1). He therefore considers the question, in dealing with the work of historical figures, whether one ought to describe their work in terms of their own conceptions of logic or from the contemporary conception. Lenzen does not here dwell upon the historiographical issue which he raised. Instead, he provides a brief examination of "traditional" interpretations of Leibniz's logic, particularly and most notably that proffered by Louis Couturat in *La logique de Leibniz* [Couturat 1901]. But he then moves on to reconstruct the "*formal core*" of Leibniz's logic. This reconstruction is the centerpiece of Lenzen's essay. The remainder of the essay demonstrates how the syllogism becomes provable in the logical calculus so reconstructed. Only then does Lenzen move on to undertake the issues in the traditional treatment of Leibniz's logic, in which Leibniz's metaphysics can be reconstructed within Leibniz's logic.

Ivor Grattan-Guinness ([Grattan-Guinness 1988] and elsewhere) has defended the distinction which had been embedded into the historiography of logic, namely the dismissie attitudes towards algebraic logic that has been so forcefully expressed by van Heijenoort and exemplified by Bocheński and the Kneales. For van Heijenoort (see

[Van Heijenoort 1967a]) and those who followed him, the difference was one of logic as calculus and logic as language. Specifically, Frege and the logicists such as Russell created a logical system which not only provided a foundation for mathematics but did so by developing mathematical logic as both a *lingua characteristica* and a *calculus ratiocinator*. Frege's *Begriffsschrift* was more than a means for manipulating propositions or equations, a formal deductive system. It was also a universal language, and contained both a syntax (the function-theoretical syntax) and a semantic, the latter being such as was applicable to a universal domain, the *Universum*, which contained objects, all that there is (later coming to contain two objects, The True and The False). The algebraic logicians, on the other hand, merely constructed a logic on the basis of a symbolical algebra, one whose semantic was limited to a specific universe of discourse rather than to the universal domain. (On the grounds that the algebraic logicians conceived of their logical system as applying to a particular and defined universe of discourse rather than to the universal domain, it can and has been argued that they constructed logics, rather than a logic; thus, one encounters a logic of number, a logic of relations, a logic of projective geometry, etc., according to the universe of discourse or domain specified by the axioms of the system.) In short, whereas the mathematical logic of Frege was the foundation of mathematics, the algebraic logics rested upon the foundation of mathematics, specifically on the symbolical algebra which provided the rules for constructing and manipulating the mathematical objects belonging to the chosen universe of discourse as given by the axioms.

Other differences were stressed by Grattan-Guinness, but also noted by van Heijenoort: *viz.* that whereas algebraic logic rested upon and grew out of algebra, mathematical logic grew out of analysis and set theory. Finally, for van Heijenoort in particular, who unlike Grattan-Guinness took almost no notice of the work of Charles Peirce and his student Oscar Howard Mitchell, or of Schröder, the syntax of algebraic logic was devoid of quantifiers. The function-theoretic syntax of Frege's *Begriffsschrift*, and subsequently of Whitehead and Russell's *Principia Mathematica*, made possible the use of quantifiers.

The kind of distinction between "algebraic logic" and "mathematical logic" asserted by van Heijenoort and Grattan-Guinness was, according to van Heijenoort, largely the result of the introduction into the latter by quantification theory. Nevertheless, within four years of Frege's construction in his *Begriffsschrift* of a quantification theory, Peirce had introduced a workable first-order quantification theory for his logic of relatives, and, a few years thereafter, completed

a second-order theory as well. Indeed, whereas Frege's work was set aside after the handful of reviews that appeared within a year of its appearance, the quantification theory devised by Peirce and his student Oscar Howard Mitchell was systematically elaborated by Schröder, and it was through the Schröderian account that it was picked up and applied by Leopold Lowenheim (1878–1957) and then Thoralf Skolem (1887–1963) to the logic of *Principia Mathematica*, by virtue of which first-order functional logic became the canonical mathematical logic (see, e.g., [Moore 1987, Moore 1988, Brady 2000, Anellis 2004]).

Since, in van Heijenoort's words [Van Heijenoort 1967b, vii], "Mathematical logic is what logic, through twenty-five centuries and a few transformations, has become today," the quantification theory of Frege's *Begriffsschrift* and the fixed, universal domain which enabled it to operate both as calculus and as language, made Frege's work the origin of mathematical logic, and left algebraic logic as an historically interesting but otherwise insignificant, back alley of the history of logic.

In their brief preface to the present volume, Gabbay and Woods take cognizance of the limitation of the view that "the mathematization of logic was, in all essentials, Frege's accomplishment" (p. vii). The present volume, which covers the period from 1685, beginning with the work of Leibniz, to 1900, seeks to redress the imbalance. Of eleven chapters in the volume, four deal specifically with algebraic logic, and one each of these is devoted to the contributions of Peirce and Schröder respectively, while a fifth chapter, that by Grattan-Guinness on "The Mathematical Turns in Logic" (pp. 545–56), explores the question of the relationship and differences between algebraic logic and mathematical logic, and of their respective historical significance. Together, these five chapters occupy approximately 45% of the present volume, or 335 pages out of a total of 750 pages of text. Theodore Hailperin provides a general survey of the evolution of algebraic logic in "Algebraical Logic 1685–1900" (pp. 323–88), beginning with Leibniz and ending with Whitehead's *Treatise*, noting only that the latter was the last extended treatment of the algebra of logic. The most prominent figures in his account are Leibniz, De Morgan, Boole, and Peirce, with Whately, Hugh MacColl (1836–1909), Jevons, Frege, Schröder, and Whitehead making cameo appearances. Victor Sánchez Valencia focuses detailed attention on "The Algebra of Logic" of the second half of the nineteenth century, and in particular of the work of Boole, as it grew out of the symbolical algebra and the influence which the algebraic approach of the French analysts had on him and his colleagues; on Jevons's logic of absolute terms; on Peirce's development of the monadic predicate logic; on Schröder's grand synthesis in his *Vorlesungen über die Algebra*

der Logik; and on De Morgan's, Peirce's, and Schröder's contributions to the logic of relations (pp. 389–544). Finally, Volker Peckhaus and Risto Hilpinen turn a microscope on the work in logic of Schröder and Peirce respectively (pp. 557–609, 611–58).

In Lenzen's reconstruction a hierarchy of calculi are set forth, each one more powerful than the previous. The first four are term logics. The fifth, obtained by mapping concepts and conceptual operators onto the set of propositions and propositional operators, is derived from the third calculus, and yields a propositional logic. This third calculus is the full algebra of concepts, which, according to Lenzen (based on his earlier controversial study "Leibniz und die Boolesche Algebra" [Lenzen 1984]), is deductively equivalent or isomorphic to the ordinary algebra of sets (p. 3). Since Leibniz provided a full set of axioms for this third calculus, Lenzen argues (p. 3) that Leibniz "'discovered' the Boolean algebra 160 years before Boole." Leibniz's other efforts at arithmetization of the syllogistic yielded the "Plus-Minus Calculus" and its weaker subsystem, the "Plus Calculus."

As we consider Lenzen's contribution, we come face to face with a crucial historiographical problem. Having noted that the definition of logic has changed over time, and that one can deal either with describing the work of historical figures or schools in terms of their own conceptions of logic, or from the contemporary conception, Lenzen opted for the latter and chose to "reconstruct" Leibniz's logic. Moreover, his claim that Leibniz had a Boolean algebra long before Boole illustrates a tendency of the "modernist" or "reconstructionist" approach and makes patent the distinction between this and the "traditionalist" approach that seeks to understand the work of the past on its own terms. I would argue first that there is a place in history for both approaches. But also that there is a danger in excluding the one in favor of the other. I am not specifically concerned here about the correctness of Lenzen's reconstruction. For I hasten to add that he is a master at reformulating Leibniz's contributions to logic in terms of current conceptions, and in some respects has far surpassed Jan Łukasiewicz's (1878–1956) [Łukasiewicz 1951] reinterpretation and reconstruction of Aristotle's logic in terms of modern mathematical logic.

Nevertheless, this historigraphic issue and the way in which we answer it is critical to our understanding of our subject. In seeking to pinpoint antecedents of the work current to the stage of development of the field of study in which he works, the "modernist" searches for what he has determined to be the earliest such result which is important for that field. Working from his retrospective vantage, the

"modernist," upon finding a proposition which, in his own contemporary terms, "looks like," or seems to mean, the same thing as the familiar, the contemporary, theorem T, will identify that proposition as the earliest formulation of Theorem T. Such an intepretation or identification will be made regardless of how it may have in actuality been understood by its first formulator or the first researcher to prove it. In this respect, "modernist" reconstructionism, taken as the sole purpose of sole perspective from which an historian comes at a the history, will in fact have—unwittingly, we can perhaps concede—been falsifying the history. For it is not difficult to think that the author of Proposition P may well have had something else in mind than what the modernist historian had by Theorem T. While this is a potential danger, there is the even graver danger in this approach, which Herbert Butterfield [Butterfield 1931] labelled as "Whiggism," in which the historian not only ignores the conceptions with which the past operated in formulating a thesis or undertaking an action, but ignores conceptions or events which, however, are significant for themselves and for the development of the field, yet not in the final analysis, considered directly relevant for the theory or theorem from the standpoint of the present.[2]

I do not deny that this sort of reinterpretation or reconstruction has its place. Rather, I would argue, with Peckhaus, that, in a handbook on the history of logic, a crucial function is to aid the reader in understanding the role of past contributions to the field within the context of the general development of the field, and that a healthy part of this can be accomplished only by attempting to present to the reader how the contributors of the past understood their own work, what they knew of the field as it stood in their own day, what they added to that, and how they conceived their work as it related to the work of their predecessors and contemporaries. It would seem to be especially appropriate to take a "traditionalist"—as opposed to "modernist"— approach when Lenzen starts off, in his very first paragraph, by asserting (p. 1) that:

> The meaning of the word 'logic' has changed quite a lot during the development of logic from ancient to present times. Therefore any attempt to describe "the logic" of a historical author (or school) faces the problem of deciding whether one wants to concentrate on what the author himself understood by 'logic' or what is considered as a genuinely logical issue from our contemporary point of view.

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[2]Recent defenses of modernist reconstructionism include [Bashmakova & Vandoulakis 1994] and [Barabashev 1997].

Thus Lenzen is clearly aware of the modernist vs. traditionalist approach. He deliberately adopts the former, namely the "systematic reconstruction of Leibniz's logic" (p. 2); and he admits (p. 9) that he has undertaken a "radically new evaluation of Leibniz's logic . . . ." The current essay is merely an expansion of this modernistic reconstruction as previously undertaken in Lenzen's [Lenzen 1990] *Das System der Leibnizschen Logik.*

In "Kant: From General Logic to Transcendental Logic (pp. 85–130), Mary Tiles picks up the history of logic with Kant. She sets about the daunting task of recovering Kant's reputation in the history of logic, which entails coming to terms with Kant's declarations, in the second (1787) edition of his *Critik der reinen Vernunft* [Bvii][3] and in his lecture notes on logic [Kant 1800], edited and published by Jäsche, that logic had been essentially completed and perfected by Aristotle, and that since then, only a few minor clarifications and tinkerings were required. Risto Vilkko [Vilkko 2002, 20] argues that Kant's judgment about the completeness of logic divided philosophers, such as Johann Friedrich Herbart (1776–1841), interested in formal logic, from those who were interested in transcendental logic, while at the same time diverting philosophers away from efforts to continue the development of formal logic towards the study of applications of logic in the areas of cognition.

In his early professional career, Kant was influenced by Christian [von] Wolff (1679–1754) and Wolff's efforts to axiomatically develop an architectonic covering all of philosophy, a system which up to Kant's day was extremely popular with academic philosophers, and has come down to us under the rubric of the Leibnizo-Wolffian philosophy. In defense of both Wolff and Kant, it must be said that the bulk of Leibniz's writings, and especially including the writings in logic, in which Leibniz undertook to algebraicize Aristotelian syllogistic, were largely unpublished until the mid-nineteenth century, when major systematic publications were undertaken by Johann Eduard Erdmann (1805–1892) [Leibniz 1839–40] and Carl Immanuel Gerhardt (1816–1899) [Leibniz 1849–63, Leibniz 1887] and therefore neither Wolff nor Kant could have been aware of Leibniz's efforts to reform logic. Whether Kant was aware of the work of Johann (1654–1705) and Jacob Bernoulli (1647–1748), and in particular of Jakob Bernoulli's *Parallelismus ratiocinii logici et algebraici* [Bernoulli 1685] is an open question.

