ℵ Modern Logic ω

The Peirce Edition Project, *Writings of Charles S. Peirce: A chronological edition*, volumes I – IV (Bloomington, Indiana University Press, 1982 – 1989).

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Victor Lenzen, recollecting [*1965*, p. 7] his Christmas-time 1914 trip on behalf of the Harvard University philosophy department to the Milford, Pennsylvania home of the Peirces to prepare the library of Charles Peirce for shipment to Harvard, recalls receiving in response to his remark to Peirce's widow that Peirce "was especially interested in logic," the reply that "He loved logic." The volumes under review clearly attest to this fact. Indeed, Peirce had his first introduction to logic at about age twelve, when in 1851, he happened upon and borrowed his brother's copy of Richard Whately's *Elements of logic* [*1845*], then the logic textbook used at Harvard, and read it cover-to-cover in several days, after which time he forever thought of every other subject as an exercise in logic (PEP[1], 1, *Introduction*, xix). Of course this is not to say that one will find something of interest to logicians or historians of logic on every page of the works being considered, nor even very much in every volume, since Peirce's interest ranged far, and he contributed to many fields, from linguistics to philosophy to psychology to geodesy, and this is quite naturally reflected in the volumes.

The first attempt to organize and publish Peirce's writings dates from 1915. Shortly after Peirce's death in 1914, his *Nachlaß* was deposited with Harvard University under the custodial care of the philosophy department. There, Josiah Royce, with the aid of his graduate assistant W. Fergus Kernan, undertook the organization of the Peirce papers. In a

[1] Peirce scholars commonly designate the volumes published by the Peirce Edition Project as *Wn*, where *n* is the volume number.

letter of 4 January 1915, to the Columbia University philosopher Wendell T. Bush, Royce (quoted by [Kernan *1965*, 91]; quoted in [Houser *1990*, 1]) wrote that:

> We have just received at Harvard the extant logical manuscripts of Charles S. Peirce, a gift from his widow, and, as I hope, a real prize. I look forward to editing them. They are certainly fragmentary but also certainly inclusive of some valuable monuments of his unique and capricious genius.

On 23 September 1916, following Royce's death, James Houghton Woods, chairman of the Harvard philosophy department, wrote to Bertrand Russell, inviting him to Harvard to teach a seminar on Peirce and offering him the opportunity to serve as editor of the Peirce papers, with Henry Sheffer to be assigned as his assistant. However, as a result of Russell's anti-war activities and consequent imprisonment, a visa was denied, and Russell was unable to accept the offer. Meanwhile, Kernan, who worked exclusively on the papers during the academic year 1915–1916 in lieu of taking scheduled graduate courses, continued the cataloguing and preservation of the collection until he joined the U.S. Army after America's entry into World War I. In 1920, after the failed attempt to bring Russell to Harvard, Clarence Irving Lewis was brought to Harvard specifically to engage in the task of preparing an edition of the Peirce papers. Lewis undertook the responsibility for reading, identifying, organizing, and collating the miscellaneous piles of papers of Peirce manuscripts that were stacked in the large room which became Lewis's office. The state of this storeroom and its contents were described by Lewis in his autobiography [*1968*, 16]. The vast stacks of manuscripts, he wrote:

> ...seemed to include everything Peirce had ever written – with the notable exception that there was no final draft of anything he had ever published. One could easily conclude that Peirce had no wastebasket, and had never discovered such conveniences as files. By far the greater part of these papers were simply loose sheets, now piled on shelves and tables and around the room.

In this, Lewis echoed the descriptions of the disorder of the papers previously given by Kernan, and even by Peirce himself. Lewis spent two years in an effort to bring some order out of this chaos before moving on to other things, being unable and unwilling to devote the many additional years to the gargantuan task which it would still clearly take to complete the remaining work.

Lewis and Kernan made a solid beginning in the organization of the Peirce papers, and their work was completed by Paul Weiss and Charles Hartshorne, who had the assistance of a clerical team to handle the mechanical details of the job. Hartshorne, a former Harvard philosophy graduate student, joined the effort in the late 1920's; after completing post-

graduate work in Europe, he was hired specifically for this purpose. Hartshorne's description of the room to which Lewis brought him to first see the papers is reminiscent of Lewis's own description while also suggestive of the work that had already been done. In an interview with Irwin C. Lieb [*1970*, 150], Hartshorne said of the manuscripts that:

> they were in a small number of big piles on a large table. I don't remember whether there was any label on the top of each pile. I think there were about eleven piles. On the shelves there were, I seem to recall, just fifty-two empty boxes, each of which was labeled, showing that the manuscripts had been sorted under fifty-two categories, but my predecessor on the job had evidently taken them out of these boxes and resorted them into a much smaller number of piles. ...

