ABDUCTION AND THE TOPOLOGY OF HUMAN COGNITION

Review of Ansgar Richter, *Der Begriff der Abduktion bei Charles S. Peirce*, Europäische Hochschulschriften 20, Vol. 453, Frankfurt a. M./ Berlin/New York/Bern/ Paris/Wien, Verlag Peter Lang, 1995. 209pp.

HELMUT PAPE

Philosophisches Seminar
Universität Hannover
Welfengarten 1
D-30167 Hannover, Germany

I. *Abduction: Logic, Methodology and the Dimensionality of Human Cognition.*
    What is the problem that the conception of a non-deductive and even non-inductive variety of hypothetical inference suggested by Charles S. Peirce (1839 – 1914) was to solve or at least to address? This, as Ansgar Richter's recent book *Der Begriff der Abduktion bei Charles S. Peirce* shows, is far from obvious, although the notion of abduction has found its heyday and rather wide-spread application in artificial intelligence and proliferation in recent approaches in literature theory and hermeneutics. At the very least, the problem of abduction can be given a historical or a systematical reading and both readings depend on what we identify as Peirce's conception of abduction. For mathematics, logic, and epistemology the systematic problem raised by abduction is the question whether, besides the formal standards for deductive and inductive logic, there is an irreducible type of abductive logic with its own, autonomous standards of formal validity. But what exactly is this logic of abduction dealing with? It seems that there are at least three options. Abduction can be seen as:

    1.) **a logic of discovery**: The logic of generating (discovering, creating) a new hypotheses,

2.) **a logic of hypothesis preference**: The logic of selecting (justifying, evaluating) a new hypothesis in comparison with other rival hypothesis, and, finally, as

3.) **a theory of experiential plausibility**: The theory that describes those criteria and conditions upon which a new hypothesis is accepted as a plausible account of an unexplained (surprising) experience.

The trouble with Peirce's conception of abduction is that he insisted on not distinguishing task (1.) and (2.). Even (3.), the experiential plausibility of abduction, is sometimes described as a part of a justification for (1.) and (2.) giving us e.g. the evolutionary conditions and the psychology of discovery that lend support to a logic of discovery or of hypothesis preference. Furthermore, only very late in his career, from 1901 onwards, he used the concept of abduction systematically but had nevertheless developed quite a number of very different notions of hypothetic inference at least since 1865. Richter shows that at different moments of his philosophical career Peirce claimed that abduction (hypothesis, retroduction, conjecture, guessing) is a non-necessary but in its own way valid form of inference that yields one, or a combination of the following results: (1) reduction of a number of predicates to one predicate, (2) introduction of a new idea that explains a surprising observation, (3) increase in the qualitative content of our knowledge, (4) conjectures which are justified only by predictions derived from them, (5) starting a self-correcting cycle of dedutive and inductive steps of an empirical inquiry, (6) assigning plausibility to a hypothesis by an instinctive reaction that is supported by a natural affinity of the human mind to truth.

In this situation some scholars have opted that Peirce's concept of hypothetical inference was simply confused and ill-conceived (e.g. L. Laudan) and could savely be ignored. Recently, in what seems to be the definite treatment of this topic, Tomis Kapitan [*1992*] has argued that there is no such thing as an autonomous logic of abduction because every step of the operations that belong to the tasks performed by (1.) a logic of discovery or (2.) a logic of hypothesis selection, can be accounted for by deductive or inductive steps. Kapitan [*1992*, 3] concludes that "'abductive' methods for generating and preferring hypotheses fail to be autonomous from either a logical and epistemological point of view". Rather, abduction is a practical inference whose only inductively or deductively irreducible component is not a logical operation at all. The introduction of a new hypothesis is a pragmatical or practical component that cannot be eliminated because it has to do with "the *conduct* of inquiry and, consequently, with procedures for evaluating inferences to practical directives." [Kapitan *1992*, 3] Ansgar Richter also denies that there is a logic of abduction or that other formal characterizations are possible. He

concludes his study with the remark that though Peirce does not give a definitive justification for abduction, Peirce's proposal of a type (3.) theory of abduction as an instinctive evaluative reaction towards a hypothesis as plausible might prove "the starting point of a comprehensive logic of discovery" (p. 180), if such an instinctive plausi-bility can be justified. But how can we justify an appeal to instinctive cognitive reactions of feelings of plausibility as a part of a logic of discovery? Is Richter's proposal equivalent to giving up the search for distinctive formal — logical and mathematical — features of hypothesis-generating cognitive processes? Even those who think that there is a "dynamic logic of abduction", like J. R. Josephson [*1994*, 4] who holds that "abduction is a distinct form of inference, interesting in its own right" and has programmed a number of abductive problem-solving expert systems which "work well and give objectively good answers, even in complicated cases where the evidence is ambiguous" [Josephson *1994*, 3], assign a vague and all-comprehensive scope to abduction. Josephson thinks that abduction is omnipresent in science and everyday life and has important general philo-sophical consequences. Among other things, abduction "leads to a form of Realism about the objects of theory and perception; it leads to the view that Truth is attainable but extremely high certainty is not; it extends the detailed conception of Reason to better accommodate fallibility and uncertainty; it loosens the bounds on what can be known" [Josephson *1994*, 1].

In this essay I will answer the conflicting claims about the subject matter and scope of abduction by advocating an approach that generalizes and neutralizes this conflict: When we discuss abduction, we cannot avoid raising the general question what the scope of formal logic is. A theory of abduction argues for a wider scope of formal logic in relation both to other formal theories and to conditions of application, and assigns a logical status to those representational mental states which Peirce calls "perceptual judgment". In fact, we will see that Peirce's concept of abduction developed as a consequence of his very comprehensive view of logic. Consequently, if we take a closer look at Richter's detailed account of the development of Peirce's ideas about hypothetical reasoning and add some of the logical, mathematical and ontological considerations which Richter did not take into account, we will arrive at a quite different result: Athough there is no complete and autonomous abductive logic with its own standards of inferential validity, abduction has a logically unique status of its own. Abduction exhibits a number of intriguing mathematical and formal pro-perties that are not adequately captured, if they are described as practical strategies for generating directives, as Kapitan wants to have it. The philo-sophical and logical problems raised by abduction go well beyond the three-fold alternative described above: Abduction addresses the problem of logical discontinuity and singularity in the dynamics of human cognition by showing that there are always non-formal contents that have to be integrated

into the smooth structure of logical relations. Abduction also has meta-
logical importance. By an abductive inference hypotheses are marked off as
singularities and separated from other deductively derivable sentences
because they are generated by phase transitions that connect entities
belonging to different ontological categories. That is to say, abductive
hypotheses are entry points into the space of logical relations whether
within a theory, or into a system of logical relation between linguistic
representations or between mental contents.