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[3]Kant scholarship designates the first (1781) edition of the *Critik der reinen Vernunft* (or K.d.r.V.) as "A," the second (1787) edition as "B," using a composite text, and renders citations accordingly.

In *Critik der reinen Vernunft*, Kant [A36] defined transcendental logic in the following terms: "The science of all the principles of sensibility *a priori*, I call transcendental aesthetic. There must, then, be such a science forming the first part of the transcendental doctrine of elements, in contradistinction to that part which contains the principles of pure thought, and which is called transcendental logic." It is distinguished from pure general logic by its attention to *a priori* conditions of thought, and therein lies the later developments among Kantians who took epistemology and psychology to belong to the realm of logic. Kant expanded his characterization and delineation of transcendental logic a few paragraphs after formulating the definition. Thus we read [A54–56 = B78] that:

> General logic . . . makes abstraction of all content of cognition, that is, of all relation of cognition to its object, and regards only the logical form in the relation of cognitions to each other, that is, the form of thought in general. But as we have both pure and empirical intuitions . . . in like manner a distinction might be drawn between pure and empirical thought (of objects). In this case, there would exist a kind of logic, in which we should not make abstraction of all content of cognition; for or logic which should comprise merely the laws of pure thought (of an object), would of course exclude all those cognitions which were of empirical content. This kind of logic would also examine the origin of our cognitions of objects, so far as that origin cannot be ascribed to the objects themselves; while, on the contrary, general logic has nothing to do with the origin of our cognitions, but contemplates our representations, be they given primitively a priori in ourselves, or be they only of empirical origin, solely according to the laws which the understanding observes in employing them in the process of thought, in relation to each other. Consequently, general logic treats of the form of the understanding only, which can be applied to representations, from whatever source they may have arisen.
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> [. . .] A science of this kind, which should determine the origin, the extent, and the objective validity of such cognitions, must be called transcendental logic, because it has not, like general logic, to do with the laws of understanding and reason in relation to empirical as well

> as pure rational cognitions without distinction, but con-
> cerns itself with these only in an *a priori* relation to
> objects.

The Kantian distinction between, and emphasis upon, transcenden-
tal logic diverted the attention of those philosophers who had no in-
terest in "general" logic, and felt that the inherited Aristotelian logic
was both sufficient and complete, to concentrate on the role and inter-
dependence of formal reasoning and cognition in general, in particular
to epistemology, psychology, and methodology and philosophy of sci-
ence. As the influence of Kantian philosophy developed through the
nineteenth century and into the early twentieth century, some Kantian
philosophers began to conflate formal logic and and cognitive science.

One of the few philosophers and mathematicians who continued to
insist upon a clear distinction and separation of the theory of inference
(formal logic) and the theory of judgment (transcendental logic) was
Bernard Bolzano.

If, as Whitehead asserted, the history of philosophy is a commen-
tary upon Plato, we very well might suggest that much of nineteenth-
and early twentieth-century philosophy was a commentary upon Kant.
One could with equal justification assert that nineteenth- and early
twentieth-century philosophy was a commentary upon Kant and Hegel,
and that the great debates upon "die logische Frage" and "Was ist
Logik?" of the period, so far as philosophy of logic was concerned,
was a commentary upon the Kantian and Hegelian conceptions of
logic, and where and how Aristotelian logic fit into this scheme. With
but few exceptions, the algebra of logic, as developed by De Morgan
Boole, Peirce, and Schröder, remained the domain of mathematicians
rather than of philosophers, and the efforts of philosophers such as
Alois Riehl [Riehl 1877] to interest his German-speaking colleagues in
the "die Englische Logik" bore fruit only in terms of the question of
where and how to draw a line of demarcation between logic and math-
ematics. The same thing may be said regarding the efforts by Louis
Liard (1846–1917) to teach his French colleagues about the new alge-
bra of logic [Liard 1877a, Liard 1877b, Liard 1878]. Again: in Russia,
Fëdor Ivanovich Kozlovskii's [Kozlovskii 1882] "Symbolical Analysis of
the Forms and Processes of Thought, Structured According to Formal
Logic," in which he introduced his readers to Boole's logic through a
criticism of Jevons, bore its fruit among mathematicians rather than
philosophers. For the time being, at least, we shall have to rely, de-
spite Gratttan-Guinness's contribution to this volume of the *Handbook*,
for investigation of aspects of this episode in the history of logic, with

all of its philosophical nuances, not upon the Handbook, but upon such writings as [Peckhaus 1997], [Pulkkinen 1994, Pulkkinen 2005], and [Vilkko 2002].

In *Von der falschen Spitzfindigkeit der vier syllogistischen Figuren* [Kant 1762], Kant took the position that general logic was deficient. He argued, along the lines of Descartes and the Cartesians Arnauld and Nicole, that general logic is concerned with universals and the formal relations between propositions (pure general logic) or that between cognitions (applied general logic), and hence adds nothing to our knowledge. His transcendental logic is intended as the foundation for an *ars inveniendi*, an art of discovery, and its task is to examine the cognitive basis for judgments, according to a list of categories that he enumerates as the essential, and inherent, structure required for establishing critical determinations of the truth or falsity of judgments. It does this by associating the terms of judgment with the mental representations of the objects of judgment. The categories which Kant took were borrowed from Aristotelian syllogistics, as based on one hand upon the moods and figures of syllogisms, and on the other hand upon the types of syllogism, with respect to relation as either categorical (existent/nonexistent), hypothetical, or disjunctive, and with respect to modality whether assertoric (necessary/contingent), apodeitic (necessary/contingent), or problematic (possible/impossible). These Kantian categories, Tiles makes implicitly clear, are not those which would satisfy Frege or the Russell of the *Principia*, since propositions for Kant still retain the subject-predicate syntax. But Tiles (p. 105) explicitly argues at the same time that, in introducing his table of categories and requiring judgments to concern concepts as representations of objects, i.e., by requiring a transcendental supplement to general logic, Kant has anticipated Frege's requirement, in his [Frege 1891] "Funktion und Begriff" that object, concept, and judgment are semantically inseparable to the extent that a proposition is to have meaning. Frege [Frege 1891, 19] says that the meaning (*Bedeutung*) of the formula such as $2 \cdot 2^3 + 2$ without specific content could, e.g., be either 18 or $3 \cdot 6$. But: "Die Gleichung $2 \cdot 2^3 + 2 = 18$ wird ausgedrückt, daßdie Bedeutung der rechtsstehenden Zeichenverbindung dieselbe sie wie die linksstehenden. . . . Es liegt dieser Meinung wieder jene Verwechslung von Form und Inhalt, von Zeichen und Bezeichnetem zugrunde." Moreover (pp. 117–8), Kant's requirement that transcendental logic take account of representations of objects in a judgment, and his famous declaration [A51 = B75] that "Gedanken ohne Inhalt sind leer, Anschauungen ohne Begriffe sind blind" anticipated Frege's extensional approach in locating

the semantic interpretation of propositions of function-argument syntax in the truth-values obtained by the "falling under of a concept" of objects as extensions of the concept. As Frege wrote in "Funktion und Begriff" [Frege 1891, 30]: "Wertverläufe von Funktionen sind Gegenstände, während Funktionen selbst es nicht sind. ... Auch Begriffsumfänge sind also Gegenstände, obwohl die Begriffe selbst es nicht sind."

In the end, however, Tiles says that Frege fell into the trap against which Kant specifically warned. For Kant held that one cannot directly know the *Ding-an-sich*, the object behind the representation, but only the representation, given through perception and as mediated by apperception or the understanding. Frege required the totality of objects falling under a concept to define a completed, or saturated, function. He "treats all concepts, even those defined using quantifiers over objects and over concepts, as if they defined corresponding objects (classes), and ends, foreseeably from Kant's perspective, in contradiction" (p. 118).

To summarize Tiles's assay, we may say that Kant's value lay not so much in his own contributions to logic, but in the logico-linguistic and philosophical problems that he raised and bequeathed about the character and extent of propositions and judgments, about the demarcation between them as well as the nature of the relation between them; and finally, by his finitism, for after all, his *Critik der reinen Vernunft* was a *critique*, an effort to define the limitations and proper extent of pure reason. What matters about Kant was the influence he had, direct and indirect, upon Frege's philosophical thought about logic and the theory of meaning; and upon intuitionists and constructivists such as Poincaré and Brouwer, through his finitism. Moreover, in warning against saturated functions, ignored by Frege and Russell, he anticipated Gödel's incompleteness theorem, Tiles thinks (pp. 118, 126).

There is one serious gap in Tiles's treatment, however—and a rather surprising one. She does not discuss the importance of Kant's treatment of the analytic/synthetic and *a priori / a posteriori* distinctions in classifying propositions of logic, arithmetic, and geometry. This omission is especially evident because of the centrality of the discussion of the character of these propositions of logic and arithmetic as tautological or not, of geometrical propositions as empirical or not, in philosophical debates throughout the nineteenth century. Well into the twentieth century among members of the Vienna Circle and analytic philosophers generally, these distinctions were a matter of discussion and controversy. And, once non-Euclidean geometries came to the attention of philosophers, the issues of whether geometrical propositions

were tautological (analytic) or contingent (synthetic), empirical (*a posteriori*) or immutable and universally give and fixed in reason (*a priori*) became of supreme concern.[4]

As Tiles does with respect to Kant, so John W. Burbidge, in "Hegel's Logic" (pp. 131–175) notes (p. 131) that Hegel is not generally considered to be a "major figure in the history of logic." This is because Hegel gave but scant attention, in his [Hegel 1812–3] *Wissenschaft der Logik* and his [Hegel 1817] *Enzyclopädie der philosophischen Wissenshaften*, despite their titles, to the traditional concerns of logic: terms, propositions, and syllogisms. Indeed, he is curt and dismissive of traditional logic. Nevertheless, Hegel was a potent influence in the nineteenth- and early twentieth-century discussions of "die logische Frage" (see, e.g., [Trendelenburg 1842, Trendelenburg 1843]).

For Hegel, says Burbidge (p. 131), logic is thought thinking thought;[5] and the topics which he considers are topics that philosophers since classical times regarded as properly belonging to metaphysics, the "science of Being." Burbidge explains (p. 131) that Hegel sought, in his works, to investigate "the processes that characterize all thought."

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[4]Let us recall, for example, that some of Russell's earliest publications were part of a debate with Poincaré on the nature of geometrical propositions (e.g., [Russell 1896a, Russell 1896b, Russell 1898, Russell 1899]), and that his first published book [Russell 1897] was a Kantian treatment of geometry; that, moreover, it was his concerns over the nature of geometrical propositions and his growing dissatisfaction, in light of the development of non-Euclidean geometries, with the Kantian conception that geometric propositions were synthetic *a priori*, that contributed to the abandonment of his own neo-Hegelian idealism and his work first on the *Principles of Mathematics* [Russell 1903a] and finally to the *Principia Mathematica* [Whitehead & Russell 1910–3] (see, e.g., [Anellis 1991, Anellis 1995a], [Bonfantini 1970], and [Gross 1974]), including the fourth, never completed volume which was to have been devoted to geometry (see [Harrell 1988]).

[5]It is not surprising that Husserl adopted the name "phenomenology" for his philosophy as given in his [Husserl 1900–1] *Logische Untersuchungen*, since, in his description of his purpose, he wrote of an "eidetic reduction" in which the intent is to examine the nature of thought without reference to the contents of thought. The expression "thought thinking thought" is Husserl's. When Peirce wished to examine logic from the standpoint not of a "normative science" but from the standpoint of what we, with Husserl or Hegel would call "phenomenology," he coined the term "phranoscopy," but it amounted to a phenomenological investigation of rational thought. See [Petrick 1972] for an account of Peirce on Hegel, and [Santucci 1970] on Hegel and Peirce on the categories; see [Mullin 1966] for a comparative study of Peirce and Husserl on the nature of logic. For a view from the other side of the coin, see [Townsend 1928] on pragmatism in Hegel and Peirce.

Perhaps the most famous line in all of Hegel's writings, that the real is rational and the rational is real, equates Reality (Werden) with Reason (Vernunft).[6] We should not be surprised, then, that in distinction from their nineteenth-century Kantian colleagues, who found logic to belong to either epistemology or to psychology, the Hegelians of that day held logic to belong to metaphysics. For most of the nineteenth and into the early twentieth century, most of the debate on "die logische Frage," of "Was ist Logik?," concerned the provenance of the subject—epistemology, psychology, or metaphysics? When nineteenth-century philosophers discussed "the laws of thought" they were identifying themselves as holding a Kantian conception of logic; when they discussed the "art" or "science of reasoning" they were identifying themselves as holding a Hegelian conception.

