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# LOGICAL MACHINES

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In the "Voyage to Laputa" there is a description of a machine for evolving science automatically. "By this contrivance, the most ignorant person, at a reasonable charge, and with little bodily labor, might write books in philosophy, poetry, politics, laws, mathematics, and theology, without the least assistance from genius or study." The intention is to ridicule the Organon of Aristotle and the Organon of Bacon, by showing the absurdity of supposing that any "instrument" can do the work of the mind. Yet the logical machines of Jevons and Marquand are mills into which the premises are fed and which turn out the conclusions by the revolution of a crank. The numerous mathematical engines that have been found practically useful, from Webb's adder up to Babbage's analytical engine (which was designed though never constructed), are also machines that perform reasoning of no simple kind. Precisely how much of the business of thinking a machine could possibly be made to perform, and what part of it must be left for the living mind, is a question not without conceivable practical importance; the study of it can at any rate not fail to throw needed light on the nature of the reasoning process. Though the instruments of Jevons and of Marquand were designed chiefly to illustrate more elementary points, their utility lies mainly, as it seems to me, in the evidence they afford concerning this problem.

The machine of Jevons receives the premises in the form of logical equations, or identities. Only a limited number of different letters can enter into these equations — indeed, any attempt to extend the machine beyond four letters would complicate it intolerably. The machine has a keyboard, with two keys for the affirmative and the negative form of each letter to be used for the first side of the equation, and two others for the second side of the equation, making four times as many keys as letters. There is also a key for the sign of logical addition or aggregation for each side of the equation, a key for the sign of equality, and two full stop keys, the function of which need not here be explained.[1] The keys are touched successively, in the order in which the letters and signs occur in the equation. It is a curious anomaly, by the way, that an equation such as A = B, which in the system of the transitive copula would appear as two propositions, as All A is B and All B is A, must not be entered as a single equation. But although the premises outwardly appear to be put into the machine in equations, the conclusion presents no such appearance, but is given in the form adopted by Mr. Mitchell in his remarkable paper on the algebra of logic. That is to say, the conclusion appears as a description of the universe of possible objects. In fact, all that is exhibited at the end is a list of all the possible products of the four letters. For example, if we enter the two premises All D is C, or D = CD, and All C is B, or C = BC, we get the conclusion in the following shape, where letters in the same vertical column are supposed to be logically multiplied, while the different columns are added or aggregated:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| A | A | A | A | a | a | a | a |
| B | B | B | b | B | B | B | b |
| C | C | c | c | C | C | c | c |
| D | d | d | d | D | d | d | d. |

The capital letters are affirmatives, the small letters negatives. It will be found that every column containing D contains B, so that we have the conclusion that All D is B, but to make this out by the study of the columns exhibited seems to be much more difficult than to draw the syllogistic conclusion without the aid of the machine.

Mr. Marquand's machine is a vastly more clear-headed contrivance than that of Jevons. The nature of the problem has been grasped in a more masterly manner, and the directest possible means are chosen for the solution of it. In the machines actually constructed only four letters have been used, though there would have been no inconvenience in embracing six. Instead of using the cumbrous equations of Jevons, Mr. Marquand uses Professor Mitchell's method throughout.[2] There are virtually no keys except the eight for the letters and their negatives, for two keys used in the process of erasing, etc., should not count. Any number of keys may be put down together, in which case the corresponding letters are added, or they may be put down successively, in which case the corresponding combinations are multiplied. There is a sort of diagram face, showing the combinations or logical products as in Jevons's machine, but with the very important difference that the two dimensions of the plane are taken advantage of to arrange the combinations in such a way that the substance of the result is instantly seen. To work a simple syllogism, two pressures of the keys only are necessary, two keys being pressed each time. A cord has also to be pulled each time so as to actualize the statement which the pressure of the keys only formulates. This is good logic: philosophers are too apt to forget this cord to be pulled, this element of brute force in existence, and thus to regard the solvet ambulando as illogical. To work the syllogism with Mr. Jevons's machine requires ten successive movements, owing to the relatively clumsy manner in which the problem has been conceived.

One peculiarity of both these machines is that while they perform the inference from (A + B)C to AC + BC, they will not perform the converse inference from AC + BC to (A + BC). This is curious, because the inference they refuse to perform seems to be merely syllogistic, while the one they do perform, and in fact continually insist on performing, whether it is wanted or not, is dilemmatic, and therefore essentially more complicated. But in point of fact neither of the machines really gives the conclusion of a pair of syllogistic premises; it merely presents a list of all the possible species in the universe, and leaves us to pick out the syllogistic conclusions for ourselves. Thus, with Marquand's machine, we enter the premise All A is B in the form a + B, and the premise All B is C in the form b + C; but instead of finding the conclusion in the form a + C, it appears as

$$\begin{aligned}&\mathrm{ABCD + ABCd}\\ &+ \mathrm{aBCD + aBCd + abCd + abCD}\\ &\qquad\qquad + \mathrm{abcd + abcD.}\end{aligned}$$