After about a year, Hartshorne was joined by Weiss, also a Harvard philosophy graduate student, and in 1931 they began the final editorial work and selection of the papers that were included in their edition of Peirce's *Collected papers*. Hartshorne and Weiss on occasion received advice from numerous people, including, for example, Henry S. Leonard, and on occasion, Whitehead. Houser [*1990*, 5] has indicated that there is evidence that the arrangement of volume III: *Exact logic (Published papers)* was Lewis's. Weiss recalled [Bernstein *1970*, 166] that when he was doing logic for the anthology, he wanted his work checked, and asked the advice of Leonard, and that in his judgment [Bernstein *1970*, 174] Leonard deserved to share with him credit as editor of that logic volume. It is possible that Sheffer also contributed particularly to the preparation of the volumes in logic, although it is unclear that he actually did. The Peirce *Nachlaß* now resides in Harvard's Houghton Library.

The project of publishing the works of Charles Peirce which is currently being undertaken by members of the Peirce Edition Project (PEP) at Indiana University–Purdue University at Indianapolis is a monumental enterprise, far more ambitious than the project carried out in the 1920's and 1930's by Weiss and Hartshorne. The incentive for the new undertaking came in 1959, when Max H. Fisch was asked to prepare an intellectual biography of Peirce to "round out" the Hartshorne-Weiss edition (H–W). By 1973, Fisch and his colleagues were fully convinced that the selection and organization of H–W was grossly inadequate, and that the organization of the manuscripts begun in 1915 was even more seriously deficient. In 1974, experts began the task of scientifically dating, organizing, and transcribing the manuscripts using the latest available technologies, and in 1975 the Peirce Edition Project was organized. The leader in bringing together the original PEP team was Edward C. Moore, who discusses his organizational work in [Moore & Burks *1992*, 101-106]. The actual work of preparing a new critical edition began in 1976, and the first volume appeared in 1982. (A considerably more detailed discussion of the history of the Peirce papers and the efforts undertaken to publish them, and in particular a

history of PEP, is given by PEP member Nathan Houser [*1990*]. [Houser *1990*] also describes in detail the purposes and editorial procedures of PEP.[2])

Work at PEP continues to this day, with the fifth volume scheduled to appear in 1992. By the time the first volume appeared, a total of twenty volumes were projected; by the time the fourth volume went to press, that estimate had grown to a projected thirty volumes. Even so, this will contain only a fraction of the available materials; as is stated on page xii of the "Preface" of volume 3, "any edition in fewer than sixty-five volumes might fairly be called 'Selected Writings'." The microfilm edition of Peirce's *Nachlaß* comprises 38 reels, and it has been estimated that a complete edition of all of Peirce's writings, published and unpublished, and including several thousand manuscript pages of discarded computations and scratch-work, would require over a hundred volumes.

Readers will already be familiar with the older project, which produced the eight-volume *Collected papers of Charles Sanders Peirce* published by Harvard University Press between 1931 and 1958.[3] Readers will be familiar in particular with the second, third, and fourth volumes of the *Collected papers*, which contain the material from Peirce's work in logic and mathematics. By Hartshorne's recollection ([Lieb *1970*, 154]), it was Weiss who "largely took charge of the logic papers," having done more work in logic than Hartshorne, and that it was Weiss, too, who had "discovered that Peirce had anticipated Sheffer in the discovery of the two Sheffer functions." This recollection is confirmed by Weiss [Bernstein *1970*, 161], who recalled having learned of the Sheffer stroke as a graduate student in logic before coming to Harvard to work with Hartshorne. We may add that the notion for the Sheffer stroke could not have come directly from Sheffer's studies of the work of Peirce in which the Peirce arrow (along with what possibly became the Sheffer stroke) was developed, since Sheffer's [*1913*] paper *A set of five independent postulates for Boolean algebras with application to logical constants* was published before the arrival at Harvard of the Peirce *Nachlaß*. Since Sheffer had been Royce's student, we may suppose that, he would have had some contact with some of Peirce's ideas through

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[2] I am grateful to Nathan Houser for providing me with a preprint of this valuable historical discussion, to which I have made frequent reference.

[3] The first six volumes, published from 1931 to 1935, were edited by Hartshorne and Weiss; the seventh and eighth volumes, edited by Arthur W. Burks, were published in 1958. Burks and E.C. Moore give a brief outline of the history of editing the work of Peirce in [Moore & Burks *1992*]; Burks also discusses his recollections of working on the H-W project in [Moore & Burks *1992*].

Peirce scholars commonly refer to these volumes as "CP". The microfiche edition of Peirce's published work is comprised of 161 (= 149 + 12 supplemental) fiche. Information on the microfiche edition was kindly provided by PEP director Christian Kloesel.

Royce.[4] Despite this, Dreben [*1990*, 89] declares without hesitation or equivocation that "not until 1933 was it known that Peirce had discovered the adequacy of joint denial in 1880 and of alternate denial in 1902."

The older project served as a model of Peirce scholarship for five decades and was the greatest source of published material from Peirce's hand throughout that period. One is naturally led to inquire why another such venture should seem necessary. The two projects certainly invite comparison, and in fact the best way to answer the question of why a new edition of Peirce's work is warranted is to examine the similarities and differences between the old edition and the current one.