In section 4, on the "Concept and Traditional Logic" (pp. 147–156), Burbidge undertakes an exposition of Hegel's analysis of the Aristotelian syllogism, and in particular Hegel's critique of syllogistic for its inadequacy in capturing the kaleidoscopic nature of reality, in which the pre-Socratic world of Herakleitos all is in a constant state of flux, a constant state of Becoming, of coming to be and passing away, and, like the pre-Socratic world of Parmenides, Being and Nonbeing pass into one another in a state of Becoming. This fluidity Hegel saw as a phenomenology. In this reality, the Law of Excluded Middle and

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[6]He wrote in the preface to the *Phänomenolgie* [Hegel 1807], e.g., that: "die *Verständigkeit* ein Werden, und als dies Werden ist sie die *Vernünftigkeit*."

One is here reminded of Parmenides (*ca.* 515/540–*ca.* 480 B.C.), who, as translated by John Burnet (1863–1928), wrote: "The thing that can be thought, and that for which the sake of thought exists, is the same; for you cannot find thought without something that is, as to which it is uttered" [Burnet 1930, 176, ll. 34–36]. The same words of Parmenides may put us in mind of Alexius von Meinong's (1853–1920) opening words in "Ueber Gegenstände höherer Ordnung" [Meinong 1899]: "Es gibt keine Vorstellungen ohne etwas zu vorstellen," which led him to postulate various levels of ontological states, Sosein and Außersein, along with Dasein, admitting of the Golden Mountain and Round squares—or Pegasus or the present king of France—that so exercised Bertrand Russell and led to his [Russell 1905] "On Denoting" and the multitude of logical and logico-linguistic enterprises in theory of meaning and the ontological commitment of one's syntax. On Parmenides, see, e.g., [Mackay 1924] and [Heinrichs 1974].

Husserl's phenomenological solution was, of course, the "edietic reduction," to strip away the specific contents of thought, to examine thought itself, "thought thinking thought," so as to avoid ontological commitment to anything other than the pure mental act apart from the subject-content of the mental act.

and the Law of Non-Contradiction are inadequate for a formal assay of reality.[7]

In *Die Phänomenologie des Geistes* [Hegel 1807], which Burbidge does not discuss, Hegel traced the evolution or progress of "Spirit" or Mind as it worked itself out in the world by a progression of becomings into Being, into ever more abstract Reality.

Burbidge fails to provide Hegel's famous dialectical logic as the proper tool or method for characterizing this evolution of reality, the triad of Thesis-Antithesis-Synthesis, which is the proper logic for expressing the unfolding of Reality. Simply stated: a particular moment expresses its own reality (*Zeitgeist*) in a thesis, which in turn gives rise to its own opposite, its antithesis. The collision of thesis and antithesis gives rise ("*Aufhebung*") to a new, higher and more abstract level of reality, a synthesis (or "unity of opposites") which in its turn becomes a thesis for its own engendered antithesis. The Aristotelian proposition $A = A$ expresses a static reality, whereas the Hegelian proposition $A \neq A$ expresses the dynamic process of reality as Becoming. It is unfortunate that Burbidge did not see fit to have provided at least a brief

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[7]Charles Peirce throughout his philosophical studies had been steeped in the writings of Hegel, and his pragmatism was not totally unrelated to Hegelian idealism (see, e.g., [Fairbanks 1962, Fisch 1974]). At the same time, he was an important contributor to mathematics, especially to linear and multilinear algebra, and familiar with the non-Euclidean geometries that came to the attention of the mathematicians of his own day. His own contributions to formal or mathematical logic contributed particularly to the development of algebraic logic. Yet we may suppose that it was at least in part the influence of Hegelian dialectical logic, with its rejection of the Laws of Excluded Middle and Non-Contradiction as the negations of Euclid's parallel postulate and the examples of noncommutative and nonassociative algebras, that led him to contemplate the possibility of non-Aristotelian logics, that is, of formal logics without either or both the Law of Excluded Middle and the Law of Non-Contradiction. See [Carus 1910a, Carus 1910b], and especially the footnote at [Carus 1910b, 158] quoting a letter from Peirce, on Peirce's thoughts on investigating alternative logical systems; see [Lane 1997] for an account of Peirce's toying with nonclassical logics, and especially of logics without either of both the Laws of Excluded Middle or Non-Contradiction, or with their negations. It was at least in part the influence of Peirce in raising the question of the possibility of such "non-Aristotelian" logics which led Nikolai Aleksandrovich Vasil'ev (1880–1940) to begin developing paraconsistent logics as the ancestor of modern formalizations of modal and multiple-valued logics (see [Bazhanov 1992]).

For an account of the logic of Hegel's *Phänomenologie*, see, e.g., [Heinrichs 1974]. For another account of Hegel's role in the formalization of logic, see [Lachterman 1987].

exposition of the Hegelian dialectical logic, since the triad of thesis-antithesis-synthesis is the core of Hegel's logical dynamics and suggested to logicians such as Peirce, and others, the possibility, provided in Aristotelian logic but neglected after the collapse of the scholastic tradition of the medieval philosophers, of paraconsistent logics.

Burbidge wraps up his exposition with a brief survey (pp. 168–172) on Anglo-American Hegelianism. (For a detailed account of the British neo-Hegelian logic of the nineteenth century, we shall have to wait for a future volume of the *Handbook*.) But what remains missing in Burbidge's account is a discussion of the the influences which Hegel's logic had upon the nineteenth-century discussions of the nature, scope, and provenance of logic, as much as an account Kant's influences on "die logische Frage" in the nineteenth century is missing from Tiles's exposition.

Followers of Kant and Hegel, we saw, tended to conflate logic with either epistemology, psychology, or metaphyscs. Leibniz aimed at mathematicizing Aristotelian syllogistic logic. But despite his coinvention of the calculus and his array of contributions to other areas of mathematics, he did not have the mathematical tools that were available to Boole, De Morgan, and their successors. Despite Lenzen's brave reconstruction of Leibniz's logical calculi, we must agree with Volker Peckhaus [Peckhaus 1997] that Leibniz's role for nineteenth-century researchers was primarily as an inspiration.

Until the development of algebraic logic by De Morgan, Boole, Peirce, and Schröder, the central figure contributing to both logic and mathematics in the period between Leibniz at the inception of mathematical logic and the "Booleans" and Frege at its birth was Bernard Bolzano (1781–1848). The aim of Rusnock and George, in their essay on Bolzano, is to show that he "developed his logic in conjunction with his mathematical research" (p. 177). His greatest contribution was to insist upon a separation of logic from psychology. Not until his Harvard lectures of 1865 did Charles Peirce express the idea of an unpsychological basis for logic.[8] But, as one surveys the history of the philosophy of logic and the discussions of the nineteenth and early twentieth centuries in the efforts to resolve the question of "Was ist Logik?," one perforce concludes that Bolzano had but scant influence or impact upon these discussions. In this regard, one can envision him as a man ahead of his time, regardless of any specific technical contributions he made to logic, mathematics,

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[8]Charles S. Peirce, Harvard Lecture 1, Spring 1865 (MS 94); `http://members.door.net/arisbe/menu/library/bycsp/earlymss/ms94harvard1.pdf`; printed: [Peirce 1982, 162–75].

or foundations and philosophy of mathematics. This is not to disparage his actual contributions; it is merely to set him within the context of the sentiments of his age, in particular as viewed in retrospect by historians of mathematics and logic.[9] Bolzano's influence centers about his demarcation between psychology and logic, and is detectable in the antipsychologism of Frege's logicism and in Husserl's conversion from the psychologism of his teacher Franz Brentano (1838–1917), in his own *Philosophie der Arithmetik* [Husserl 1891] and "Psychologische Studien für elementaren Logik" [Husserl 1894] to the phenomenology of his *Logische Untersuchungen* [Husserl 1900–1], following Frege's vociferous critiques of the *Arithmetik*.[10]

Rusnock and George (p. 180) note that Dedekind, Cantor, and Weierstrass were all familiar with and influenced by Bolzano's work, Cantor and Dedekind in particular with the *Paradoxien des Unendlichen* [Bolzano 1851]. They also argue (p. 180) that Brentano's students, especially Husserl, Meinong, Kazimierz Twardowski (1866–1938), and Benno Kerry (né Benno Bertram Kohn, 1858–1889), helped reestablish a serious interest in Bolzano's work, and that Husserl's antipsychologism was reenforced, if not engendered, by Bolzano's influence.[11] At

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[9]By way of example, we have the corrective analyses that were undertaken in the twentieth century to reverse the views of earlier historians of philosophy and of logic. Specifically, Walter Dubislav [Dubislav 1931] was prepared to consider Bolzano as a forerunner of mathematical logic, and [Schubring 1993] was concerned with how well Bolzano's contemporaries were acquainted with his work—which suggest that the common view was that his work was largely disregarded. That Friedrich [Frederik] Anton van Hartsen (1837–1877) provided an exposition, analysis and critical examination of logic from the standpoint of recent and contemporary writers such as Apelt, Hegel, Kant, Mill, and many others—including Bolzano— [Van Hartsen 1869], suggests that Bolzano's work merited serious consideration, as much so as his contemporaries, including even Kant, Hegel, and their followers. [Grattan-Guinness 1970–1] argues that the similarities in the work in analysis of Bolzano and Cauchy were serendipitous; yet the need to confute the charge of plagiarism of Bolzano's [Bolzano 1817] *Rein analytischer Beweis* by Augustin-Louis Cauchy (1789–1857) for his *Cours d'analyse* [Cauchy 1821] speaks to the perception that Bolzano's work was not widely disseminated and that made it possible for someone of Cauchy's wide repute to have gotten away with presenting, without detection, Bolzano's work as his own.

[10]See, e.g., [Barbosa de Oliviera 1994], [Mortan 1961], [Mohanty 1982], and [Drummond 1985] on the impact on Husserl of Frege's antipsychologism; [Kluge 1980] on Bolzano's influence on Frege. Frege's critique of Husserl's psychologism is given in [Frege 1894]; see [Hill 1994] for a discussion of Frege's attack on Husserl's psychologism.

[11]They completely ignore Frege's influences upon Husserl's dramatic shift from psychologism to the antipsychologism of Husserl's phenomenology.

the same time (p. 180), they note that Brentano rejected the interpretation which his students took of Bolzano's work, and they quote Brentano's letter to Shmuel [Samuel] Hugo Bergman (1883–1975) of 1 June 1909, in which Brentano called Meinong's and Husserl's conceptions, as they interpreted Bolzano's work, "bizzare and absurd" (p. 180, quoting [Brentano 1966, 308]). In his letter to Bergman, Brentano attributed to Robert Edler von Zimmermann (1824–1898) of the University of Vienna the misconstruals of Bolzano by Meinong and Husserl of Bolzano's belief—referring to Bolzano's *Satz-an-sich*—in ideas and propositions that "exist from eternity."[12] These Brentano called "astonishing abberations."

For Bolzano, one may quantify over classes, which are extensions. An extension is a concept having reference to objects. Thus, one distinguishes between an extension and its content. 'Pegasus' is an extension having no content; 'even prime' is an extension having one content or element; 'natural number less than n' has n-many contents; and 'natural number' has infinitely many. Logic is for Bolzano concerned with the relations between extensions as expressed in propositions. The relation is expressed in propositions having subject-predicate syntax. But, as Rusnock and George note (p. 191), this does not limit Bolzano to traditional formal logic, or syllogistic, which is equivalent to no more than the first-order monadic predicate calculus. Bolzano is saved from this limitation by his concept of *Gegenständlichkeit*, or predicate reference, that is, to the property having objects. For Bolzano, 'There are A's' is equivalent to 'The idea A has a reference', which Rusnock and George note can be rendered as '$(\exists x)\, Ax$'. Similarly, 'Some A are B' is equivalent for Bolzano to 'The idea of an A which is also a B has a reference', and this can be rendered as '$(\exists x)(Ax \cdot Bx)$'. Accepting also negation, Bolzano can, using the subject-predicate syntax, establish a relational logic. Although they do not say so, we justifiably conclude that Bolzano has carried out a crucial aspect of Leibniz's program, namely of treating syllogistic as a relational logic.