I talk about hypotheses as "singularities" and about "phase transitions"
and "the space of logical relations" because I am going to show in this
essay that Peirce developed in his post-1894 writings a topological way of
viewing logical relations between representations with categorically different
logical status: topological connectivity provided for him a comprehensive
model of a space in which the individual logical sequences, e.g. those started
by an abduction, could be embedded. (For a study that treats Peirce's whole
logic from the point of view of his topology, see Burch [*1991*].)
Consequently, he demanded that in a metaphysics based on logic "the first
inquiry concerning any general must be . . . what its dimensionality is"
[Peirce, 6.211, 1898].[1]

II. Developing Concepts of Hypothetical Inference: Peirce's Sequential Realism in Formal Logic and in his Theory of Cognition.

Only late in his life did Peirce become aware of what peculiar meta-
logical and topological properties an adequate account of hypothetical
inference has to deal with and the decisive experience was his formulation of
a logic-based evolutionary metaphysics from 1890-93. By 1898, when
Peirce planned to give a lecture series at Harvard on his evolutionary
metaphysics, the title "The Consequences of Mathematics" or "The Logic of
Events" reflects a change in the style of reasoning: To some extent the
biological, evolutionary, physiological and psychological concepts of the
1891 version of his evolutionary metaphysics were either replaced or
interpreted by logical and mathematical notions. In particular he applied and
developed topological concepts — adapted from the early topology by
Benedict Listing [*1862*] — to the cosmological development of an onto-
logical phase transition necessary to initiate cosmology evolution. This
ontological transformation is not only described as a hypothetic inference
but is described in the language of topological conditions for changes in and
between dimensions of shape. Peirce describes singularities by the
restriction in the type of movement in which a singular point or line can be
reached or transformed. He constructs his description for the layout and the

---

[1] The various editions of Peirce's writings are quoted in abbreviated format
that is spelled out in detail in the Peirce entry in the bibliography.

changes in the zero-state of cosmological development, which is a state of possible qualities, by drawing on topological notions. I will be arguing that for Peirce's notion of abduction the logical relation of singularities to qualities is crucial. Peirce tries to account for qualities in terms of one-dimensional continua with a hyperbolic shape. Of course, there may be singularities that break them, and so he has to explain when and how singularities occur. In doing so, he describes a tension between the consequences of his topological account and constraints that have to be added to the cosmological picture for external, non-topological reasons. This tension arrises, because "unless we suppose the continuum of possible quality to have topical singularities — which is quite inadmissible at the present stage of development, though they may be evolved later, — then we must admit that when a quality diminishes in intensity to *zero* it can then continue the same line of change and increase in intensity in a definite quality, a *contrary quality*" [NEM IV, 130]. Even worse, in this model the contrary quality would have its maximum exactly in the zero-point of its counterpart. But there is no evidence for such a strange connection between qualities. But why are we allowed to treat contrary qualities as disjoint and how do we get singularities after all while still using the topological model? What Peirce appeals to is the fact that we do indeed know a case where continua of qualities and singularities are connected and this connection creates a psychological discontinuity "because we know that sensation as a *limen* which is a point of discontinuity . . . . This singularity cannot exist in the possible quality itself . . ." [NEM IV, 131] But before we can explain and understand this suggestive move that defines the backdrop of Peirce's mature, topologically based concept of abduction in 1901 in more detail, let us take a look at some of the features and function of hypothetic inference and let us ask how he does account for the changes that took place.

      But even before we can deal with this suggestive move that defines the backdrop of Peirce's mature concept of abduction in 1901 in more detail, let us take a look at Richter's account of the other features of hypothetic inference and let us see how he accounts for the changes that took place. In the 209 pages of this study and in the 548 (!) footnotes — which sometimes are very long, substantial, and carefully worked out and should in some cases have been made part of the text — Ansgar Richter tries to give a detailed chronological account. His analysis and evaluation concentrate on the differences in detail between the varying approaches and developments towards a logic of hypothetic inference which have been designated by the term "Peirce's notion of abduction".[2] But he also describes

---

[2] Richter uses all the newly published texts and the translations now available in German which contain material unpublished in English. But he did not consult the microfilm edition of Peirce's manuscripts which has been available

an overall shift between the components of the approach: From an earlier formal logic and syllogistic theory of hypothesis to a methodological, psychological and anthropological account of the conditions of plausible guesses. The chrono-logical structure of the book is interspersed with several non-chronological, systematical surveys about pragmatism and abduction, on Peirce's analysis of Kepler's discovery of the ellipsoid form of the planetary movements as an abductive inference, on semiotics and abduction and on the relation between Peirce's economy of research as connected with his mathematical eco-nomical theories and his ideas about hypothesis selection. The best of these systematic sections is the one on Peirce's economy of research and his mathematical theories of economics which presents some new material; the weakest is the one on the semiotical classification of abduction.