The most obvious similarity is that much of the material which had already appeared in the H-W edition has again appeared in the PEP edition or will ultimately appear in future PEP volumes. Thus, among the papers of particular interest to logicians, one finds, for example, Peirce's 1867 papers *On an improvement in Boole's calculus of logic* and *Upon the logic of mathematics*, along with his 1870 paper *Description of a notation for the logic of relatives, resulting from an amplification of the conceptions of Boole's calculus of logic*, in H-W volume III and in PEP volume 2. The differences are more profound. The most important differences are that (a) the PEP edition is arranged chronologically, while the H-W edition is topical, but within each topic manages to integrate a chronological approach; (b) the PEP edition is not simply the collection of writings that one finds in the H-W edition, but includes such valuable editorial apparatus as "Textual Notes" and "Emendations" which mark Peirce's errata and differences in various versions of his manuscripts; annotations, or "Editorial Notes" by the editors and their expert contributing editors which help to elucidate obscure references and historical and technical points, which signal connections between Peirce's work and the work of his predecessors and successors; a bibliography of the works to which Peirce referred in the writings included in each volume, with those items known to have been owned by Peirce indicated by an asterisk; a chronological list of Peirce's writings for the period covered by each volume; an index; and an introduction in each volume that consists largely of either a biographical sketch of Peirce for the period covered by that volume; or one or more essays by specialists discussing the most significant aspects of Peirce's work during that period; most of these scholarly devices are missing from the H-W edition; (c) the writings in the H-W volumes have sometimes been heavily edited or abridged – in some instances manuscripts had been broken up and excerpts distributed through several volumes, while in other cases fragments from various manuscripts were pasted into other, often unrelated, manuscripts – in most cases without these facts being stipulated, whereas manuscripts in the PEP volumes are presented *in toto* and in accordance with the most exacting modern critical standards, and use technical analysis of watermarks, inks, and similar techniques to

[4] We learn from Dreben [*1990*, 82] that Sheffer was Royce's student. His dissertation was completed in 1908 (see Dreben [*1990*, 83]). Dreben's paper sketches the history of logic studies in the Harvard philosophy department through Quine's doctoral thesis of 1932 and summarizes Quine's thesis.

help determine the precise chronology of manuscripts and the relationship among papers; and (d) the PEP edition will, when finished, be much more complete in its representation of the total output of Peirce's pen.

Some of the anomalies that occur in the writings appearing in H-W surely represent the best scholarly judgment of their day, and many of the differences between versions of papers in H-W and PEP must be under-stood in the light of the more sophisticated technologies available today for identifying chemical properties of inks, watermarks, and other means for dating materials, supplemented by a naturally improved scholarship over the last fifty years.

There are of course advantages as well as disadvantages to some of the differences which have been enumerated. The most obvious disadvantage to the chronological approach taken by PEP, in which writings occur in the order in which they were written (according to the best judgments of the editors), so that papers on philosophy, meteorology and physical sciences, psychology, semiotics, logic, matrix theory or other areas of algebra, and numerous other subjects on which Peirce wrote, all occur without any organization according to topic, is that it is necessary for those interested in logic to search through all of the papers in all of the volumes to locate the logic papers, rather than have several volumes devoted specifically to logic, within which the papers are arranged chronologically. The purpose of this approach, as stated in the "Preface: of the first volume (pp. xi–xii) "is to facilitate the study of the historical development of Peirce's thought," not simply in the separate subject areas within which Peirce worked, but in the "systematic unity of his thought."

Despite the anomalies already mentioned, with the editing and abridging that was involved, H–W can still be a useful starting point for further exploration of Peirce's writings. Study of the papers in H–W provides a skeletal structure for understanding Peirce's achievements in logic and a global outline of his development as a mathematical logician. For a more detailed view of Peirce's contributions and a more precisely articulated picture of his development, one has to look at the PEP edition. Here, the chronological approach is a decided advantage. Moreover, the care taken in the presentation of the textual notes and annotations is a valuable asset to the reader. These notes, taken together, provide additional evidence of Peirce's intellectual struggles in advancing his ideas in logic (and the other subjects with which he was concerned) from inception to articulation, while the annotations help define the relevance of Peirce's work to contemporary work in logic as well as help link that work to the history of logic and to the contributions of Peirce's colleagues, contemporaries, and immediate successors.

Modern Logic readers will be most familiar with volumes II: *Elements of logic* (1932), III: *Exact logic (Published papers)* (1933), and IV: *The simplest mathematics* (1933) of the H–W edition, which contain writings principally on history and philosophy of logic, traditional logic, Boolean algebra, the logic of relatives, and set theory. Within each volume, however, the presentation is chronological (as could best be determined by the

scholarship of that day). The advantage (to both the historian of logic and the research logician wishing to take advantage of Peirce's contributions) in having all of Peirce's logical and mathematical papers collected in one place, or in one series of volumes within the larger multi-volume collection, while retaining a chronological presentation, in comparison to the PEP approach, is obvious.