Bolzano's endeavor to create a relational logic (which is called a *variational logic*) is completed by his conception of logical consequence and equivalence. Bolzano's logical consequence, *Ableitbarkeit*, or Deducibility, is a triadic relation in which a set of propositions, M, N, O, are deducible in terms of variations i, j, k from propositions A, B, C if and only if A, B, C are *compatible* with respect to i, j, k and every substitution of ideas i', j', k' for i, j, k renders all of A, B, C true and also makes all of M, N, O true. Deducibility fails if any of A, B,

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[12]See [Winter 1975]; see also [Van Hartsen 1869].

C are not *compatible* with respect to i, j, k. Equivalence is mutual deducibility, such that M, N, O is deducible from A, B, C and A, B, C is deducible from M, N, O.

In "Husserl's Logic" (pp. 207–321), Richard Tieszen gives a close textual exposition of Husserl's writings on logic and mathematics, following the chronology of the evolution of Husserl's thought through Husserl's major publications, as well as some of the material in the Husserl *Nachlaß*.

Although Husserl studied as a mathematician, writing his doctoral thesis of 1881 on the calculus of variations,[13] his entire professional career was devoted to the investigation of the psychological and philosophical foundations of mathematics and logic, and in particular the epistemology, ontology (metaphysics), and methodology of logic and mathematics. Tieszen admits (p. 207) that Husserl neither proved any theorems nor created a formal logical system. Rather, Husserl's importance was in his demand, in the wake of Frege's criticisms of his psychologism, in his own rejection of psychologism. Apart from his major works, his literary debates with Frege and Hilbert on the nature of mathematics exerted what influence Husserl had outside of and beyond the circle of his students and the adherents of his phenomenology.[14] We may suppose, from our reading of Rusnock and George, that had Husserl undertaken to construct a formal logic, it would have been something akin to the mereology of Stanisław Leśniewski (1886–1939), a formal logic of the whole-part relation, founded upon an ontology.

There are some minor typographical errors in the reference section of Tieszen's essay: "Bar-Hillerl" should be "Bar-Hillel," and "Kung" should be "Küng."

The articles with which we have been dealing so far might more properly belong in a Handbook of Philosophy of Logic. This is not to say that philosophical issues did not play a role in the attitudes towards logic held by many of those who wrote on logic throughout the history of the subject. Nor is it to say that such attitudes to an important degree did not determine whether one chose to continue to adhere to syllogistic logic rather than "mathematical" logic, or, in the latter case, prefer algebraic logic to the newer "symbolic" calculi. Finally, there is no call to assert that the project of "reconstruction" of

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[13]*Beiträge zur Theorie der Variationsrechnung*, Ph.D. thesis, University of Vienna, 1881; advisor Leo Königsberger; according to the Mathematics Genealogy Project (`http://www.genealogy.ams.org/html/id.phtml?id=82584`). Tieszen (p. 207) gives the year of Husserl's thesis as 1883.

[14]See, e.g., [Hill 1995], [Mahnke 1923, Mahnke 1976], and [Majer 1997] on the Husserl-Hilbert debate, especially on completeness.

historical attempts to devise systems of logic in contemporary terms is without merit or has no significance for our understanding of the history of logic. It is to assert, however, that reconstruction—or interpretation, if one will—is not equivalent to exposition. Moreover, it is one-sided: reconstruction, or retrospective philosophical interpretation tends to fail to ask or answer the question: "what did he know and when did he know it?" and the related question "how did he get there from where he started?," i.e., what was the relation of the advance made by the researcher to the state of the field at the time he began working on his contributions. To redraw the problem in terms initiated by Volker Peckhaus (e.g., [Peckhaus 1989]), the history which leaves out the historical-contextual aspect of a logician's work is an incomplete history and ignores the question of how a researcher understood and used the knowledge of the field which he inherited in the process of making his own contributions. Reconstruction, on the other hand, tells us merely what import a logician's work had for our contemporary knowledge of the field and our retrospective understanding of that contribution.

To this end, a discussion of how eighteenth-century mathematicians and philosophers sought to carry out Leibniz's efforts to place syllogistic on a mathematical or at least symbolical footing would have been much more helpful than a reconstruction of Leibniz's systems or arguments for the 'relevance' of Kant and Hegel to the history of logic. And concomitantly, even the efforts in [Styazhkin 1967, Styazhkin 1969] to consider the work of the Bernoullis, of Lambert and others are superior to the absence of such an effort in the present volume. However stunted or halting the work of the eighteenth-century Leibnizians, their endeavors in logic form, I would suggest, a crucial bridge between Leibniz's work and the work of Boole and his cohorts. Consider, for example:

- the recognition by the Bernoullis, and in particular of Jakob Bernoulli, of the *parallelismus ratiocini logici et algebraici* [Bernoulli 1685]
- Gottfried Ploucquet (1716–1790), who developed a logical symbolism and a combinatorial logical calculus for the syllogism (see, e.g., [Ploucquet 1763]) and was influenced by Lambert
- Georg Jonathan von Holland (1742–1784), who carried on Ploucquet's work, employing Ploucquet's logical system and symbolism for use in mathematics (in [Von Holland 1764])
- Johann Heinrich Lambert (1728–1777), who devised a notation sufficient for formalizing syllogistics with four figures (see, e.g., [Lambert 1764–5])

- Georg Joachim Darjes (1714–1791), a student of Leibnizian systematizer Christian Wolff (1679–1754), and author of *Introductio in artem inveniendi seu logicam theoretico-practicam* [Darjes 1742] and *Via ad veritatem commoda auditoribus methodo demonstrata* [Darjes 1755], who introduced logical symbolism for syllogisms, e.g., $+S-P$ is a universal negative proposition in which the subject S is affirmed and the predicate P denied
- Francesco [Friedrich Adolf; Frédéric Adolphe Maximilien Gustav] de Castillon (1747–1814), a student of J. H. Lambert, who formalized syllogistic by using arithmetical operators to represent syllogistic relations for categorical propositions and devised an intensional formalization of syllogistic; read his paper on the new logical algorithm in [Castillon 1803, 16]: "Notre algorithme semble également le prouver par sa simple inspection: $S = A + M$, $S = -A + M$ indique tout uniment, que penser S, c'est penser M avec ou sans A."
- Joseph-Diez Gergonne (1771–1859), who introduced symbols for logical relations such as includes and included in [Gergonne 1816–17, Gergonne 1818]

from among the more conspicuous examples. We should likewise mention in this connection Leibniz's contemporary, Arnold Geulincx (1625–1669), who advocated mathematicizing logic and the use of deductive proofs [Geulincx 1662].

These early efforts admittedly proved to be incomplete and abortive.[15] The algebraicization of syllogistic logic had to await the development of a more sophisticated and supple algebra than was available in the eighteenth century, and it is therefore unsurprising that Boole, who with his colleagues developed the "symbolical" algebra in early years of the nineteenth century, would be among the leaders, along with De Morgan, thereafter in the development of algebraic logic, along with their

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[15]Contemporary efforts to arithmeticize or algebraicize Aristotelian syllogistic still persist, most notably and successfully by two of my former teachers, Edward A. Hacker (see [Hacker 1967], [Hacker & Parry 1967], and [Parry & Hacker 1991]) and Frederic Tamler Sommers (see, e.g., [Sommers 1970, Sommers 2000]). On Parry, see [Anellis 2005a]. A considerable amount of attention has been given to Sommers's arithmeticization of syllogistic, his calculus of terms; see especially [Engelbretsen 1981, Engelbretsen 1987]; see also [Anellis 2005b] on Sommers and his work.

N.B.: The Library of Congress incorrectly lists William Thomas Parry (b. 1917) as coauthor with Hacker of [Parry & Hacker 1991], confusing him with William Tuthill Parry (1908–1988).

German contemporaries who were taking a combinatorial approach to treat logic as an "algorithmistic" calculus. It is, however, a legitimate, important, and even essential, historical question to inquire what influence, if any, and to what extent, the work of the eighteenth- and early nineteenth-century Leibnizian logical symbolizers, from the Bernoulli brothers to Gergonne, as well of course of Leibniz himself, may have had on the work of Boole, De Morgan, and their contemporary German combinatorial logicians. Even if only to the extent that it set a goal for the algebraic and combinatorialist logicians in their work, even if only to keep alive the knowledge of and inspiration of the "Leibniz program."

Theodore Hailperin (p. 323) defines "algebraic logic" as "a *style* of doing logic, a style in which concepts and relations are expressed by mathematical symbols." Particularly those of algebra, rather than as a *kind* of logic."[16]

When we reach Hailperin's account of "Algebraical Logic 1685–1900" (pp. 323–88), we arrive at historical exposition rather than reconstruction. Moreover, Hailperin fills in some of the gaps that we have noted, for although much of his section on "Early Efforts" (pp. 323–42) is concerned with the work of Leibniz, it begins with an account of the parallelism between logic and algebra that the Bernoullis (see [Bernoulli 1685]) detected,[17] and considerable attention is also given to Lambert's work as Leibniz's successor, along with a recognition, if not a detailed account, of the work of Ploucquet and von Holland as these impacted on Lambert's work. Thus, even when Hailperin translates the algebraicization of syllogistic by the Bernoullis, by Leibniz, and by Lambert into modern notation, we are not dealing with an hypothetical reconstruction, but witnessing the actual struggles of these logicians in developing an adequate notation.

To be more specific: in dealing with Leibniz and others, Hailperin offers not only careful exposition, but an analysis of the work being

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[16]Likewise, Hailperin [Hailperin 1981] and others, e.g., [Green 1994], have been keen to distinguish Boolean algebra from the algebraic logic developed by Boole.

The question of whether the results of the Leibnizo-Boolean efforts to algebraicize syllogistic logic is or is not the same thing as algebraic logic as it has come down to us from Peirce and Schröder through Tarski and his students is, I suggest, a related, but separate, issue.

[17]Hailperin correctly lists Jakob Bernoulli as the sole author of the *Parallelismus ratiocinii logici et algebraici* [Bernoulli 1685], although the results represent their joint work, and is typically credited to both Jakob and Johann. The *Parallelismus . . .* was first published as a pamphlet. In his bibliography for his essay, Hailperin gives the year for the *Parallelismus ratiocinii logici et algebraic* as 1684, whereas in the text he gives it as 1685.

considered, and translating, in the meanwhile, that work into more familiar, modern, notation. The change of symbolism, however, does not carry over into a fanciful reconstruction of the logician's work into a contemporary reinterpretation of that work. Instead, Hailperin suggests how that work was seen or used by later logicians. Nor does analysis lead in Hailperin to modernization in the sense of "whiggish" reinterpretation. Rather, it is an evaluation; thus, for example, Hailperin in setting out Leibniz's treatment of the syllogisms notes (p. 331) that in Leibniz's proof of *Cesare* there exists what to the eyes of a modern logician would appear to be a "gap," but which, to Leibniz himself did not appear to be such; he also points out (p. 327) "an apparent slip-of-the-tongue" of Leibniz, on which was left uncorrected by Louis Couturat in his production of Leibniz's work [Couturat 1903].

It is fair to insist that one of the reasons that Leibniz, Lambert, and others prior to De Morgan and Boole failed in their efforts to do logic in algebraical "style" is that it took the work of Boole and his contemporaries and associates of continental Europe and the British "analytical school" to develop a symbolical algebra capable to the flexibility to render the project feasible.[18] Therefore it is not surprising that a sketch of some of this work is included in Hailperin's essay (pp. 344–5). The focus here is in particular on the development of symbolical algebra and operator algebra by Duncan Farquharson Gregory (1813–1844), specifically Gregory's [Gregory 1844] "On the Real Nature of Symbolic Algebra," and George Peacock (1791–1858), in particular his [Peacock 1845] *Symbolical Algebra*. For Peacock, and his colleagues, among whom were George Boole, symbolical algebra was the science which treats the combinations of arbitrary signs and symbols by means defined through arbitrary laws, to provide a logical presentation of algebra, and to be distinguished from numerical algebra. They argued that one can assume any laws for combination and use of symbols, provided the assumptions are independent of one another and therefore not inconsistent with each other. It is also important to note that a significant reason that Leibniz's work in logic had little direct impact on the work of the handful of mathematicians and logicians beyond the "Leibnizo-Wolffian" school centered in Germany, was that so much of

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[18]On the development of "symbolical" algebra and its role in the development of algebraic logic, especially from the perspective of British work in the field, see, e.g., [Pycior 1981, Pycior 1983] and [Richards 1980, Richards 1987]. See also, e.g., [Anellis & Houser 1991], [Bashmakova & Rudakov 1995], [Becher 1980], [Clock 1964], [Dubbey 1977], [Koppelman 1971], [Maddux 1991], [Merzbach 1964], [Murawski 1990], [Nový 1968a, Nový 1968b, Nový 1968c, Nový 1973], and [Parshall 1985–6].

this work remained unpublished until around the middle of the nineteenth century (see [Leibniz 1839–40, Leibniz 1849–63, Leibniz 1887]). Thus, for example, we know that it was not until about a year after publication by Boole of his *Laws of Thought* that Robert Leslie Ellis (1817–1859) informed him that Leibniz had anticipated some of his notations.[19] Giving some due to the "whiggish" historiographers, we might remark as did Louis Couturat [Couturat 1901, 354], and noticed by Bertrand Russell [Russell 1903a], [Russell 1994, 548],[20] that if Leibniz "had less respect for scholastic logic, . . . the Algebra of Logic would have been constituted some 200 years sooner." But the point is at least worth having in the back of one's mind when reading Hailperin's exposition. Hailperin is too careful, however, to merely whiggishly speculate, but simply factually states (p. 337) that his direct contributions to logic were nil, his manuscripts "unknown until too late to have any influence," whereas his project of devising a *calculus ratiocinator* as a *characteristica universalis* was, through his programmatic publications and letters, a spur to others.