Already in 1854 Peirce was convinced that reasoning about empirical matters places a constraint on knowledge in general and is autonomous: "man's truth is never absolute because the basis of Fact is hypothesis" [W 1, 7]. In the Harvard Lectures of 1865 and later Peirce claims that every judgement has the logical function of an explanation of something unknown presented in the phenomena. The Harvard Lectures introduce a distinction between inference apriori, aposteriori and induction. Typical for the entire early phase (until about 1870) is an account of the difference between the epistemological status of necessary or deductive and non-necessary, that is inductive or abductive reasoning by a permutation of premisses and con-clusion in the framework of a slightly extended syllogistic logic. Thus, in 1865, the *a posteriori* inference has the conclusion of the *a priori* inference as its minor premise; the minor premise of the aprioi inference is its con-clusion. The criterion of demarcation between the three types of inference is degree of certainty: Only apriori inference, that is deduction, is apodictively certain. For the Lowell Lectures of 1866, where for the first time prob-ability theory is applied to the logic of inductive reasoning, Richter shows that Peirce now distinguishes between the twelve modes of syllogistic reasoning which are all deductive and the inductive or hypothetical variations of their form which can be created by the permutation of premisses and conclusion which Peirce calls "rule", "case", "result". Richter spends three pages (pp. 33–35) listing the results of executing Peirce's scheme of the derivation of the twelve forms of hypothetical variants of syllogistic modes of inference which Peirce did not explicitly list. All in all, we would get a classification of 36 syllogistic forms such that for every deductive form there is a hypothetical and inductive one, generated by changing the order of rule, case and result. This is what Peirce calls the "triangular relation between deduction, induction and hypothesis" [W 1, 438] and remarks that the discovery of the true validity of

---

since 1960 and still contains about 70% of Peirce's writings. But Richter is well

induction and hypothesis is connected with this relation. This remark leaves Richter completely at a loss and he only comments on it by saying that Peirce did not develop it any further. But this is not all the case. In many places Peirce defended the thesis that the validity of arbitrary, non-monotonic reasoning stands under the following condition: The logic of induction and hypothesis complement each other in an iterative, self-corrective process of applying logic in scientific inferences such that deductive derivations establish truth-preserving connections between the temporally distinct stages of hypothesis generation and testing. The assumption that there are such logical and functional relations by which different types of inference complement each other is invariant in most of Peirce's different approaches to philosophy of science, mathematics and logic.

About 25 years later, another feature of Peirce's early treament of hypothetical reasoning, the realistic interpretation of logical form, will become constitutive for the program of his evolutionary metaphysics. For the evolutionary metaphysics is designed by interpreting the theorems and rules of logic as constitutive principles of being. In 1865 Peirce's logical realism consists in viewing especially aposteriori inferences as representations of a relation that exist in reality. Such a hypothetical inference infers an antecedence from a consequence that is presented to us in experience because it is determined by something without the mind. This realistic interpretation implies that the antecedence-consequence relation exists independently of its representation by the inference *in re*. Let us try to describe one of the overall general features of Peirce's logic which Richter's account misses: Later on, from 1866 onwards, this realistic view of the antecedence-consequence relation was transferred to the conditional. Furthermore, Peirce extended his realistic reading even to transitivity properties of all inferences and all true conditionals so that all logic can be developed as a generalization of the theory of the antecedence-consequence relation. This relation highlights the crucial feature of formal logic (and of his defintion of the sign-relation as well) because only this relation establishes a representation relation for which the transitivity property holds which connects us with the world. The transitivity of representation in traditional aristotelean logic for terms is described by the *nota notae* principle: if a first term is used as a predicate of a second term, then everything that is predi-cated of the second term is also predicated of the first term.

As A. N. Prior [*1976*, 127] has pointed out, Peirce generalizes the *nota notae* into a principle that characterizes the validity of all logical principles of inference by two features, universality and implicativeness. For all prin-ciples of logical inference it holds that "the conclusion follows from the premiss, because the premiss is only applicable to states of things to which the conclusion is applicable" [4.480n1, 1903]. This generalized and realisti-

cally interpreted version of inferential transitivity, sometimes in conjunction with the thesis that we know all empirical, experiential relations by knowing first consequences and than infer their (probable) antecendences, is what I will call Peirce's *sequentialist view of logical realism*, or *sequentialism*. This sequentialism maintains that the process of sign-transformation involves *some* real sequential cognitive process: ". . . all my own writings upon formal logic have been based on the belief that the concept of Sequence, alike in reasonings and in judgments . . . could in no wise be replaced by any composition of ideas. For in reasoning, at least, we first affirm, or, affirmatively judge, the conjugate of the premisses, the judgment of the conclusion has not yet been performed. There then follows a real movement in thought." [MS 300, 1908, 47*f.*]

Sequentialism is one of the logical ideas behind the invention of the philosophy of pragmatism in 1878. The logical operation of introducing a hypothetical antecedence from which an already known consequence can be inferred, is a feature of Peirce's anti-cartesian model of logic in his epistemology in 1868. In "Certain Consequences of Four Incapacities" two of the main theses are that all knowledge about mental states "is derived by hypothetical reasoning from our knowledge of external facts" and that "every cognition is determined logically by previous cognitions" [5.265] — which is a direct result of applying sequentialism across the board to all types of cognition. The invention of pragmatism in 1878 is only a further step in the direction of the generalization of the idea that the hypothetical adoption of an explaining antecedence is a logically independent step.

Another feature which Richter does not discuss sufficiently is that logical and epistemological aspects of hypothetical inference are crucial for a sequentialist theory of the logical status of mental qualitative contents and that this problem has stimulated the development Peirce's theory of ab- duction. In the metaphysics of mind we distinguish three categorical types of elements of experience in all dimensions of mental activity: Whatever else there is, there are always monadic qualities of feelings or qualia, dyadic sensation of reactions, namely perception and volition, and triadic general conceptions — mental habits manifest in concept, thoughts and reasonings — habits relating elements of experience to one another. Dyadic and triadic elements are relational and can be analysed in the sequentialist model. But on a first glance it may seem that the monadic states of consciousness, in metaphysics Peirce calls it "quale-consciouness", has no connection with it, because this aspect of consciousness is logically unrelated. It has "in itself, . . . no object and is attached to no object. It is a mere tone of con- sciousness" [7.530]. So in a way, the notion of quale-consciousness and of a hypothesis that cannot be inferred from another premiss, raise the same logical problem for the sequentialist approach: How are simple, non- relational states like the begin or end of a sequence, to be accounted for? Obviously, the sequentialist cannot allow that all logical sequences consist

of the same type of finite steps, because "if by an argument we mean an attempt to state a step in reasoning, then the simplest step in reasoning is incapable of being completely stated by any finite series of arugments" [7.536].