Logicians and historians of logic will find very little of interest in PEP volume 1, which is devoted to the period 1857–1866, before Peirce began serious work in logic. The paper *The axioms of intuition after Kant* (MS 50: May 1859), pp. 31-33 contains a philosophical "proof" of the "axiom" that *Space has three dimensions*, of the euclidean "axiom" that *A straight line is the shortest way between two points*, and the mathematical correspondent to the first "axiom" that *Two lines cannot enclose space*. It is interesting to note that Peirce should want to provide "proofs" of "axioms". There is no evidence that Peirce meant anything different from "axiom" than we do today; so that the attempt to prove axioms shows that Peirce, very much like Russell at about the same age, was seeking a certainty, and that this led to the attempt to provide a foundation for mathematical foundations. Russell's *Essay on the foundations of geometry* (1896) was a search for the over-arching philosophico-foundational truths for all geometries, euclidean and noneuclidean, metric and projective; like Peirce's little manuscript, it was also an attempt to give mathematical credibility to the intuitions of spatial experience, and Peirce, like Russell, took his basic view concerning spatial intuition directly from Kantian philosophy. It is also noteworthy that Peirce "discovered" logic at the same age that Russell "discovered" geometry – and in much the same way, having, as we noted, read Whately at about age twelve.

The two related papers *The conception of infinity* (MS 53: August 1859), pp. 40-42 and *Why we can reason on the infinite* (MS 53; 23 October 1859), pp. 42-43 are equally and purely philosophical and have little to offer the mathematician, although they may be of some interest to philosophers of mathematics. Peirce's Harvard Lectures of 1865 *On the logic of Science* is comprised of a series of manuscripts devoted to scientific methodology and inductive logic. Scattered throughout the pages of these lectures, however, one can find discussions which may be of interest to logicians and to historians of logic; for the most part, these discussions pertain to traditional logic, and to such questions as the role of syllogistics for inductive logic and hypothetical reasoning. The text of Lecture VI, however, on *Boole's calculus of logic* (MS 100: March–April 1865), pp. 223–239, will be of special interest to historians of logic, not so much for its very brief historical remarks but because it is apparently Peirce's first professional (expository) treatment of the technical work of Boolean algebra and the application of Boole's calculus for probability. We may foresee in this lecture the hint of Peirce's interest and work in Boolean algebra that led Peirce to develop his logic of relations. With this perspective, Lecture VI comes to foreshadow the pioneering work which Peirce began in 1867 on the logic of relations, and takes on the aspect as the historical root of cylindric and polyadic algebras as well as of relation algebras. There follow a number of other papers, most of which primarily treat what we

might today call philosophy of logic rather than logic, but which already suggest that Peirce was beginning to play with new and old ideas in a manner that generated his first attempts to surpass traditional syllogistic and to expand or "improve" Boolean algebra into the logic of relatives. The *Memoranda concerning the Aristotelian syllogism* of November 1866, pp. 505–514, a privately printed handout for Peirce's Lowell Institute lectures, however, is strictly in the realm of traditional logic; it classifies categorical syllogisms by their mood and figure, summarizes the relationships among them and examines the Aristotelian techniques for reducing second and third figure syllogisms to the first figure.

Among the most well-known of Peirce's logical writings to appear in volume 2, which covers the period 1867–1871, are Peirce's 1867 papers *On an improvement in Boole's calculus of logic* (pp. 12–23) and *Upon the logic of mathematics* (pp. 59–69), along with his 1870 paper *Description of a notation for the logic of relatives, resulting from an amplification of the conceptions of Boole's calculus of logic* (pp. 359–429). The first section of the *Introduction* to this volume, entitled *The decisive year and its early consequences*, is by Max H. Fisch and includes a brief survey of Peirce's work in logic during this period (pp. xxx–xxxi). The third section of the introduction to this volume includes Daniel D. Merrill's essay *The 1870 logic of relatives memoir* (pp. xlii–xlviii), which focuses on Peirce's paper *Description of a notation for the logic of relatives, resulting from an amplification of the conceptions of Boole's calculus of logic*, and calls the paper "one of the most important works in the history of modern logic" (p. xlii). Peirce's 1870 paper retains what Merrill calls the "substructure" of Boole's logic of classes while developing Peirce's logic of relations as an extension of Boole's work. In many ways, the 1867 paper *On an improvement in Boole's calculus of logic* (pp. 12–23) is the most historically important and interesting, since it is Peirce's first professional publication in logic and signals the first serious attempt to undertake the extension of Boole's algebra of classes and marks the beginning of the development of the algebra of relations. There are many other works in this volume which will be of interest to logicians and historians of logic, including the 1871 manuscript *Of the copulas of algebra* (MS 175; 27 April 1871) (pp. 451–456), as well as material from Peirce's logic notebook and other notes, the text of various lectures and lecture notes on logic, published and unpublished writings on history of logic, book reviews – for example a review, originally published in *The Nation* in 1870, of Alexander Bain's *Logic* (PEP 2:441–444), a letter from Peirce to William Stanley Jevons of 25 August 1870 comparing the algebraic operations which they use in their logical systems (PEP 2:445–447), and published necrologies in memory of De Morgan (pp. 448–450) and Charles Babbage (pp. 457–459), and in the "Appendix" a letter from J.E. Oliver to Peirce concerning notations employed for symbolizing algebraic operations (pp. 492- 497). Of the nearly 500 pages of Peirce's writings in this volume, logicians and historians of logic will find something of interest and value in all or many of the 400 of those pages devoted to logic.