Following upon the sketch of the work of Leibniz and his direct successors, but before examining the development of algebraic logic made possible by the development of the "symbolical algebra," Hailperin turns to an account of the "Revival of Formal Logic in England" (pp. 343–4), noting the work in particular of Henry Aldrich (1648–1710) and Richard Whately (1787–1863).

Aldrich's [Aldrich 1691] *Artis Logicae Compendium* had a long and flourishing career, well into the nineteenth century, its last edition appearing in 1850; his related [Aldrich 1650] *Artis Logica Rudimenta* being published as late as 1862 (see [Aldrich 1820]). As Hailperin noted (p. 343), Aldrich's *Logic* was a prime exemplar of the worst of neoscholastic logic, a pastiche of psychology, epistemology, and rhetoric, along with a treatment—in Aldrich's case comparatively original—of the valid and invalid syllogisms. But it was in any case a defense of Aristotelian logic, and to that extent therefore a defense of formal logic.

Hailperin does not explicitly take up the oft-contentious issue in this study of *who* deserves the credit for the phenomenon which his section heading names, of revitalizing the study of formal logic in England.[21] But he credits Richard Whately's [Whately 1826] *Elements of Logic*,

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[19]As noted in a footnote to the introduction to the second edition of Jevons's *Principles of Science* [Jevons 1877].

[20]In a comparative double review of [Cassirer 1902] and [Couturat 1903] on Leibniz.

[21]E.g.: [Kneale 1948] delivered the honor to Boole; [Jongsma 1983] credited Whately; [Panteki 1993] argues for Thomas Solly (1816–1875) as the "unknown

which, he says (p. 343), "marked an abrupt change," namely to the view that logic is not, as it was for Aldrich, an *art* of reasoning (or for empiricists such as John Stuart Mill—as found in [Mill 1843]—an ars invienendi), but an "abstract *science* relating to linguistic structure."

Quoting the revised second American edition (p. 35, 1852) of Whately's [Whately 1826] *Elements*, Hailperin takes Whately's treatment of the syllogism 'Every X is Y; Z is X; therefore Z is Y' to show that Whately had an extensional, rather than an intensional, conception of the syllogism as dealing with classes, although he notes that the use made of a symbol for class inclusion, in which the cited syllogism is formulated as 'If $X \subset Y$, and $Y \subset Z$, then $X \subset Z$', does not occur until much later. He does remark, in a footnote (p. 344, n. 9), that De Morgan, in his *Formal Logic* [De Morgan 1847, 234–5], argued in favor of the superiority of extensional to intensional treatments.

By the early 1870s, Alexander John Ellis (1814–1890) was able to go much further than did the Bernoullis in speaking of a parallelism between algebra and logic. He was able, thanks to the work of British symbolical alagebraists and those who, following upon the work of Boole, De Morgan, and other algebraic logician, to unequivocaly to assert the claim [Ellis 1873] that there were "algebraical analogues of logical relations." The remainder of Hailperin's essay, without explicitly citing Ellis in this regard, traces this development, beginning with De Morgan as the "last of the traditional logicians" and winding up with mention of Schröder's [Schröder 1890–1905] *Vorlesungen über die Alebra der Logik* and Whitehead's [Whitehead 1898] *Treatise of Universal Algebra* as the last of the works in algebraic logic.[22] For the remainder of Hailperin's essay, then, and indeed for the remaining essays in this *Handbook*, it will generally suffice to provide an outline of the material herein presented, as much will be already familiar to historians of logic. Thus, for example, Hailperin's essay can in essence be considered as a much-expanded and far more detailed account of the history of algbraic logic from the 1840s to 1900 than is presented in Houser's [Houser 1994a] survey.

Much of the account of De Morgan's early work (pp. 346–9) is concerned with quantification of the propositions of the syllogism, as well

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pioneer" in the mathematicization of logic in England, referring to his [Solly 1839] *Syllabus of Logic.*

[22]See, e.g., [Nový 1976] for a more favorable retrospective estimation of Whitehead's work.

as the priority dispute between De Morgan and William Stirling Hamilton (1788–1856) over the origin of quantification over predicates.[23] This is followed by an exposition of Boole's [Boole 1847] *Mathematical Analysis of Logic* (pp. 349–54) and next of Boole's pivotal [Boole 1854] *Laws of Thought* (pp. 354–64). Hailperin then turns to the origin of the logic of relations (pp. 361–6), citing (p. 361) De Morgan and Charles Peirce as the originators. Peirce's 1870 American Academy of Arts and Sciences lecture "Description of a Notation for the Logic of Relatives, ..." (see [Peirce 1870]) is introduced (p. 363) as the work in which Peirce "introduces into the study which De Morgan initiated, a format with algebraic symbols extending that which Boole had impressed into service for class terms." One of the more significant differences between De Morgan and Peirce which Hailperin notes (pp. 364–5) is that whereas De Morgan restricted his operations to well-defined universes of discourse, Peirce permitted, and indeed had no hesitations in letting his operations range over both the universe [universal domain] and the empty domain. This leads him (p. 364*f.*) to question, Peirce's conviction notwithstanding, whether multiplication in Peirce's algebra of relatives is precisely De Morgan's composition of relations. [Merrill 1990, n. 16, p. 361] is cited as the source for an amplified account of De Morgan's work.

In considering the simplification of Boole's algebra, Hailperin turns first to Jevons (pp. 367–78), in which the question of whether logical addition should be interpreted as inclusive or exclusive disjunction becomes a major issue, and Jevons's choice of the former is associated with his preference for an intensional over an extensional understanding of class terms.[24] A few short pages are given over to Robert Grassmann

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[23]See, e.g., [Hamilton 1847]. So many of the British logicians of the era (and historians of our own day) became embroiled in the dispute and charges of plagiarism that it would be unedifying to provide a list of those involved and the charges and countercharges. A survey of the essence of the dispute, and its influence upon Boole, should suffice; therefore, see, e.g., [Laita 1979].

[24]As Hailperin notes, discussions of the preference for inclusive and exclusive disjunction date back to Chrysippus of Soli (279 or 281–206 or 208 B.C.) and was a bone of contention between Stoic and Aristotelian logicians and among the Stoics themselves (see, e.g., [Mates 1961, 51–5]). Claudius Galen of Pergamon (129 A.D.– *ca.* 199 A.D.) was among the first, if not the first, to find the distinction between inclusive and exclusive disjunction troublesome. The use of inclusive disjunction in place of exclusive disjunction was among the most salient and substantial alterations made by Jevons to the logical system developed by Boole, while John Venn, who initially adopted Boole's exclusive disjunction for the first edition of his *Symbolic Logic* [Venn 1881], but came to accept Jevons's inclusive disjunction in the second, revised edition of that work [Venn 1894], partially because of the strong influence

(pp. 370–1) and Schröder (pp. 371–2), culminating with Schröder's axioms for the class calculus.

The next major section treats propositional logic, with attention to Boole (pp. 373–5), MacColl (pp. 375–8), Frege (pp. 378–9), and Peirce (pp. 379–81). Much of the final section, titled "Odds and Ends," is concerned with truth-functional logic, with attention to Peirce and Schrüoder, and with contrasting the truth-value ideas of Peirce and Gottlob Frege (1848–1925). It is admitted (p. 378) that inclusion of Frege within the context of a discussion of algebraic logic is somewhat anomalous. But Hailperin justifies the intrusion on the ground of the importance of Frege's work for the history of mathematical logic in general. Moreover, Frege's work provides Hailperin with a semantic foil to the syntactic approach of the algebraic logicians.[25]

What is glaringly absent from Hailperin's extensive survey is an account of the introduction by Peirce and his student Oscar Howard Mitchell (1851–1889) of a quantificational theory within the algebra of relatives, and its systematic elaboration by Schröder. This may be a reflection of the prejudice of the generation of logicians weaned on Whitehead and Russell's *Principia* and indoctrinated into the Frege-Russellian conception that algebraic logic is deficient and incomplete, being a mere calculus and not a language; devoid of a quantification theory; and the logicist philosophy that requires logic to be the foundation for mathematics, rather than as a simulacrum of algebra. In any case, Hailperin promises (p. 378) that the primary contribution of Frege to logic, quantification theory, will be dealt with in another essay of the present *Handbook*. He argues, in nearly the same breath (p. 378) that, unlike MacColl and Peirce, Frege strongly and carefully distinguished between inference and the conditional. He ends on the note that with Schröder's *Vorlesungen* and Whitehead's *Treatise* the algebraic style of logic came to an end. This he attributes directly to the "fact" (p. 386) that: "With the development of quantifier logic the inadequacies of the 'algebra of logic' as a foundation for logic became apparent," and retains a value today only for the specialized field of algebraic

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Jevons's modifications to Boole's system and and the long-lived popularity of his system and his textbooks, partly because Venn came to recognize the advantages of that notation; Venn explained his change of mind at [Venn 1894, 46]. John Stuart Mill was among those who sided with Jevons in this. Hailperin does not, however, attend to the dispute as it was carried on in the literature of late nineteenth-century English logic.

[25]This theme was expressed in [Van Heijenoort 1967a] and more systematically expanded, from a philosophical perspective, in [Hintikka 1997].

logic and its applications. By contrast, historians such as van Heijenoort (see especially [Van Heijenoort 1967a, Van Heijenoort 1986]), Hintikka [Hintikka 1997], and Sluga [Sluga 1987], and others of the Frege-Russellian historiographic school would attribute the ultimate rejection of the "Boolean" tradition in favor of the logicism of Frege and Russell in terms of the failure of the former to provide a full *lingua rationalis*, but merely a *calculus ratiocinator*, so that unlike the latter, it failed to develop a *characteristica universalis*; or in other, older terminology, algebraic logic was merely a *logica utens*, whereas the logistic of Frege and Russell was a *logica magna*.

This blindness to the development within what Peirce called the "algebra of relatives" and Schröder's algebra of logic of quantification theory cannot be historically sustained. There are numerous accounts of the full development of a first-order theory, and of at least an incipient second-order theory in the work of Peirce, beginning as early as 1867 in "On an Improvement in Boole's Calculus of Logic" [Peirce 1867], in which Peirce first undertook to make the distinction, generally lacking in neo-Aristotelian theories of syllogistic, between individuals or singular popositions, and universal propositions, and finding fulfillment, utilizing the improved notation of Mitchell [Mitchell 1883], for first-order theory in Peirce's [Peirce 1883a] "The Logic of Relatives" and a first-order theory, together with an at least rudimentary second-order theory in Peirce's [Peirce 1885] "On the Algebra of Logic: A Contribution to the Philosophy of Notation." The historical literature well attests to this development.[26] A full exposition and amalgamation can be found in Schröder's *Vorlesungen.* Moreover, it is precisely the notation and presentation of quantification as found in Schröder that Löwenheim and then Skolem employed in their formulation of the Löwenheim-Skolem Theorem.[27] The most that one could say is that, whereas it was admittedly Frege in his [Frege 1879] *Begriffsschrift* who developed a fully articulated quantification theory, it was the quantification theory of Peirce and Mitchell, especially as found in Peirce's [Peirce 1883a], that proved influential into the earliest years of the twentieth century, and which proved to influence the course of the subsequent development of logic at least through the period when the Löwenheim-Skolem Theorem was of pivotal influence.