The concept of phenomenal qualities, qualia or the way in which things feel for us, e.g. what it is like to be in pain, even as used in contemporary philosophy of mind, originates from Peirce via C. I. Lewis and Nelson Goodmann. In contrast with the contemporary use and to that of Lewis and Goodmann, Peirce develops an account of the logical and ontological status of qualia that denies that we know or have introspective access to qualia as immediate contents of consciousness: "We have no power of Introspection, but all knowledge of the internal world is derived by hypothetical reasoning from our knowledge of external facts" [5.265, 1868]. The thesis that we cannot access qualia as they are in themselves entails that they cannot be experienced immediately. That is, we only know qualia by forming abductive hypotheses.

The inaccessability of the immediate character of qualia is of logical and ontological importance. The quale-consciousness is never an individualized or concrete content and for this reason, we experience qualia by experiencing a represention to a possible range of that a monadic aspect of mental states may have. (We will see towards the end of this essay that in Peirce's metaphysical account a quality and its representations may be states of one continuum.) But besides this relation between indeterminate grades of quality no intuition into a more immediate quality of our experience exists: The denial of an ability of immediate introspection and the definition of cognition as "determined logically by previous cognitions", is one Peirce's main theses from 1868 onwards [5.265]. But if the sequentialist model is to succeed as a comprehensive model of cognition and consciousness, how does it account for a cognition that introduces a new quale of a perceived state of affairs into the sequence of logically determined cognitions? From Peirce's representational conception of qualia, it follows that, if a quality of feeling is not immediately grasped, it is represented. Qualities are the determinable and representable aspects of what Peirce calls a thought-sign or, an idea. So when Peirce sometimes talks about directly known "feeling" or "quality of feeling", he should be read to mean a mental represention or "idea" of a range of possible qualitative determinations.

Beside the three categorical types of elements, sequentialism gives us a law of relating mental entities. Peirce develops this in the form of a associationist — or what we would today rather call a "connectionist" — law of mind as a rule for the dynamical relations of ideas in time. In 1892 he gives it the following formulation: "Logical analysis applied to mental phenomena shows that there is but one law of mind, namely, that ideas tend to spread continuously and to affect certain others which stand to them in a peculiar relation of affectibility. In this spreading they lose intensity, and especially the power of affecting others, but gain generality and become

welded with other ideas." [6.104, 1892] This law of mind is an indeterministic principle of mental development that describes a probability increase for a specific connections between ideas as a function of the pre-existing structural layout, frequence and time of activation. What is important about this associative law of mental action is that what a sequence of related activations of an idea results in is a general pattern or habit of affectibility and behavioral reactions. For short, every idea is experienced by the associated inner or outer reactions it leads to. Therefore, what we can know about feelings and ideas, are habits or habitual connections, or individual instances of such connections.

Therefore, whenever we know or are conscious of an idea, we are conscious of a habit or its individual instance. (Of course, we will need general conceptions to represent habits and more specifically, individualized conceptions or ideas to represent individual experiences of activating a habit, but we will ignore this complication for the time being.) The tension to which abduction is to provide an answer is that it is not clear that ideas (thought-signs) represent *qualia*, as Peirce assumes in his concept of quale-consciousness and, at the time, be general conceptions or habits. For, if ideas are habits and if habits are connections — "mental commissures" [6.301, 1893] — between parts of the brain represented by ideas, then ideas exist only as connections between ideas. It follows from this connectionist analysis of mental activity that ideas can be known only as relations to other ideas. A quality of feeling was described by Peirce as something which is *in itself* without any relation to anything else and It follows that no quality of feeling *in itself* is similar to any other quality. Consequently, no idea — in sofar as it is a mental habit — can represent a quality directly.

But if there is no directly represented resemblance, in what does the similarity between ideas of qualities consists in? Peirce does not want to deny that, e.g. my sensation of red today is like my sensation of red yesterday. Holding on to the denial of intuition, Peirce accounts for likeness without treating it as a relation between the immediately intuited qualities of different experiential states. In his sequentialist approach to the mind similarity can only consist in a structural property of the mental system, "in the physiological force behind consciousness which leads me to say I recognize the same [sensation of red] as the former one, and so does not consist in community of sensation" (both quotes [5.289n1]). Does it follow from this, as e.g. G. Lynn Stephens says, that qualia are in Peirce's philosophy "a reality whose ultimate nature thought cannot express: noumena where one would least expect to find them, in the content of consciousness itself"? [Stephens *1985*, 107]. If Stephens' analysis were correct, the noumenality of the qualities of feeling would imply that abduction and Peirce's sequentialist approach to the philosophy of mind would turn the qualitative content of consciousness into some unknowable flux of *noumena*. But Stephens' interpretation does not consider an option that may

make qualia knowable just because of their singular logical and epistemo-
logical status: It is not the *immediate* knowledge of qualia that secures their
reality and our acquaintance with them. A topologically based mental onto-
logy may distinguish elements of consciouness by their logical and topo-
logical properties too. Even direct knowledge of qualia does not become
impossible because one rather dubious traditional epistemological require-
ment, namely an immediacy of knowledge, is shown to be futile. This sort
of traditonal immediacy would have to fall out of the scope of all kinds of
representations and would be different even from the representation of a
quality e.g. the color red by an iconic sign, let us say, a sheet of paper
showing a sample of red. Instead, the feature of immediacy simply shifts to
sequences: *What consciousness consists of immediately are relations
between its elements.* The noumenality of qualities of feeling is limited to
their immediady. Our knowledge about qualia and their logical function can
be explained because qualia have:

    a) modal properties, that is, the degree of reality which they appropriate
is that they *constitute a range of real possiblities underlying the
representational function of all mental phenomena;*

and,

    b) topological properties, that is, in the mental realm they function as
singularities that are realized in a range possibilities of lower dimensionality
than any other mental element; such that, topologically speaking, *qualia
can be represented only adequately as unidimensional continua which
allow for an open, indeterminate number of determinations.*