While logicians and historians of logic will find PEP volume 2 indispensable for understanding Peirce's mature work, because it includes the three most crucial papers of Peirce's early period as a mature logician and covers the period which may arguably mark the start of Peirce's most prolific and creative stage as a professional logician, the historian of logic who is looking for the first glimmers of Peirce's future intellectual development may well wish to begin studies of Peirce with PEP volume 1, however, despite the paucity of material on logic in that volume, because of the prefiguring nature of the paper *Boole's calculus of logic* and the suggestive character of the papers following it in which Peirce begins to experiment with the ideas, new and old, which led to the seminal 1867 paper *On a improvement in Boole's calculus of logic.*

The third volume, like the first, has very little that will be of interest to logicians. Thus, for example, only a handful of the ninety plus pages of Peirce's manuscripts collected and printed under the title *Toward a logic book, 1872-73* even make use of Peirce's innovations in logical notations and results in the algebra of relations, and these offer nothing of technical or historical interest; the work as a whole is simply philosophical. Of greater interest are the several short papers on linear and multilinear algebras, *Linear associative algebra: improvement in the classification of vids* (MS 227: between 3–7 April 1873) (pp. 161–163), *On the application of logical analysis to multiple algebra* of 1875 (pp. 177–179), which was included in H-W, volume 3, *Notes on the fundamentals of algebra* (MS 287: Winter–Spring 1876) (pp. 186–188), *Sketch of the theory of non-associative multiplication* (MS 294: Spring–Summer 1876), (pp. 198–201), and *Note on Grassmann's calculus of extension* of 1877 (pp. 238–239), which also may be found in H-W, volume 3. This work in linear and multilinear algebra, based in part on the work of his father Benjamin, opened the way to Charles Peirce's treatment, in the notes and appendices to the 1881 edition in the *American Journal of Mathematics* of his father's *Linear associative algebra*, of various systems of algebra as interpretations of matrix logic based upon the logic of relatives. Here we may find the conceptual basis for Whitehead's work on universal algebra, in which Boolean algebra and "symbolic algebra", that is, turn-of-the-century mathematical logic, are taken to be among the many symbolic systems whose axiomatic structures universal algebra studies and unifies. There are also two short papers on a problem for syllogistics, *Logical contraposition and conversion* (pp. 191–194), first published in the journal *Mind* in 1876, and the associated, hitherto unpublished manuscript *Addition to the note for Mind* (pp. 195–197), in which Peirce takes exception to the method for making the inference from 'All S is P' to 'No not-P is S'.

The fourth volume, the newest volume to appear, is, for the logician and the historian of logic, by far the richest and most rewarding in the series to appear to date. (It should be noted that a typographical error occurs in the copyright date given in this volume: "1986" should read "1989".) This volume covers the period from 1879 to 1884, one of Peirce's most active and innovative in logical research. It is during this period that he held his academic post at Johns Hopkins University and was able to interact with and influence

such brilliant students as Christine Ladd-Franklin, Fabian Franklin, Allan Marquand, B.I. Gilman, and Oscar H. Mitchell, each of whom made their own important contributions to logic, and with such colleagues as James Joseph Sylvester. It was in this period that Peirce came into contact with Schröder's work. It was also during this period that Peirce, borrowing an idea first presented by Mitchell, successfully developed his quantification theory for the algebra of relatives to which Schröder owed so much and discovered binary operations of the Peirce arrow and what was later to become the Sheffer stroke. During each semester of these five years, Peirce taught logic courses at Hopkins, including elementary and advanced, as well as special courses in the logic of relatives and on the history of medieval logic (Houser's *Introduction*, p. xli). It was also during this period that Peirce's reputation as one of the leading logicians of the day took concrete shape, John Venn and William Stanley Jevons among those who attested to the quality of his researches at that time.

This is also the period that has had the most long-lasting effect on Peirce's reputation as a logician and which was the most prolific for his publications. Peirce's voluminous output in logic is all the more remarkable in consideration of the fact that he was also at this time carrying out research in geodesics at his job with the U.S. Coast and Geodetic Survey's Bureau of Weights and Measures. Thus, this volume also includes Peirce's lengthy research reports in this area, such as, for example, his 1879 report *Measurements of gravity at initial stations in America and Europe* (PEP 4:79-144). These scientific researches provided the experimental data and techniques upon which Peirce tested and refined his theoretical work on probability theory. Thus the report on measurement of gravity was soon followed by a manuscript from the Winter-Spring 1880 on *A large number of repetitions of similar trials* (PEP 4:145-147).