In many respects, Victor Sánchez Valencia's essay on "The Algebra of Logic" (pp. 389–544) is a recapituation of the topics dealt with by

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[26]See, e.g., [Beatty 1969, Berry 1952, Brady 1997, Byrnes 1998, Martin 1976, Merrill 1997], to cite but a few; see e.g., [Thiel 1988] on Schröder's contribution.

[27]See [Brady 2000] and [Anellis 2004].

Hailperin. But Valencia offers a closer reading of those same sources, and delves into more detail on many of the theses and topics which in Hailperin were merely highlighted. In particular, Sánchez Valencia examines the work in continental analysis, centered on the work of Lagrange, that proved crucial to Boole and fellow members of the British Analytical Society that led to Boole's work in logic. For example, the work of the members of the Analytical Society, and of Boole in particular, in the calculus of operations and the algebra of functions as part of the development of the symbolical algebra, is noted and sketched (pp. 393–402). Sánchez Valencia also goes into some depth on the work of Whately, and of Gerge Bentham, Thomas Solly, and William S. Hamilton on quantification over predicates (pp. 402–13).

Besides dealing with the contributions of Boole to propositional logic and truth-functional logic, Sánchez Valencia notes Boole's contributions to the monadic predicate calculus as a result and extension of his work on the algebra of functions (pp. 413–52).

In turning to Jevons (pp. 452–61), the choice between extensionality and intentionality is considered, and this is followed by a sketch of Jevons's treatment of the mechanization of logical operations, both the logical abacus, which is generally neglected in accounts of the mechanization of reasoning and graphical methods, and of the logical piano.

Sánchez Valencia next turns (pp. 462–77) to Peirce and his work in monadic predicate logic and moves on to detail the chronological development of Peirce's work, beginning in 1867, through the crucial work of 1870 when Peirce abandoned identity as the fundamental logical relation and sets inclusion in its place. Peirce's "claw" ($\prec$) is understood as implication, but its wider significance is as a "quantified copula". Further development in Peirce is considered as Sánchez Valencia contrasts Peirce's [Peirce 1870] "Description of a Notation for the Logic of Relatives ...," and in particular the negated implication ($\overline{\prec}$) and remarks on Cayley and MacColl as they interpreted Peirce's negated implication, in comparing the algebraic view, as developed by Peirce throught the 1880s with Frege on negative copulas. The section on Peirce carries the development through the early 1890s and continues to explore interpretations of the copula and ways in which those interpretations impacted Peirce's handling of terms, open formulae, and bound variables of quantifiers.

A section on the chronological development of Schröder's logic of absolute terms (pp. 477–87) considers primarily *Der Operationskreis* [Schröder 1877] and the *Vorlesungen über die Algebra der Logik* [Schröder 1890–1905]. Sánchez Valencia deems the discovery of dulaity to be the central feature of Schröder's work in the *Operationskreis.*

Schröder's logical universe consists of two terms, 0 and 1. No other terms have logical value. The contributions of the *Vorlesungen* are manifold [pun intended]. We are presented with a calculus of domains (Gebiete), subsets of a given manifold (Mannigfaltigkeit) and Subsumption ($\in$), the latter a binary relation which is reflexive, transitive, and asymmetric. Revising the concept from the *Operationskreis* that only 0 and 1 are logically meaningful terms, in the *Vorlesungen* they become privileged, but not exclusive: $0 \, e \, \alpha$ and $\alpha \, e \, 1$, for all domains α. Given product $\alpha\beta$ and sum $\alpha + \beta$, defined by the partial order of subsumption, $\alpha\beta$ as the g.l.b. of α and β, $\alpha + \beta$ as the l.u.b., Sánchez Valencia concludes that Schröder's theory of domains is a lattice theory, in all but name (p. 483).

The central focus in Sánchez Valencia's discussion of the *Vorlesungen* is on the class calculus of Schröder and his hierarchy of classes, including the universal class. But first and foremost is an account of Schröder's proof of distributivity. Schröder argued that distributivity does not hold for all domains. Peirce originally held the opposite view, and developed a proof, which he never published, although many years later he did provide a sketch for Huntington in a letter, that all lattices are distributive, and which Huntington reproduced in a footnote to his [Huntington 1904] "Sets of Independent Postulates for the Algebra of Logic."[28] For Sánchez Valencia (p. 438), the issue is not whether Peirce or Schröder was correct, but whether Schröder was "justified in asserting that Peirce thought distribution to be demonstrable from what amounts to his own theory of domains." Sánchez Valencia (pp. 483–4) thinks that Peirce's change of mind was precipitated by considering Schröder's domains to be precisely like his own as he originally intended them in Peirce's [Peirce 1880] "On the Algebra of Logic." But therein lies the problem. No clear consideration was, evidently, given, initially by either Peirce or Schröder, to whether their domains were the same; more importantly, Peirce's "claw" is not Schröder's Subsumption.

The sections on the logic of relations (pp. 487–538) begin with an account of the work of De Morgan with respect to treating categorical syllogisms as inferential relations, and traces the evolution of De Morgan's use and understanding of the copula, when he began noting in the third, 1836, edition of *On the Study and Difficulties of Mathematics* [De Morgan 1836, 203], two distinct uses of the copula: as equality; and as a "signal" of predicability. In 1847, in *Formal Logic*

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[28]See especially [Houser 1991, esp. pp. 21–2 and nn. 59–62, pp. 29–31], Huntington's reproduction of Peirce's proof is to the footnote of Theorem 22 on p. 330 of [Huntington 1904].

[De Morgan 1847], he added a third use, which Sánchez Valencia calls "modal." In examining the common characteristics of those three uses, De Morgan, in Sánchez Valencia's rendering (p. 503), are:

(1) $\alpha \mathbin{C} \beta = \beta \mathbin{C} \alpha$
(2) If $\alpha \mathbin{C} \beta$ and $\beta \mathbin{C} \gamma$, then $\alpha \mathbin{C} \gamma$
(3) For any α and β, either $\alpha \mathbin{C} \beta$ or α not-C β

Thus: "[a]ny predicate that fulfills these conditions makes, in De Morgan's view, 'all the rules of logic true.'"

From these and further considerations over the course of his research, De Morgan's traditionalist approach evolved into a full-blown logic of relations. In [De Morgan 1851] the copula can be both transitive and symmetric, and treated as a binary relation. It is then the turn of Peirce (pp. 515–31) to work through and finesse De Morgan's logic of relations by devising improved quantification rules to develop monadic predicate relations, and then of Schröder (pp. 531–8), to take the theory of relations founded by De Morgan and Peirce, and to "systematize Peirce's theory of dual relatives as 'the algebra and logic of binary relations'" (p. 531).

The glaring typographical errors in Sánchez Valencia's essay are the constant misspelling of "Operationskreis" in the discussion of Schröder as "Operationkreis"; the backwards rendering of Schröder's Subsumption symbol; and in the references for Sánchez Valencia's essay, it is "Baggage" for Babbage.

There is little to add to what has already been said here about Grattan-Guinness on "The Mathematical Turns in Logic" (pp. 545–56): it explores the question of the relationship and differences between algebraic logic and mathematical logic, and of their respective historical significance; and it reiterates the point, frequently made by Grattan-Guinness, that a sharp distinction must be drawn between algebraic logic, whose roots are in algebra, and "mathematical logic," whose roots are in analysis and is based upon a function-theoretical syntax.

The two essays by Volker Peckhaus, "Schröder's Logic" (pp. 557–609), and Risto Hilpinen, "Peirce's Logic" (pp. 611–658), provide details of the work of Schröder and Peirce respectively which were inadequately, only sketchily touched upon, if at all, in previous essays of the present *Handbook*.

Peckhaus, an acknowledged expert on Schröder and his work, provides one of the most detailed modern biographies on Schröder thus far produced (pp. 559–64), working with material not available to Randall Dipert for his important [Dipert 1990–1] biography. More importantly,

Peckhaus examines in detail the intellectual influences upon Schröder's work, in particular his background in the theory of equations. This includes the combinatorial analysis of Martin Ohm (1792–1872), for whom logic is a mathematical system for combinatorial manipulation of concepts, and especially Ohm's [Ohm 1822] *Versuch eines vollkommen consequenten Systems der Mathematik*. It includes the algebraic and combinatorial analysis of Ohm's intellectual predecessor, Carl Friedrich Hindenburg (1741–1808). It includes the *Lineale Ausdehnungslehre* [Grassmann 1844] of Hermann Günther Grassmann. It includes Robert Grassmann's efforts to develop his brother's work in a systematized reorganization of science [Grassmann 1872a, Grassmann 1872b, Grassmann 1872c, Grassmann 1872d, Grassmann 1872e, Grassmann 1872f, Grassmann 1875, Grassmann 1890], and in particular his work in applying Hermann's analysis to logic, arithmetic, and number theory. And it includes the *Theorie der complexen Zahlensystem* [Hankel 1867] of Hermann Hankel (1839–1873).[29] It should first be noted that Hermann Grassmann belonged to the combinatorial school. It should likewise be noted that the primary, if not the sole, justification for having the essay on Schröder appear before that on Peirce is that Schrüoder did not become aware of Peirce's work until after he had already begun his own effort, from within the context of the combinatorial school of Hindenburg-Ohm-Grassmann, to develop his own "absolute Algebra," and that the pedigree of his research stretched slightly farther back into mathematical history than did Peirce, who took his initial cues from the British symbolical algebraists, and Boole and De Morgan. It was only after he had already begun his work and had begun formulating and working out his resarch project in logic that the importance and influence of Peirce's work became crucial for Schröder.[30]

Peckhaus's essay concentrates on Schröder's concept of a domain (*Gebiet*), the quantification of equations of the algebra of logic, especially the algebra of relatives, and the role of axiomatics, as well in particular of pasigraphy, or, in Leibnizian terms, a general script or *characteristica universalis*. What is missing—from a Boolean perspective—to render it a full *lingua rationalis* is a *calculus ratiocinator*. The latter is supplied by Schröder's algebra. But from

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[29]On the importance for the work of the Grassmanns on Schröder's development of logic, see especially [Peckhaus 1996] and [Grattan-Guinness 1996]. See [Heath 1917] for a view of the connection between Hermann Grassmann's *Ausdehnungslehre* or calculus of extension and Leibniz's *characteristica universalis*.

[30]See [Houser 1990–1] on relations between Peirce and Schröder, as based upon their correspondence.

the standpoint of Frege (and Peano), as indeed for most historians of logic who adopt the historiography of the Russello-Fregeans, the separation of the *characteristica* and the *calculus* in Schröder's algebra (and those of the algebraic logicians generally) cause the Algebra der Logik to be a mere *calculus* and not a *lingua*. This was the gist of Frege's [Frege 1880–1, Frege 1882, Frege 1883, Frege 1895] critique of the "Booleans" in general and of Schröder in particular, and indeed of the debates between Frege [Frege 1895] and Schröder [Schröder 1880], as well as of the discussion of pasigraphy in Schröder's [Schröder 1898a, Schröder 1898b] account of Peano's approach as compared with his and Peirce's, rendered an indelible part of the historiography of logic by [Van Heijenoort 1967a, Van Heijenoort 1986], and by Hintikka [Hintikka 1997], who expanded into the the main philosophical theme of twentieth-century analytic philosophy.[31]

The comparatively unique viewpoint of Hilipinen is stated at the very outset. It is a rejection of the view, vigorously defended by Grattan-Guinness, that algebraic logic is distinct from, and had no role in, the development of "mathematical" logic, where by the latter is meant the logistic quantification theory of Frege (and Russell, and, to at least some extent Peano). It is likewise a refutation of the dichotomy, endorsed by van Heijenoort and, following him Hintikka, of the notion that only in the union of logic as calculus and logic as language can a true *characteristica universalis*, and consequently logic as a formal concept-script (Begriffsschrift), become the foundation and apparatus for rigorous mathematics. This dichotomization is owing to Frege, and to Frege's critique of Schröder in particular and of the Boolean conception in particular (pp. 611–2). Those who, like van Heijenoort, accept the Fregean analysis and critique, that algebraic logic provides only a syntax but not a semantic, for a formal language, deny the status of *mathematical* logic to algebraic logic, and consequently to the entire development from Boole and De Morgan to Peirce and Schröder. One may even go so far as to apply to the decline of the status of algebraic logic in the eyes of the "logisticians" or Russello-Fregeans the explanation given for the decline of influence, at the same time to Peano's school. As [Borga & Palladino 1992, 40] wrote, that, believing that they had fulfilled their goal of fully developing logic as the instrument to "reach the highest possible rigor in mathematics," they lost interest in continuing the study of new developments in mathematics,