Why can these two sorts of formal properties not be ignored by epistemo-
logical considerations justifying an abductive logic of discovery? Peirce's
several and widely different suggestions for a logic of hypothetical reasoning
are meant to capture the insight that in every realistic account of the
structure of human reasoning there have to be premisses, in particular
perceptual judgments, that one has to introduce a reference to qualia charac-
terized by these two properties. In empirical matters every abduction
introduces in an abductive argument some "idea". That is to say, the model
is an empirical assertion, an abduction that suggests a predicate representing
some qualia which, for some sensory processes, plays an irreducible role in
a perceptual judgment. An assertion expressing that a certain predicate refers
to a perceptual judgment is, by the same token, a mental event of repre-
sentation. In this way it occupies a unique position in the dynamics of
human cognition and in its relation to all ensuing argumentation: To
introduce into a context of theoretical, that is, logical relations, a premiss,
or, in empirical matters, a hypothesis taken from perception, adds a new

element to the space of empirical reasoning. In this way an interpreted perceptual judgment and an abduction assume a particular logical status as entry points or end points for verificational and experimental processes. Beyond the answer, "I saw that the rose was dark-red and you may as well come with me and take a look", there is no room for doubt after asking somebody, "Why do you think the rose is dark-red?". The queer logical singularity of an abductive hypothesis must be preserved to allow for a relation between different ontological types of representations being connected to one another. Granted that all reasoning has to start (and to end) somewhere, it is by no means arbitrary but a genuine problem for a logic of abduction which type of mental representations are appropriate to execute the start-up (or the stop) function. In this way the logic of abduction is part of a logical account of the verificational, experimental or observational steps of the epistemological process of inquiry.

III. The Logical Link: A Sequentialist View of Mental Process Connects Pragmatism and Abduction.

In producing hypotheses and in checking their deductive consequences, we represent by inferential relations between cognitions, the meaning of the antecedence-consequence relations which determine our conception of the object to which our perceptions and actions have to relate. Whereas the early formulation in 1878 emphasizes "sensible effects" and "practical bearings", a late formulation of the pragmatic maxim from 1905 stresses that the hypothetical acceptance of consequences performs a logical role for the clarification of theoretical concepts and statements: "In order to ascertain the meaning of an intellectual conception one should consider what practical consequences might conceivably result by necessity from the truth of that conception; and the sum of these consequences will constitute the entire meaning of the conception." [5.9]

Richter (pp. 61–72 and 134–145) acknowledges only a vague and "undeveloped" — his most frequent term to characterize Peirce's various claims and arguments about hypothetical inference — link between hypothetical inference and pragmatism in its early and late stages. He points out that in the 1878 paper "Deduction, Induction and Hypothesis" [2.619–644] the early pragmatism connects the logical, psychological and methodological accounts of hypothetical inference. But he does not see what was described above as the sequentialist view that constitutes the framework for the development of *early and late pragmatism*. It is in the realistic framework of the sequentialist approach that the inferential process and its transitivity properties justify Peirce's claim that hypothetic inference must have an independent status. Accordingly, when Peirce in 1903, in his "Lectures on Pragmatism" states that "the maxim of Pragmatism . . . covers the entire logic of abduction" [5.196] and describes the pragmatic

maxim as a rule for the selection of hypotheses, even claiming "that this is *all* that the maxim of pragmatism really pretends to do, at least so far as it is confined to logic" [5.196], Richter is not able to accept this claim and rejects most of the other general claims by which Peirce connects abductive logic and methodology with pragmatism and his epistemology.

In view of the fact that the "Lectures on Pragmatism" present the most extensive discussion of abduction and is representative for Peirce's mature conception of abduction, I will at this point discuss Richter's criticism in more detail. Richter objects as I will show mistakenly that:

(1.) there is no "systematic justification" for Peirce's "basic belief" (p. 141) that in abductive generation and selection of a hypothesis "man has a certain Insight, not strong enough to be oftener right than wrong, but strong enough not to be overwhelmingly more often wrong than right, into . . . the general elements, of Nature. An Insight, I call it, because it is to be referred to the same general class of operations to which Perceptive Judgments belong" [5.173].

(2.) the 1903 lectures develop a concept of abduction and pragmatism that deviates radically from the early approaches because abduction is de-scribed as an instinctive, spontaneous guess, stressing its similarity to perceptual judgments and assigning to pragmatic maxim the role of con-trolling the verifiablility of deducible predictions while largely ignoring problems of its logical form, reducing abduction to a sort of reductive inference of the type "P is an observation. If O were true, P would also be true. Therefore, it is reasonable to guess that O is true" [*cf.* 5.189].

More particularly, Richter argues for (2.) by claiming that the early and late concepts of abduction and pragmatism cannot be compared because there is a major break in the subject matter of the two versions of the pragmatic maxim: Whereas the 1878 maxim was designed to clarify "conceptions", the 1903 is supposed to evaluate "hypotheses", that is sentences. Let us take up the much more general second objection first: Is there a change in the subject matter of Peirce's treatment of pragmatism and abduction between 1878 and 1903? Without doubt there are a number of important changes. But does, as Richter assumes in his argument, the difference between concept and sentence actually invalidate Peirce's 1903 claim that in interpreting the pragmatic maxim as a rule for the selection of hypotheses he only explicates his 1878 maxim? If it would, then already in 1878 Peirce would have committed this equivocation of concepts and sentences. The paper which contains the pragmatic maxim is titled "How to Make Our Ideas Clear" and it continues and refers to the first of this series titled "The Fixation of *Belief*". Indeed many of the examples and the argumentation of

the second paper too, deal with and concern beliefs, that is to say, sentences. So much worse for Peirce, you might say, he simply was confused.