Peirce's most important logic papers in this period appeared in the *American Journal of Mathematics* (AJM). The AJM was founded by Sylvester in 1878, and Sylvester was its editor during his American sojourn, which ended in 1884 with his return to England and a post at Oxford University. Peirce's first contribution to the AJM was *On the algebra of logic*, pp. 15-57 in the third volume of AJM, published in 1880 (PEP 4:163-208). Peirce's publications in AJM ended in 1885, after Sylvester's position as editor was assumed by Peirce's supposed friend Simon Newcomb. Peirce's paper *On the algebra of logic: a contribution to the philosophy of notation*, having been accepted for publication in AJM by Sylvester prior to relinquishing his editorship, appeared in AJM 7 (1885), 180-202. The second part of this paper was to have followed in the next issue and was submitted by Peirce, but Newcomb vetoed its publication, arguing that it should be rejected because it was a logic paper rather than a mathematics paper (according to Houser's introduction, p. xl). The loss was a significant one, marking the decline of Peirce's creative contributions to mainstream developments of contemporary logic, and coinciding with personal misfortunes that began in 1884 with his forced retirement from Johns Hopkins. Peirce's work from 1867 to 1885 was an expansion and improvement on Boole by the attempt (tentative

in 1867, successful in 1885) to introduce quantifiers into Boole's logic. In AJM 1885, we in fact find quantifiers and a fully developed first-order functional calculus. We know that Russell read both AJM papers, *On the algebra of logic* and *On the algebra of logic: a contribution to the philosophy of notation*, when he began his own work on the logic of relations, and took detailed, if not extensive, notes [ca. 1900-1901] on these two papers in preparation for his own work on the subject.

The loss of the teaching position at Johns Hopkins increased the growing financial difficulties which Peirce faced following his dismissal. The death of his father in 1880 and Sylvester's return to England in 1884 left Peirce without influential supporters and protectors. The loss of the teaching position at Johns Hopkins was in particular a blow to Peirce's creativity in logic inasmuch as Peirce had used his lectures to elaborate the ideas that would eventually comprise his published papers. Houser (*Introduction* to PEP 4, p. xliv) notes that Peirce's papers *On the logic of number* (AJM **4** (1881), 85-95; PEP 4: 299-309) and *On the algebra of logic* (AJM **3** (1880), 15-57; PEP 4: 163-208) correspond to the contents of Peirce's courses. The manuscript *On the algebraic principles of formal logic* (PEP 4: 21-37, Fall 1879) is presumably an early, albeit incomplete, sketch of *On the algebra of logic*, and it proposes a systematic presentation of the subject, far in advance of the work that Peirce produced in his 1867 paper *On an improvement in Boole's calculus of logic* and his 1870 manuscript paper *Description of a notation for the logic of relatives, resulting from an amplification of the conceptions of Boole's calculus of logic*. At the beginning of this fruitful period, Peirce's *On the algebra of logic* is important because it launches Peirce's sustained creation of the logic of relations as a complete deductive science which forms the basis not only for logic but also for universal algebra. In this paper, too, we find all of the apparatus required to develop lattice theory, although these elements were not unified into a systematic presentation by Peirce, and it appears that Peirce in fact failed to comprehend the concept of lattices as an independent mathematical entity at this early stage. Strictly speaking, what we actually have in Peirce's paper is the concept of the poset, with sup and inf. Nevertheless, this is the work that gave rise to and fed the controversy as to whether, as Peirce said, all lattices are distributive, and it is the source for Schröder's work in the field. In *On the logic of number*, Peirce gave an axiomatic presentation of arithmetic which Shields [*1981*] has shown to be equivalent to Dedekind's axiom system as presented in *Was sind und was sollen die Zahlen?* (1888) and Peano's as found in *Arithmetices principia, nova methodo exposita* (1889). In *On the logic of number*, Peirce used the notation of the logic of relatives to define a *finite set* as a set in which De Morgan's syllogism for transposed quantity is valid; it is a cardinal definition equivalent to the modern definition of a Dedekind finite set.

The first work in the fourth PEP volume is a review, published in the magazine *The Nation* in 1879, of Carveth Read's book *The Theory of Logic: An Essay* (PEP 4:1-5). In the year from the Spring of 1879 to the Spring of 1880 there follow several manuscripts devoted to philosophy and psychology of logic in addition to the *Algebraic principles*

paper, and then the *Algebra of logic* paper for AJM. The material in *The logic notebook* (PEP 4: 214-217), written on 6 November 1880 is comprised of miscellaneous thoughts spurred by "being unable at present to write a second paper on the Algebra of Logic" (p. 214), work that was interrupted by his father's death, during the period in which he concentrated his attention on preparing his father's work for publication, and it can be seen as notes towards the writing of his 1885 paper *On the algebra of logic.* It was also during this period, in the Winter of 1880-1881 that Peirce wrote the manuscripts *A boolian algebra with one constant* (pp. 218-221) in which are developed what have since become known as Peirce's arrow and the Sheffer stroke, and *The axioms of number* (pp. 222-224), which served as a prelude to the AJM paper *On the logic of number.* The paper *A Boolian algebra with one constant* had previously appeared as the first paper in volume 4 of H–W. Weiss recalled [Bernstein *1970*, 161] that Hartshorne had initially placed it in a pile of papers marked "not to be used" or "unpublishable". It was only because Weiss had come to work on the H–W edition and had already known about the Sheffer stroke that this paper was rescued and included in H–W.