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[31]See also, e.g., [Sluga 1987] and [Ferriani 1984] on the dispute between the "Booleans" and Frege on calculus versus language. See [Peckhaus 1990–1] on pasigraphy in Peirce and Peano.

and that their work, while laying the basis for future research in logic, sloughed off interest in further developments, leaving their work—in the Peano school the *Formulaire de mathématiques*, in the algebraic school Schröder's *Vorlesungen über die Algebra der Logik* and Whitehead's *Treatise of Universal Algebra*—as the "synthesis of and capstone to the contributions of the nineteenth century." The road was thus left open for Hilbert and especially Russell to take the field abandoned by the older generation. In the case of Peirce, the wound was self-inflicted and apparently deliberate. Pressed by others to appraise the work of Russell, especially Russell's [Russell 1903a] *Principles of Mathematics*, and even the first volume of the *Principia* [Whitehead & Russell 1910–3], and to compare it with his own, he allowed the opportunity to slip.[32]

Christian Thiel [Thiel 1987] was one of the first historians of logic to challenge the Frege-Russell view of a dichotomy between algebraic logic and "mathematical" logic. [Anellis 1995b] pointed out that the dichotomy, which the Frege-Russell historirgraphy also argued, was based upon the assumption that algebraic logic was devoid of a quantification theory. The historiography which asserted a dichotomy between algebraic logic and "mathematical" logic on the grounds that the latter provided a quantification theory while the former did not, can not, however, be sustained or justified historically. For, although Frege, starting his effort toward that goal later than Peirce, was, in the *Begriffsschrift* [Frege 1879], four years ahead of Peirce in "The Logic of Relatives" [Peirce 1883a] in successfully setting forth an adequate first-order theory, Peirce and Schröder did nevertheless provide a quantification theory for their algebra of logic. Thus, [Peckhaus 2004, 3], citing [Anellis 1995b, 272], agrees that the distinction between algebraic logic and "mathematical" logic or logistic is "artificial." The difference between the algebraic logic and the logistic is one of syntax, and does not depend upon the absence of a quantification theory in the former and the presence of a quantification theory in the latter. Van Heijenoort's (and Grattan-Guinness's) distinction between algebraic logic

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[32]See [Anellis 2004–5, 80–1]. Among those encouraging Peirce to publish a critique and comparison of his work with Russell's were Peirce's student Christine Ladd-Franklin [Ladd-Franklin 1904] and Eliakim Hastings Moore (1862–1932) [Moore 1902, Moore 1903], the latter in his capacity as editor of the *Transactions of the American Mathematical Society*. Peirce was far less reticent to criticize Russell (and Whitehead) in private, writing, for example, to Ladd-Franklin [Peirce 1904] that "...a year has past since I agreed to notice Russell's vol. 1"—i.e., Russell's [Russell 1903a] *Principles of Mathematics*—"and I feel its pretensiousness so strongly that I cannot fail to express it in a notice," and writing to Frederick William Frankland (1854–1916) that in his opinion, "Russell and Whitehead are blunderers constantly confusing different questions" [Peirce 1906].

and quantification theory is best and most accurately replaced with a distinction between algebraic logic, based upon a relational syntax (which has not yet entirely abandoned subject and predicate as the relata, but also includes propositions as relata) and function-theoretic logic, based upon the function-argument syntax.

With much of Peirce's work already taken into account by Hailperin and Sánchez Valencia, it is no loss that, after stating and rejecting the dichotomy between algebraic logic and "mathematical" logic, Hipinen immediately launches into those aspects of Peirce's work largely ignored until recently by the broader community of historians of logic to take up those topics ordinarily dealt with only by Peirce specialists, both historians of logic and philosophers of logic and language. Thus, the first step in Hilpinen's expository survey is to provide an account of Peirce's work in quantification theory (but not in its evolution of Peirce's research). Included in the account of Peirce's propositional logic is a proof (pp. 618–9) of

$$(P \rightarrow Q) \rightarrow ((Q \rightarrow R) \rightarrow (P \rightarrow R))$$

rendered in more familiar notation that Peirce's. The proof is by contradiction. This is, interestingly, followed (p. 619) by a proof of the same formula using Smullyan's tableau method. The essential point behind this comparison of proof procedures is to illustrate that Peirce's "method is based on the model-theoretic conception of logical truth as truth under all interpretations (in the present case, assignments of truth-values to propositional letters); and it resembles the method of one-sided (signed) semantic tableaux," adding that: "Like the tableau method, it consists in a systematic search for a counterexample to a given formula or inference" (p. 168).

Hilpinen has been deeply influenced by Jaakko Hintikka and the tableau method which he developed and which is closely akin to Smullyan's tableau method, Hintikka's "trees" growing sideways, rather than, like Smullyan's, growing downward.[33] But the essential point is that Hilpinen follows Hintikka in using a model-theoretic approach, in which semantic interpretation of truth-values is the key. Latterly, Hintikka has come to see this model-theoretic approach in game-theoretical terms, and Hilpinen follows Hintikka in accepting this conception.[34] What is not at all evident is that Hilpinen explicitly recognizes the

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[33]See [Anellis 1990a, Anellis 1990b] for a history of the "tree" method, with special reference to the contributions of Hintikka, Raymond M. Smullyan, Richard C. Jeffrey, and Jean van Heijenoort.

[34]See, e.g., [Hintikka 1979, Hintikka 1983, Hintikka 1988] and [Hilpinen 1982]; see also [Brock 1980].

explicit presence of semantic tableau in Peirce; on the contrary, in reading Hilpinen's account, one can come away with the impression that, according to Hilpinen, they are only implicitly present in Peirce. Hilpinen does not remark, for example, of the existence of "trees" in the *Symbolic Logic* [Dodgson 1977] of Charles Lutwidge Dodgson (1832–1898, *nom de plume* Lewis Carroll) and the account of Dodgson's trees in [Bartley 1977]; nor does there appear any evidence that the inspiration for Carroll's trees came directly from his study of Peirce (see [Abeles 1990]).

The treatment of the semantic approach to logic in Peirce leads naturally to a discussion of the connections which Peirce conceived between logic and language, and in particular to Peirce's distinction between type and token, and between icon, index, and symbol. The general theory of signs was an important aspect of Peirce's effort within philosophy to establish an architectonic of science, and of logic, deductive, inductive, and abductive, as a methodology for science. For Peirce, quantifiers in the algebra of relatives play the role of introducing (and underwriting) an ontological commitment into one's semantic interpretation of the syntax of the calculus. Signs *represent*, or *point to*, that object in the universe of external reality which they designate. Thus, to use Hilpinen's example (p. 622): "The word 'cat' is a sign. Any cat is an object of the sign, and any other sign which refers to cats, for example, the Spanish word 'gato', a cat-picture, or an idea of a cat is an interpretant of the sign 'cat'."

The general theory of signs, semiotics, is thus for Peirce a major theme of his work, and he regarded logic as a branch of semiotics. The other two branches of semiotics for Peirce were, in addition to "critical" or formal logic, speculative grammar, and speculative rhetoric. Included in logic for Peirce was the question of the nature of propositions, and, as we know, it is on this question that Frege and the logicists based much of their critique, and final rejection, of Peirce and the "Booleans".

For Peirce and his colleagues, propositions were logical equations, and whether the relation was considered (depending upon its particular context) to be the traditional copula, identity, inclusion, or inference, the relation was triadic and held between two terms, subject and predicate, which stood (depending upon the particular context) for some element of the universe of discourse, either of Aristotelian (or Kantian) category, or of a class of objects, which the relational nexus (copula, identity, inclusion, inference) brought together so that the relata created in the nexus a *tertium quid*, in which a third term was indicated.

Taking a simple example: given S and P as subject and predicate respectively of a proposition, and a relational nexus R (either the copula, identity, inclusion, or inference), SRP gives us the third member of the triad, brought together by R from S and P.[35]

Peirce's interest in the theory of signs ties in quite naturally to his efforts to devise graphical methods for checking the validity of arguments and depicting the relationships between the terms or classes of logical equations. Thus it is natural for Hipinen, after examining Peirce's semiotic (pp. 622–8), to turn to a sketch of his work on "existential graphs" (pp. 628–40). Within the extent of the pagination devoted to Peirce's logical graphs, Hilpinen manages to provide an excellent, if not detailed, account. For more extensive and extended discussion, as well as providing the survey, Hilpinen relies chiefly upon the far-flung and somewhat disjointed references in Volume 4 of the Hartshorne and Weiss edition of Peirce's *Collected Papers* [Peirce 1933]. For those who would pursue the study more thoroughly, there are three major studies devoted exclusively to Peirce's graphs, [Zeman 1964], [Roberts 1973], [Thibaud 1975], and [Shin 2002], each of which appears in Hilpinen's list of references.

The unhappy fact is that Peirce's work on graphs came late in his career, and went largely ignored—as well as largely unpublished. For by this time, Peirce had effectively withdrawn from an active academic life. But even so, for those who, like Venn, and indeed all reviewers of the *Begriffsschrift*, who complained about the typographical cumbrousness of the notation of Frege's *Begriffsschrift*,[36] Peirce's graphs, much inspired by his training in chemistry and augmented by Cayley trees in algebra, as well as by Peirce's natural right-brain dominance,

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[35]If we combine Hintikka's thesis that Peirce's work led the way to a game-theoretic approach to logic with the thesis that logic for Peirce was a branch of semiotics, we can be led to conclude that Peirce had marked out the path, later taken by Ludwig Wittgenstein (1889–1951), from the *Bildtheorie* or picture theory of language, in which logical syntax or logical grammar provides the logical "scaffolding" for language" in the *Tractatus Logico-philosophicus* [Wittgenstein 1922] to the notion, in his [Wittgenstein 1953] *Philosophische Untersuchungen / Philosophical Investigations*, and *Philosophische Grammatik / Philosophical Grammar* [Wittgenstein 1969, Wittgenstein 1974] of language as a game, and in his [Wittgenstein 1956] *Remarks on the Foundations of Mathematics / Bemerkungen über die Grundlagen der Mathematik*, in which mathematics is a game, a theme which has been proposed by [Nubiola 1997]. This is a connection which Hilpinen recognizes, but does not pursue, in the present survey.

[36]See [Anellis 2004–5, 83]. The term "cumbrousness" was employed for Frege's notation in particular in [Venn 1880]; see also [Frege 1972], especially pp. 234–5.

these graphical methods, had they been better publicized in their day, would have met with stiff resistance, we may be fairly certain.[37]

Recognizing that the scientific enterprise is a corporate community effort that is built up gradually through experimentation and continual correction and increasing precision, Peirce examined the methodology of science, as well as set up an architectonic, or classification, of the sciences. In studying the methodology of the sciences, Peirce considered deduction, induction, and abduction, and Hilpinen treats these three modes of reasoning (pp. 643–55). Peirce's pragmatic philosophy underwrote the conception that truth can be relative, in the sense that agreement between the "laws" of science and the external universe must be developed and refined, even occasionally altered, over time, and that, in practical terms, depends for its standing upon the consensus of the scientific community; science is a corporate cooperative enterprise of continuing investigation. It is natural, therefore, that one of Peirce's concerns was an analysis of the role of *vagueness* (thus his encyclopedia article "Vagueness"[38]), and this led him in turn to consider modalities and possible worlds. Thus Peirce was led to consider what have come to be called paraconsistent logics. It also led him to investigate the difficulties of material implication and consider alternatives, such as counterfactual implication. Thus Peirce was a pioneer of modern modal logic, and, as we know, was a direct or indirect inspiration for logicians, especially in the years after his death, to devise modal logics as formalized system.[39] Clarence Irving Lewis (1883–1964) was

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[37]See [Cayley 1857, Cayley 1875, Cayley 1881] for Cayley on tree diagrams; for Peirce on trees, see [Peirce n.d.].

Peirce himself explained (according to [Bell 1945, 556–7]; but see [Houser 1994b]; also cited in [Anellis 1995b, 277]) that: "my damned brain has a kink in it that prevents me from thinking as other people think." For a biography of Peirce, see [Brent 1993].

[38]See [Peirce 1901a]; see also, e.g., [Chiasson 2001] and [Brock 1979] for a discussion of Peirce's logic of vagueness, and [Engel & Engel-Tiercelin 1992] or a discussion of vagueness and the logic of vagueness in the broader context of Peirce's philosophy.