But what is the reason that justifies the assumption that the logical properties of concepts have to be different in all relevant respects from the logical properties of sentences? Note, e.g. that the first complete development of first-order predicate logic by Gottlob Frege was titled *"Begriffs-schrift"*, that is *"concept*-language". But it is not (the rather weak) historical argument that the notion of concept was used quite differently and am-biguously around the turn of the century and that already in 1878 Peirce was discussing a *method for evaluating beliefs* which speaks most decisively against Richter's objection. Rather, it is the fact that Peirce's sequentialism in logic (remember that this always includes semiotics) justifies him to treat concepts and sentences as different, but sufficiently similar incomplete steps in the logical process. Already in 1868 Peirce points out that "at no one instant in my state of mind is there cognition or representation, but in the relation of my states of mind at different instants there is" [5.289]. The reason is that neither the concept nor the sentence represents the logical relation of sequence which is essential for complete representations but an argument will do so. If we run through the steps of an argument, we have to perform a *sequence of inferential steps* to arrive at the conclusion. From the point of view of sequentialistic reading of logical relations, an argument cannot be built up out of individual propositions, and a proposition cannot be built up out of individual concepts. For "the smallest constituent of a propositions is a proposition. Just as it is strictly correct to say that nobody is ever in an exact position . . . , but positions are either vaguely described states of motion of small range, or else . . . are *entia rationis* . . . invented for the purposes of clear descriptions of states of motion; so like likewise thought (I am not talking Psychology, but Logic, or the essence of Semiotics) cannot . . . be at rest, or be anything but inferential process; and propositions are either roughly described states of thought-motion, or are artifical creations intended to render the description of thought-motion possible" [MS 295, 1905, 102–103]. Whatever the truth and ultimate justification of the sequentialist approach to logic might be, it is obvious that Peirce is only arguing in agreement with his own sequentialist understanding of logical form when he neglects the difference between concepts and arguments. Therefore, Richter's objection misses a crucial point, because the sequentialist conception of logic remains, in its charac-teristic features, the same in early and late pragmatism. Surely, on the surface level, the views about the importance of logical, especially syllo-gistic forms and of the logical status of perceptual judgment has changed dramatically. What happened in the years from 1878 to 1902 that may have caused this change? Is it true that there are no logical reasons that may account for it?

This brings us back to Richter's first more general objection that Peirce gives no systematic justification for his basic belief or assumption that in the abductive generation and selection of a hypothesis we human beings show a faculty of insight into general features of reality. Richter (p. 141) observes correctly that this basic belief entails an anthropological thesis: Evolution and nature have brought it about that we have this faculty of guessing at our disposal as a part of our natural constitution. Now if we disregard all the empirical interpretations of this anthropological fact that might be given, this is, first of all, a metaphysical thesis. Richter is reversing the systematic order of Peirce's philosophy and gets completely lost when he suggests (p. 141, *fn.* 421) that the thesis about abductive cognitive abilities could only be systematically justified in the framework of Peirce's theory of categories, his synechism (i.e. that the continuous connections have an ontological priority in all universes of discourse), and even his "religious beliefs". If we would need theology or personal beliefs in God to explain a logic or methodological concept of abduction, this would surely be the end of every modern logical account of knowledge, Peirce's logical and pragmatic account included.

Richter attitude discourages any further study of any possible connection by assuming that there is no possible formal, mathematical or logical framework for metaphysics. To exclude the possibility of a formal explanation is against the grain of Peirce's notion of philosophical method. It contradicts his view of a systematical hierarchy between disciplines which relies on the fact that the more specific disciplines depends upon the more general disciplines for their principles and laws. For Peirce, mathematics and logic have a much wider range of objects and therefore a more general status than metaphysics because metaphysics is concerned with the constitution of reality accessible to experience, taken in general. Logic, being concerned with the formal properties of representation, is less general than mathematics but more general than metaphysics. Therefore, Peirce requires that "metaphysics consists in the results of the absolute acceptance of logical principles . . . as truths of being" [1.487, 1896]. So we have good reason to suppose that there are logical and, maybe, even mathematical reasons which justify the new 1903 approach to abduction and to pragmatism.

Indeed, there Peirce gives even formal arguments, relying on his logic of relatives, to justify our abductive talent of having insight into the general properties of nature. For example, he argues in the 1903 "Lectures on Pragmatism" for that element of this thesis which says that the abductive faculty to understand the general features of reality is a possible result of perception. In the VI. lecture he shows that we must have the ability to perceive directly some of the general relational properties of the sequential ordering that holds between perceived events. For example, we are able to perceive the property of transitivity of sequence for a general class of events

for which it holds that "whatever is subsequent to C is subsequent to anything, A, to which C is subsequent — which is a universal proposition" [5.157].

By now the reader of this essay will not be surprised to see that Peirce resorts to the notion of real sequence of perceived events to explain how we are able to grasp the general elements in perception. Again, Richter does not notice this argument. This observation highlights an important omission in Richter's study which, on the one hand, draws a very rich, careful and detailed historical account: Richter's study suffers from a lack of an effort to reconstruct the systematic connections which link Peirce's diverse attempts. Richter does not look for the unity that underlies Peirce's varying proposals in the search for a theory of hypothetical inference but stresses differences and contradictions. For example, he does not describe the general conception of logic which Peirce works with and of which the sequentialist realism is only the most pertinent feature for any discussion of abduction, just one of several general and basic feature of Peirce's logic. To name some other important omissions: Peirce's emphasis that all logical processes have dia-logical structure leads him to define logical operators by dialogical rules that describe decidable proof-strategies. In this way Peirce anticipated an account that introduces all logical operations (negation, conjunction, disjunction, conditionalization) of first order logic and thereby supplies an alternative constructive foundation for classical logic. This was done around 1960 by Paul Lorenzen and Kuno Lorenz, and is summarized in Paul Lorenzen and Kuno Lorenz [*1978*]. A second ignored area is Peirce's algebraic logic of relations. Hans G. Herzberger showed in a paper [*1981*] in a standard logical framework adapted from Quine that Peirce's claim that there are at least and at most three irreducibile categories of relations, if we allow only one algebraic type of operation on relations, the relative product. This operation defines a concept C as the product of two other concepts D and E in such a way that it identifies "the subject of one concept with a subject of the other" [1.294, 1902]. The algebraic type is always $(X + Y) - 2$. Furthermore, Richter does not take into account, not even in chapter XVI on semiotics and abduction, that for Peirce logic comprises three disciplines, listed in the order of their generality. First, there is semiotics as a theory of possible forms; secondly, there is a formal and critical logical theory of argumenta-tion; and, thirdly, a semantical and methodologicaly account of abduction, deduction and induction as parts of a dynamical, self-corrective process as methods operating in science and everyday life. The place of abduction and hypothetic inference in Peirce's multi-layered conception of logic can only be explicated in the framework of logical realism, of which his sequentialist realism is a part, and if one does justice to the way in which dialogical, for-mal and non-formal layers of argumentation may connect logical, methodo-logical and semiotical theories.