As we only want a description of A, we multiply by that letter, and so reduce the conclusion to ABCD + ABCd, but there is no elimination of the B nor of the D. We do not even get the full conclusion in the form ab + BC, although it is one of the advantages of Marquand's machine that it does give the conclusion, not only in the form just cited, but also, simultaneously, as

$$\begin{aligned}&(\mathrm{a + B + c + d})\,(\mathrm{a + B + c + D})\\ &(\mathrm{a + B + C + d})\,(\mathrm{a + B + C + D})\,(\mathrm{a + b + C + D})\,(\mathrm{a + b + C + d})\\ &\qquad\qquad(\mathrm{A + b + C + D})\,(\mathrm{A + b + C + d}).\end{aligned}$$

The secret of all reasoning machines is after all very simple. It is that whatever relation among the objects reasoned about is destined to be the hinge of a ratiocination, that same general relation must be capable of being introduced between certain parts of the machine. For example, if we want to make a machine which shall be capable of reasoning in the syllogism

If A then B,
If B then C,
Therefore, if A then C,

we have only to have a connection which can be introduced at will, such that when one event A occurs in the machine, another event B must also occur. This connection being introduced between A and B, and also between B and C, it is necessarily virtually introduced between A and C. This is the same principle which lies at the foundation of every logical algebra; only in the algebra, instead of depending directly on the laws of nature, we establish conventional rules for the relations used. When we perform a reasoning in our unaided minds we do substantially the same thing, that is to say, we construct an image in our fancy under certain general conditions, and observe the result. In this point of view, too, every machine is a reasoning machine, in so much as there are certain relations between its parts, which relations involve other relations that were not expressly intended. A piece of apparatus for performing a physical or chemical experiment is also a reasoning machine, with this difference, that it does not depend on the laws of the human mind, but on the objective reason embodied in the laws of nature. Accordingly, it is no figure of speech to say that the alembics and cucurbits of the chemist are instruments of thought, or logical machines.

Every reasoning machine, that is to say, every machine, has two inherent impotencies. In the first place, it is destitute of all originality, of all initiative. It cannot find its own problems; it cannot feed itself. It cannot direct itself between different possible procedures. For example, the simplest proposition of projective geometry, about the ten straight lines in a plane, is proved by von Staudt from a few premises and by reasoning of extreme simplicity, but so complicated is the mode of compounding these premises and forms of inference, that there are no less than 70 or 80 steps in the demonstration. How could we make a machine which would automatically thread its way through such a labyrinth as that? And even if we did succeed in doing so, it would still remain true that the machine would be utterly devoid of original initiative, and would only do the special kind of thing it had been calculated to do. This, however, is no defect in a machine; we do not want it to do its own business, but ours. The difficulty with the balloon, for instance, is that it has too much initiative, that it is not mechanical enough. We no more want an original machine, than a house-builder would want an original journeyman, or an American board of college trustees would hire an original professor. If, however, we will not surrender to the machine, the whole business of initiative is still thrown upon the mind; and this is the principal labor.

In the second place, the capacity of a machine has absolute limitations; it has been contrived to do a certain thing, and it can do nothing else. For instance, the logical machines that have thus far been devised can deal with but a limited number of different letters. The unaided mind is also limited in this as in other respects; but the mind working with a pencil and plenty of paper has no such limitation. It presses on and on, and whatever limits can be assigned to its capacity to-day, may be over-stepped to-morrow. This is what makes algebra the best of all instruments of thought; nothing is too complicated for it. And this great power it owes, above all, to one kind of symbol, the importance of which is frequently entirely overlooked — I mean the parenthesis. We can, of course, dispense with parentheses as such. Instead of $(a + b) c = d$, we can write $a + b = t$ and $tc = d$. The letter t is here a transmogrified parenthesis. We see that the power of adding proposition to proposition is in some sort equivalent to the use of a parenthesis.

Mr. Marquand's machines, even with only four letters, facilitate the treatment of problems in more letters, while still leaving considerable for the mind to do unaided. It is very desirable a machine on the same principle should be constructed with six letters. It would be a little more elegant, perhaps, instead of two keys to each letter, to have a handle which should stand up when the letter was not used, and be turned to the right or left, according as the letter was to be used, positively or negatively. An obvious extension of the principle of the machine would also render it possible to perform elimination. Thus, if six letters, A, B, C, D, E, F, were used, there could be an additional face which should simply take no notice of F, a third which should take no notice of F or E, a fourth which should take no notice of F, E or D; and these would suffice. With such a machine to represent AB + CD, we should proceed as follows: Put down handle E to the left. [The left hand would naturally signify the negative.] Leaving it down, put down handle A to the right and then bring it back after pulling the cord. Put down handle B to the right and pull the cord, and then restore handles B and E to the vertical. Next, put down handle F to the left and successively put down the handles C and D to the right, as before. After restoring these to the vertical, put down handles E and F to the right, and pull the cord. Then we should see on the third face