The manuscripts *On associative algebras* (pp. 225-227), *Notes on associative multiple algebra* (pp. 228-233), and *Unequivocal division of finites* (pp. 233-237) were written during the Winter of 1880-1881 and are associated with Peirce's preparation of his father's *Linear associative algebra* for publication in AJM. The intent of these papers and of the notes and addenda to his father's *Linear associative algebra* that were published along with the paper in 1881 in volume 4 of AJM, was as Copi [Copilowish *1948*, 194] suggested, to introduce matrices "partly as an aid in his classification of relations, and partly for the sake of illustrations or examples." So Peirce had developed a matrix logic in terms of which all of the algebras presented by Benjamin Peirce (as well as those presented by Cayley and Sylvester) can readily be expressed as special cases. Thus, for example, as Houser (*Introduction*, PEP 4, lii) notes, Peirce, in his privately published paper *Brief description of the algebra of relatives* of 1882 (PEP 4:328-333) also argued that Sylvester's [*1884*] universal multiple algebra is just a special case or interpretation of his own logic of relatives, the published version of Sylvester's Johns Hopkins University lectures to which Peirce was responding in *Brief description....* The work trying to show that universal algebras are interpretations of matrix logic presented within the formalization of the calculus of relations was carried out in a series of papers on linear and multilinear algebras such as those included in PEP 3, as well as in the notes and appendices to his edition of his father's AJM 1881 paper. Indeed, much of the work of both Benjamin and Charles Peirce on linear algebras may well have been inspired and initiated specifically in order to show that the algebra of relations has mathematical applications outside of logic. Charles Peirce's former student Henry Taber also showed [*1890*, 353] that "every linear associative algebra has a relational form...." Some work in this direction was also undertaken by Whitehead in his [*1901*] AJM paper *Memoir on the algebra of symbolic logic.*

There are a number of other manuscripts included here that are important for tracing the development of Peirce's work on the algebra of relations, and contributing towards the important paper on *The logic of relatives* of 1883 (PEP 4:453-466), published as "Note B", [*1883*, 187-203] in *Studies in Logic*, which Peirce edited. This is the paper that first introduced quantifiers in our modern sense into the algebra of relations, with the existential and universal quantifiers defined respectively in terms of logical sums and products. It was this paper of Peirce's which [Tarski and Givant *1987*, xv] pointed to as the starting point for Tarski's work, beginning with Tarski's [*1941*] paper *On the calculus of relations*.[5] There are also manuscripts which include the application of the logic of relatives to analysis of syllogistic logic and various other, largely minor, pieces that will be of interest to logicians and especially to historians of logic. Among these research papers are included along the way a review of Jevons' *Studies in deductive logic* (pp. 238-239) first published in *The Nation* in 1881, which sets Jevons' textbook in both its historic setting as the product of advances over the earlier work of De Morgan and Boole and in its contemporary setting as a response to Sylvester's remark that pure logic could not be "infinitely developable" in the way that mathematics could, and Peirce's own starts at writing a logic textbook (e.g. pp. 400-401) and numerous sets of lecture notes for his logic courses (e.g. his Summer 1883 manuscript *Syllabus of sixty lectures on logic*, pp. 476-489). Of 80 pieces presented in 558 pages of this volume, some 50, presented in more than 300 pages, will be of interest to logicians and historians of logic, to philosophers of logic and mathematics, to algebraists, or to historians of mathematics, and almost all of the rest will be of interest to historians of science, especially to historians of physics.

Future volumes of the PEP edition will continue the chronological presentation of Peirce's output. We may imagine that many of the later volumes will continue to contain logical gems and important discoveries, but that these will becomes less frequent with each volume as Peirce turned his attention increasingly towards philosophy. This is far from saying that nothing of interest or importance will be found in future volumes. Quite the opposite. Thus, for example, in Peirce's paper *On the algebra of logic. Part II* (MS 507) dating from the Summer of 1884, we will find a suggestion for a system, based upon developments in O.H. Mitchell's paper in the *Studies in Logic* edited by Peirce and published in 1883, that had it been fully articulated, would have looked very much like Gentzen's 1934 system of natural deduction for classical logic; a suggestion of relativization of quantifiers is detectable in the *Studies in logical algebra* (MS 519) dating from May of 1885 and contemporaneous with Peirce's vol. 7 AJM paper of 1885 *On the algebra of logic: a contribution to the philosophy of notation*, where we can find something very much resembling what today we call *Skolem normal form*; in *The logic of relatives: qualitative and quantitative* (MS 532) dating from 1886 and scheduled for

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[5] Tarski and Givant incorrectly list this work as dating from 1882, possibly because its preface is dated December 1882.

publication in volume 5, we may look forward to Peirce's use of a finite version of the Löwenheim-Skolem theorem, accompanied by a proof which is similar to Löwenheim's. Clearly, the inconvenience of finding Peirce's logical writings scattered widely through PEP's large multi-volumed set will be offset by the strictly chronological presentation of the papers. The chronological rather than topical approach affords an opportunity not only to analyze Peirce's own achievements in logic and his intellectual development as a logician; it also provides a clearer picture of Peirce's attainments against the broader background of the entire development of logic and allows us to detect more precisely where, and in what ways, Peirce anticipated ideas that were to develop only much later and where those anticipations fell short.