IV. Qualia Regained: The Topology of the Mind and the Representation of Qualitative Possibilties.

It is time to describe more precisely what the independent logical role of abduction is and to substantiate my thesis from the beginning of this essay that this independent logical role of abduction depends on the topological property of hypotheses as singularities in the space of logical relations. This raises a further question. We also have to explain why hypotheses as points of logical discontinuity can or should be treated as instinctive reactions, guesses or perceptual judgments based on an anthropological faculty to know some of the basic properties of reality.

To find some answers, let us first take a look at what happened to Peirce's logic and metaphysics between 1878 and 1903. During this interval Peirce discovered Listing's topology and Cantor's transfinite sets. Around 1895 he developed a two-dimensional graphical logic, the so-called "Existential Graphs". On the development of this graphical logic Peirce spent most of his energy generating a series of logical systems that comprise a complete first order logic, some weak modal logics and with some semantical models similar to possible world semantics. Later on, in 1905, he uses Listing's classification of topological forms and transfomations for completeness proofs for his graphical logic. But before this he developed an evolutionary metaphysics in the years 1884-1893 trying to explain the origin of the physical universe and the laws of physics as a transformation that changes the ontological modality and the dimensionality of a state of unrestricted indetermination, which Peirce described as being less than nothing. Richter mentions these developments only in passing, if at all. When, in 1898, in notes for a lecture series titled "The Logic of Events" he applied Listing's topological framework to the "objective logic" of cosmological development, Peirce assumes that there is a structural similarity between the topological status

I.) of a hypothesis as long as it is considered as inferred by an abductive inference,

and,

II.) of every more determinate state that results from the zero-state before the origin of the physical universe.

First of all, both the abductive hypothesis and the more determinate state are phase transitions from states where the transformation changes the ontological modality and the dimensionality from a 0- to a 1-dimensionality. To put it in more traditional, though not quite adequate philosophical terms, both transformations lead from the potential to the actual. To describe this transformation in a logic of cosmological events the traditional logic of

monotonic, deductive necessary inferences was neither possible nor adequate because it would have required giving the possibility of a logical connection between the state of unrestricted indeterminacy and any more determinate state. So even if "that the whole universe and every feature of it must be regarded as rational", we do have to conclude "that it is *constrained* to be as it is by the logic of events; for the logic of evolution . . . need not be supposed to be of that wooden kind that absolutely constrains a given conclusion. The logic may be that of inductive or hypothetic inference." [6.218, 1898] Although from the "boundless Nothing" of unrestricted indetermination nothing *necessarily* results, it is an unstable state that will allow for some *possible* transformation. Now the first, and for this reason most probable result, is the weakest determination possible by the introduction of what Peirce calls "*unit* of some quality". This unit, and we will see later on why it has to have this form, he described as the one-dimensional continuum of qualities. The phase-transition from a state of absolute indeterminacy to a state where minimal constraints for possible states hold, took place in such a way that "unbounded potentiality became potentiality of this or that sort — that is, of some *quality*. This was hypothetic inference. Its form was:

> Something is possible,
> Red is something;
> ∴ Red is possible. [6.220]".

This describes a process of generating a unit state (or hypothesis) that introduces a restricted specific possiblity for all further transformation (logical consequences).

This account leads to many open questions and problems which are the reason why Richter rejects the more systematic 1903 pragmatic of abduction, rejects the possibility of a logical and mathematical justification of abduction and prefers Peirce's 1908-1910 conception of free-floating plausibility to which some beliefs tend. I will concentrate on two important ones: (a) Does the topological and logical analysis in his cosmology of the origin of the universe lead Peirce to a view of hypothetical inference as a *topological* transformation, and to what extent can we directly link the topological phase-transition into units of quality on a cosmological scale to the predicate in an abductive hypothesis, guess or perceptual judgment that asserts that such a represented quality holds for some object? (b) Why is it that qualities introduced in an abductive hypothesis and the qualities in the cosmology have to have the form of one-dimensional continua?

Now (a) is the question whether Peirce not only described cosmological development as a topological transformation that has the form of an hypothetic inference which is evidenced by the quote from 6.220 given above, but also whether the topological view was applied directly and explicitly to

abduction. The second half of the question concerns the closeness of the connection between the phase-transition into qualities in evolutionary metaphysics and in logic. The second question (b) concerns the form of qualities and we will take a short look into another manuscript from the "logic of events" series to find an answer for it.

Obviously, if the second half of question (a) points to a possibile way of connection between logic and metaphysics which may be read to imply a vicious circle: If the thesis of the evolutionary metaphysics that qualities are products of a cosmological development of "objective premises" is taken to justify the validity of abduction, the concept of abduction involves a contradiction. In this case Peirce would explain why qualities may occur as the first discrete ontological units at cosmological origin of the universe, presupposing its logical validity while at the same the logical validity depends on this cosmological role. But Peirce claims at no place in his late writings that abduction is justified just because qualities have an objective place in the evolution of the universe. Sometimes, in his theologico-metaphysical moments, he compares the universe with a vast argument of God and asks for the role of qualities "in the economy of the universe" [5.119, *1903*]. But when he describes qualities as "uncaused factors" which may be compared to premises for us, namely the iconic predicates in perceptual judgments, he stresses that is *our* way of imagining them and that "what is first for us is not first in nature" [*Ibid.*].