[39]Peirce treats modal logic and modality in two manuscripts "Significs and Logic" (MSS 641, 642) [Peirce 1909a, Peirce 1909b], in the second one of which he discusses the Principle of Excluded Middle and the Principle of Contradiction, as well as in numerous other manuscripts, before and after. He also treats the distinction between assertion and proposition and between modal propositions and the psychological modals "can" and "would" in his [Peirce 1877] publication "The Fixation of Belief." He also wrote the entry on "Modality" for James Mark Baldwin's *Dictionary of Philosophy and Psychology* [Peirce 1901b], in which he notes the original of the modal syllogism in Aristotle, its treatment by the scholastics, and covers the history of modality and modal logic from Kant and Hegel through the most recent

one obvious example of one inspired by Peirce's work in examining the differences between Philonian and Chrysippian implication, to develop a logic of strict implication, in which implicational propositions are defined by modal operators (see Lewis [Lewis 1912, Lewis 1914]). In Hilpinen's essay, Peirce's own work in modalities and possible-worlds semantics is discussed, the formal expression of whih Hilpinen explains in the notation of modern modal logic (pp. 640–2).

In a similar vein Peirce was also a pioneer of modern multiple-valued logic, working especially in trivalent or "triadic" logic, and Hilpinen briefly notes his work in this regard (pp. 642–4), again treating unary trivalent connectives in the more familiar form of a Łukasiewicizan truth-table. Indeed, Łukasiewicz's work in many-valued logics was directly inspired by a study of Peirce (see [Hiz 1997]). Even earlier, however Nikolai Aleksandrovich Vasil'ev was directly inspired by Peirce, in particular by a footnote to a paper by Paul Calvin Carus (1852–19191) in which Peirce was quoted as writing that he had from time to time toyed with the question of what a logical system might look like if one or more of the traditional Aristotelian laws, in particular either the Law of Excluded Middle or the Law of Noncontradiction, or both, were either negated or omitted (see [Carus 1910a, Carus 1910b], especially [Carus 1910b, 158]; see also [Bazhanov 1992]). Hilpinen specifically names Łukasiewiciz and Emil Leon Post (1897–1954)—and unnamed "others"—(p. 644) as developing trivalent logic independently of Peirce, and sends readers to [Fisch & Turquette 1966] and [Turquette 1967, Turquette 1969] for fuller accounts of Peirce's triadic logic.[40]

Although not indicated by Hilpinen, Peirce in all likelihood was led to consider the possibility [pun intended] of nonclassical or "non-Aristotelian"—or more properly, nonbivalent—logics by his study of the history of ancient and medieval logic, including the modal logic of Aristotle, the logics of the Stoics and Megarans, and the modal logic of William of Ockham and the Ockhamites.[41] Additionally, his scientific

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contemporary work of Trendelenburg, Sigwart, and Lotze. See [Zeman 1973, v, 14, 129–31, 295] for discussions of Peirce and history of modal logic.

[40]Hilpinen refers readers to [Łukasiewicz 1920a, Łukasiewicz 1930] and to [Post 1921]. In the case of [Łukasiewicz 1920a], Hilpinen wrote "[Łukasiewicz 1920a]," but the more appropriate reference is the second, a translation of [Łukasiewicz 1920b], in the series of papers [Łukasiewicz 1920a] on the notion of possibility. See *supra*, n. 7, on Peirce's role in inspiring Vasil'ev's work in paraconsistent logic.

[41]The best recent treatments of Aristotle's modal logic are [McCall 1963] and [Patterson 1995]. On modal syllogistics in the middle ages, see, e.g.,

training (as a chemistry major at Harvard, and his work thereafter in geodetics, metrology and related fields) would have led him to consider statistics and probability theory, in his efort to develop logic as an aspect of, and foundation for, scientific methodology. And last, but not necessarily least, the concerns for many of the algebraic logicians whose work form a significant background to and development of Peirce's own work was the application of logic to probability theory. We find those concerns made explicit, for example, in De Morgan's [De Morgan 1847] *Formal Logic*, subtitled *The Calculus of Inference, Necessary and Probable*, Boole's [Boole 1854] *Investigation of the Laws of Thought*, subtitled *on which are founded the Mathematical Theories of Logic and Probabilities*, and in Venn's [Venn 1866] *Logic of Chance*, to mention only the most prominent.[42]

Peter M. Sullivan's account of "Frege's Logic" (pp. 659–750) begins with a brief account of the history of the influence which Frege's work has had both during his own lifetime and after. The account is a familiar one: Frege's work was largely ignored, until Russell made the Russell Paradox, based upon Frege's Basic Law V in Volume 2 of Frege's [Frege 1903] *Grundgesetze* the subject of his own *Principles of Mathematics*, the famous appendices on Frege's work and on the Russell paradox to be found therein ("Appendix A. The Logical and Arithmetical Doctrines of Frege" [Russell 1903a, 501–22]), and the appendix presenting the theory of types as a means for overcoming the Russell paradox ("Appendix B. The Doctrine of Types," [Russell 1903a, 523–7]). What Sullivan does not do is record the reasons for the general, if not universal, dismissal of Frege's work by such reviewers of the *Begriffsschrift* as Venn and Schröder. Nor are there discussions between Frege and Hilbert, or between Frege and Husserl, given much attention by Sullivan, important as these were for helping to understand the nature of an axiomatic system and a formal deductive system, the philosophical issues between formalism and logicism, or, in the case of the Frege-Husserl discussions, Husserl's final rejection of psychologism.[43]

[Lagerlund 2000] and [Thom 2003]. For a discussion of Peirce's acquaintance with scholastic logic, see [Michael 1976].

[42]On Boole's contributions to probability theory, see, e.g., [Hailperin 1976] as it relates to his work in logic. See [Hailperin 1988] for a general history of the logic of probability from Leibniz to MacColl.

[43]Two articles, one of an earlier date [Stroll 1966] and presenting the older, accepted view, the other of more recent vintage [Vilkko 1998] and advancing a less "whiggish," contextual-comparative, and hence more historically realistic view, look

In view of Sullivan's largely philosophical interest in Frege's work, it is astounding that he simply follows the old canonical view that Frege's work was, in essence, laughed out of court by Venn, Schröder, and their fellow reviewers, and not seriously picked up again until Russell aroused the curiosity of philosophers in his treatment of the Russell Paradox. Sullivan's lack of treatment of the accounts of Frege's scant early influence, and of the sources for the alleged reversal of Frege's impact is all the more notable by virtue of the amount of attention which he devotes to Frege's philosophy in his essay,[44] issues which are largely of the nature of philosophy of language and subsidiary interests to philosophers of language issues in related areas of metaphysics, epistemology, and philosophy of mathematics.[45]

Only in the early expository section on "Elements of the system of Begriffsschrift" (pp. 662–74) do we get an examination of the technical aspects of Frege's work, and this is limited, with but minor forward

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at aspects of the attention or lack of attention to Frege's work prior to its "rediscovery" by Russell, and the recent publication of Carnap's [Carnap 1931] notes on Frege's lectures and his personal recollections (see [Carnap 1963, 4–5], and as reported, for example, by [Hintikka & Hilpinen 1997, ix]) help elucidate the impact which early criticisms of his work, especially by Russell, had on Frege. For a brief, but incomplete account of reviews of Frege's work, the *Grundlagen* [Frege 1884] and the *Grundgesetze* [Frege 1893, Frege 1903] as well as the *Begriffsschrift*, see also [Anellis 2004–5, n. 2, 69–70]; for a fuller account, see [Anellis 2006, 175–6]. Moreover, there is much literature available on whether the "mistake" in Frege's *Grundgesetze* was as Russell described, whether it was due to Basic Law V (see, e.g., [Bynum 1973] and [Sternfeld 1966, 131–6, 163–8], among many others), and whether it could have been "rescued" had Frege not been resistant to applying Hume's Principle (see, e.g., [Boolos 1999, 135–341], which brings together Boolos's work on showing how Frege could easily have avoided the contradiction that destroyed his program, as well as other efforts by, e.g., [Burgess 1998, Burgess 2005]). For details and a fuller discussion on where, if anywhere, Frege's mistake lay and what it was, as well as a discussion of proposed fixes, see [Anellis 2006, 201–9]. Finally there is a vast literature available on many of the philosophical issues raised by Frege's work; see, e.g., [Thiel & Beaney 2005], which includes a bibliography.

[44]Certainly Pulkkinen's [Pulkkinen 1994] account of the philosophical debates of Frege's day concerning the "logische Frage" was available to Sullivan, even if Vilkko's [Vilkko 2002] and Pulkkinen's [Pulkkinen 2005] appeared too late to be of use.

[45]Much of the attention given by philosophers to issues in Frege's philosophy of language and to Frege's contributions to other areas of philosophy rather than to his technical contributions to logic, has been due to the influence of Michael Dummett, and Sullivan is among those who have been influenced in this direction by Dummett. Among Dummett's most important works in this regard are his [Dummett 1973, Dummett 1978, Dummett 1981a, Dummett 1981b, Dummett 1981c, Dummett 1991a, Dummett 1991b].

glances to the *Begriffsschrift* [Frege 1879] rather than to *Die Grund-lagen der Arithmetik* [Frege 1884] and the two published volumes of the *Grundgesetze der Arithmetik* [Frege 1893, Frege 1903]. Even in the subsection "A formal system" (pp. 674–81) and the section on "Guiding Conceptions" (pp. 681–701), the technical apparatus of Frege's work is considered only in terms of essentially philosophical issues, such as relations of concepts to objects, and of judgment to truth. In the remainder of the essay (pp. 701–48) we are dealing exclusively with philosophical and linguistic questions, such as the problem of *Sinn und Bedeutung* (Sense and Reference) that was a central concern for analytic philosdophers from Russell to the present. Indeed, one might well agree with the writer who claimed that the chief influence of Frege in the twentieth century since his "rediscovery" by Russell was for linguistic philosophy, rather than for logic.[46]

Finally, it must be noted that Sullivan unequivocally and unhesitatingly accepts the view (p. 161) that Frege's work was uniquely original, and the epitome of modern mathematical logic. He endorses the view (p. 661) that this was due largely, if not exclusively, to Frege's quantificational system, and that any difference between what one finds in the *Begriffsschrift* and what one learns in today's symbolic logic texts is one of mere emphasis (p. 661). He quotes (p. 661) Michael Dummett [Dummett 1981b, xxxv] to the effect that the *Begriffsschrift* "is astonishing because it has no predecessors: it appears to have been born from Frege's brain unfertilized by external references." The only concession that Sullivan makes is to the historical background that was part of Frege's own mathematical training, especially to Weierestrass's work in analysis and Dedekind's work in number theory. By contrast, "Peirce's innovations," including, admittedly, introduction of quantifiers into the logic of relatives, says Sullivan dismissively (p. 662), "arrive piecemeal and in response to particular inadequacies of the Boolean framework he was developing."

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[46]A sampling of works, stimulated by Dummett, dealing with Frege's role in analytic philosophy and its history in general, and in philosophy of language in particular, includes [Tait 1997], [Biro & Kotatko 1995], [Haaparanta & Hintikka 1986], [Held 2005], [Kenny 2000], [Makin 2000], [Penco 1994], [Van Heijenoort 1977a, Van Heijenoort 1977b, Van Heijenoort 1985], and [Weiner 2004].

One cannot help but recall in this context that Russell's touting, and criticisms, of Frege spawned what Joong Fang called a "Frege industry." Looking back at the Frege industry" in the philosophy of logic, and considering it in connection with the history of analytic philosophy, some might wish to argue that Frege has been more important to the history of philosophy of language than to philosophy of logic or to logic (see [Anellis 1993, 148–52]).

In summation, the quality of the contents of this volume is uneven, and the distribution of space allotted to those whose work is considered or to topics discussed is equally unbalanced, more especially as measured by the "whiggish" view of history. From the perspective of careful exposition and fullness of coverage of technical achievements of those whose work is presented in this volume, rather than from the perspective of reconstruction, interpretation, or philosophical import, the essays by Hailperin, Sánchez Valencia, Peckhaus, and Hilpinen must be singled out for particular acclamation, while Lenzen's essay must be lauded as a brilliant piece of reconstruction. As for the remainder of the essays (with the possible, grudging exception of Rusnock and George on Bolzano), they would have been better suited to a volume devoted to the history of nineteenth-century philosophy of logic, and in particular one in which the central theme was the nineteenth–mid-twentieth–century debates on "Was ist Logik?" and the foundational debates on logicism, formalism, and intuitionism.

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