(A + B + C + D) (A + b + C + D) (A + b + C + d) (A + B + C + d)
(A + B + c + D) (A + b + c + D)
(a + B + c + D)
(a + B + C + D) (a + B + C + d)

or, what comes to the same thing,

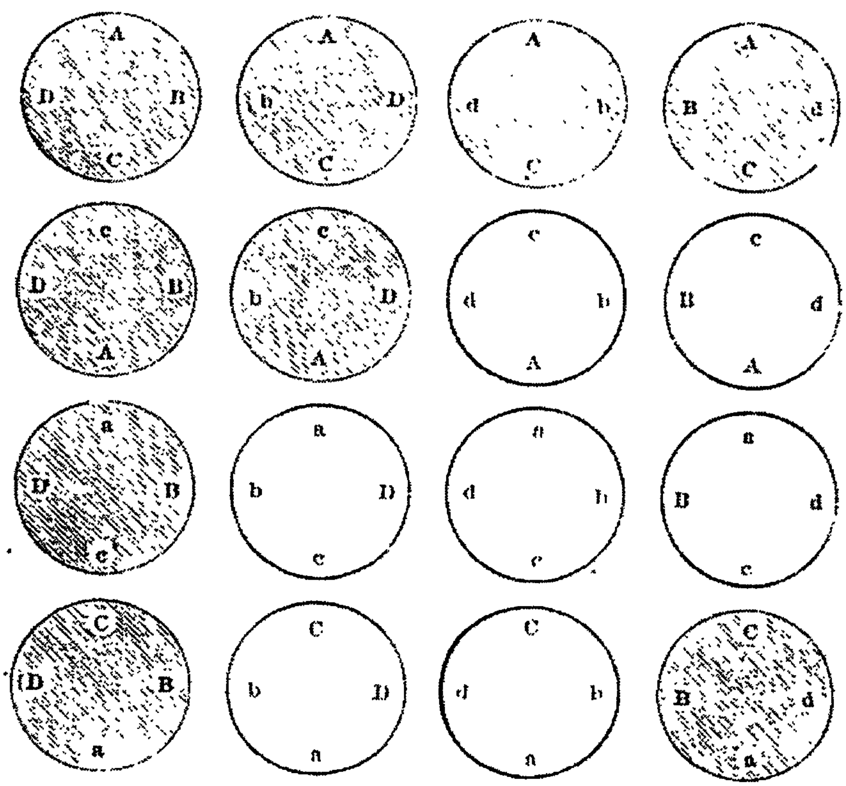
aBCD + abCD
ABCd + ABCD + AbCD
ABcd + ABcD

I do not think there would be any great difficulty in constructing a machine which should work the logic of relations with a large number of terms. But owing to the great variety of ways in which the same premises can be combined to produce different conclusions in that branch of logic, the machine, in its first state of development, would be no more mechanical than a hand-loom for weaving in many colors with many shuttles. The study of how to pass from such a machine as that to one corresponding to a Jacquard loom, would be likely to do very much for the improvement of logic.

## Notes

[1] *Phil. Trans.* for 1870.

[2] It would be equally true to say that the machine is based upon Mrs. Franklin's system. The face of the machine always shows every possible combination; putting down the keys and pulling the cord only alters the appearance of some of them. For example, the following figure represents, diagrammatically, the face of such a machine with certain combinations modified:



This face may be interpreted in several different ways. First, as showing in the shaded portions

(A + B + C + D) (A + b + C + D) (A + b + C + d) (A + B + C + d)
(A + B + c + D) (A + b + c + D)
(a + B + c + D)
(a + B + C + D) (a + B + C + d),

which is the same as what is seen on the unshaded portions if we regard the small letters as affirmative and the capitals as negative, and interchange addition and multiplication, that is, as —

$$\begin{aligned}&aBCD + abCD\\&+ ABCd + ABCD + AbCD\\&+ ABcd + ABcD.\end{aligned}$$

Or, looking at the unshaded portion, we may regard it as the negative of the above, or —

$$\begin{aligned}&(A + b + c + d)\,(A + B + c + d)\\&(a + b + c + D)\,(a + b + c + d)\,(a + B + c + d)\\&(a + b + C + D)\,(a + b + C + d),\end{aligned}$$

or, what is the same thing, as —

$$\begin{aligned}&abcd + aBcd + aBcD + abcD\\&+ abCd + aBCd\\&+ AbCd\\&+ Abcd \qquad + AbcD.\end{aligned}$$

There are two other obvious interpretations. We see, then, that the machine always shows two states of the universe, one the negative of the other, and each in two conjugate forms of development. In one interpretation simultaneously impressed terms are multiplied and successively