The PEP edition is already an important scholarly achievement, not only for Peirce studies but for the history of science and especially for history of logic. For all the new and interesting material which the volumes that have already appeared have made available in published form and as integral texts for the first time, much more, even from the periods already covered by these volumes, remains in the archives. There are also a few points of editorial methodology which the Peirce Edition Project might usefully borrow from the Bertrand Russell Editorial Project (BREP) and the Gödel Edition Project (GEP) which would enhance the construction of the PEP edition. For an intellect such as Peirce who has written in so many diverse fields as mathematics (including logic), various branches of physics, scientific methodology, psychology, and philosophy, PEP might do well to borrow BREP's approach of dividing the work to be published into more than one series of books – the Russell project has divided the material into technical, including logic, philosophy of mathematics, philosophy of science, metaphysics, etc, and non-technical, including politics, philosophy of religion, ethics, and occasional pieces, etc., all presented chronologically – within the same over-all collection. If for reasons of the integrity of the critical edition this is not as feasible for Peirce's work as it is for Russell's – for example because there are philosophical papers that are of special value for the history of logic, or some logic papers that have a special significance for the development of pragmatism – then a useful remedy would be to select out from the completed edition several new series which present topics chronologically – perhaps, for example, a four or five or six volume set of Peirce's logical or logical and mathematical writings, presented chronologically.

The introductions to the four PEP volumes that have appeared to date are of varying degrees of usefulness for informing the reader about the background and circumstances of Peirce's life and intellectual activity at the time the pieces included in those volumes were written, and of varying degrees of usefulness in informing the reader of the relationship between the more important papers of the respective volumes and of the relative merits or achievements of those papers. In this respect, Houser's introduction to the fourth volume is the most useful. For the reader who does not know much about the context in which Peirce worked or the relationships among those papers, some of this particular deficiency could be remedied by adopting GEP's use – modeled on the practice in Jean van Heijenoort's

classic and influential anthology *From Frege to Gödel: a source book in mathematical logic, 1879-1931* – of introductory articles for papers or groups of closely related papers in which the historical importance and specific contributions of the papers are sketched and which elucidate some of the concepts which are found in, developed by, or used to present the results given in, those papers; or by adopting the BREP's use of headnotes to briefly describe the intellectual context in which papers were written and the particular circumstances of their creation. The historical and explanatory notes and the "Textual Apparatus" of the PEP edition only partially fulfill the purpose of the headnotes of the BREP edition, and fulfills even less of the purpose of the introductory articles in GEP's collection or van Heijenoort's anthology, which are rich in background material and deftly set the historical context for the papers presented, linking those papers with related and general developments in logic while elucidating the concepts being utilized or developed by the papers.

Whatever these shortcomings of the PEP edition, it is unlikely that they will be reversed. More than either the Russell or Gödel projects, the Peirce project must, because of its financial and administrative circumstances, rely increasingly upon external "Consulting Editors" who assist the project members when they can and to whatever extent they wish at any particular time, absorbing with varying degrees of commitment many of the duties that had formerly been the province of PEP's regular editorial board and which remain the province of the regular editorial board of BREP and of the Gödel project. Nevertheless, the PEP edition of Peirce's work is a clear and major improvement over the H-W edition, and not only for the reasons which were enumerated at the outset and because of the greater amount of material included in the present edition than can be found in the H-W edition. The true value of the PEP volumes for the historian of logic who may not have ready access to the Peirce archives or the time to sort through the archives and carry out the difficult chronological and hermeneutical studies that would be required to reconstruct the development of Peirce's work from the archival materials, is that it offers the historian a fuller and more accurate picture of Peirce's work than was previously available. In particular, we find that Peirce was often on the cutting edge of logical research through much of his career, that he was an original research logician who understood mathematics much more deeply than did some of his contemporaries, and that he anticipated much of the work that would follow only a generation or two later, and that in some cases he actually was the first to develop new techniques or ideas that would have to be rediscovered by successors who did not take the time or effort to study Peirce's work or did not have access to Peirce's work. The PEP edition is already establishing itself as an important research tool, and will prove itself to be indispensable to scholars wishing to understand the various facets of the fields in which Peirce worked and to which he contributed mightily. It will allow us to recover much "lost" mathematics and trace Peirce's development and establish the achievements in logic while giving a greater appreciation for his contributions to logic and their originality and helping us to understand

better the connections between his work in logic and his work in algebra and more generally understand the significance of his work.

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