It needs to be stressed that it is by no means a vicious circle to claim, as Peirce actually does, that qualities have an independent ontological status as possibilities. Qualities are those elements of reality which are real and, at the same time, some of them are the constitutive elements of con-sciousness, namely of "quale-consciousness", or the way something feels for us. That qualities are real possiblities and possible contents of experience, is the basis for the applicability of formal theories and links perception with the "ideal world" of mathematical forms: "For I hold that potentialities have a being, though they are not actual; . . . I maintain that we directly contem-plate this ideal world and when we open our eyes we perceive in the world about us that which corresponds to the freedom of the ideal world. It is true that reflection is required to enable us to recognize it. But that reflection *recognizes* it, and assures us that we *saw* it from the very first impression of sense" [NEM IV, 144–145]. But Peirce does not think that consciousness is made up only by this qualitative element and that all qualities actually are consciousness in a full-fledged sense but rather he holds that a quality is a possibility and is therefore always a "potential consciousness". And when he says that his idealism is the thesis that only phenomena exist, he adds "and all that logically follows from experience by Deduction, Induction, and Hypothesis" [NEM IV, 144].

Peirce described abduction in such a way that every such hypothetical inference has many features which no deductive or inductive logical

operation can have. But a topological transformation is a kind of operation which supposes a more much general, and not deductive type of relation between the states connected by it, so it makes sense to say about the results of such a topological transformation that its "chief elements are its groundlessness, its ubiquity, and its trustworthiness" [MS 692, 1901] which *cannot be captured* by any other logical process, and that "like a flash" [5.181] it introduces a new element by bridging a gap between states of different logical status. That Peirce thinks that such a logical operation represents a connection between entities that belong to different ontological categories and which has to be established by a singular act in order to arrive at an abductive hypothesis, is evident from many passages. For example, he stresses the radical difference in character between the abductive hypothesis and the perceptual content of a sensory process it represents: ". . . I see an azalea in full bloom. No, no! I do not see that; though that is the only way I can describe what I see. *That* is a proposition, a sentence, a fact; but what I perceive is not proposition, sentence, fact, but only an image, which I make intelligible in part by means of a statement of fact. The statement is abstract; but what I see is concrete. I perform an abduction when I so much as express in a sentence anything I see." [MS 692, 1901] The abstract versus the concrete; the qualitative image as a mental event in the perceptual system of a person versus the sentence as a linguistic representation belong to different ontological categories. However, any individual act of linguistic representation or thought may easily establish some such trans-categorial connection with a mental event in the perceptual system. How is this fact to be explained? We need a general account that shows how the probability for the right connection is restricted in such a way that individual acts of cognitive activity will have some probablity to establish the correct abductive link. For every individual cognitive act of abduction can only be a "forceful connection", a dyadic relation between entities of different categories. It is true, as Peirce insists, that it will suffice if these abductive acts are less often right than wrong. The assumption of a general adjustment, or affinity brought by evolution, between our ability to form abductive hypotheses or to experience qualitative perceptual processes, like seeing the image of an azalea, would not be circular because this *general* explanation cannot be transferred to single cases. So when Peirce claims that "science would be impossible if man did not possess a tendency to conjecture rightly" [7.679, 1903] and thinks that therefore there has to be "some original connection between human ideas, and the events that the future was destined to unfold" [7.680, *1903*] this is the thesis that there is a higher general probability for earthborn man to guess correctly than for some alien intelligence or some computer.

Let us turn now to question (b), why qualities have to be introduced in the form of one-dimensional continua to perform their cosmological task of determining the concrete form of "units" at the origin of the universe. As

Richter notes, even as early as 1866 in the Lowell Lectures, Peirce describes
hypothesis as an inference which increases the information about a state of
affairs by representing qualities as the form of a thing, "an image of the true
qualities of a thing" [W 1, 485]. Now a variant of this idea, namely the
more general assumption that qualities are the only immediate repre-
sentations of formal structure, supplemented by the sequential realist
assumption leads to a concept of quality as a range of possible deter-
mination. Let us trace out Peirce's arguments. We saw that Peirce intro-
duced qualities by hypothetical inference as the first unities of determination
which have a distinct dimension: If something is possible, than some
quality is possible. Now the lowest degree of dimensionality which allows
for sequential logical relations is a unidimensional continuum. But the
hypothetical inferred quality would remain a logical singularity, if it does
not allow logical sequences between qualities. Now a unidimensional
continuum allows for a sequence of degrees of quality, and if this sequence
is continuous, the potentiality is preserved as a range of variable degrees of
qualities. However, the form of a unidimensional continuum of a quality is
already an inductive generalization. Therefore, in the cosmological ontology
of the origin of the universe ". . . the bare Nothing of Possiblity logically
leads to continuity" only because "induction arranges possible experience
after the type of a logical sequence. Hence, the first dimension of the
continuum of quality is a sequence. . . . The Logical sequence has . . . been
considered as a *tree*. . . . The ultimate antecedent is a *zero* without
extension, the ultimate consequent is a vast manifold. Hence the continuum
of possible quality in N dimensions must be in a sequence starting from a
point and expanding to a final limit of $N - 1$ dimensions. Logic radiates
like light." [NEM IV, 127–128] We cannot discuss here the topological
arguments that induce Peirce to describe the unidimensional continua of
qualities as having an absolute maximum and two regions which are just
alike, such that "these two regions thus mirroring one another are in their
infinite maxima identical." [*Ibid*, 133] What is important for us is that
these two regions of the continuum are the quality and the representation of
quality in consciousness: "The quality . . . and the quale-consciousness,
which *feels* that quality are now two, because the quality, being generalized,
and continuity we remember is generality, is capable of entering different
concsciousnesses. . . . The intensity of quality is one thing, the intensity of
the feeling that represents it is another" [*Ibid*.]. Just like the singularity at
the *zero*-point, an infinite degree of intensity of a quality or its
representation "is at a point of discontinuity" [*Ibid*.]. This explains, why
representation and quality may have identical maxima. So the answer to our
second question (b) is: Qualities in Peirce's evolutionary cosmology and in
his theory of abduction are given the form of unidimensional continua
because a sequence of such a form connects qualities and their repre-
sentations. The form of the unidimensional continuity results from the

logical requirement that there has to be a direct connection between (i) qualitative content and (ii) the inductive distribution of possible determinations of qualitative contents. (Remember that a quality is not only itself a unit of determination but allows for an unlimited number of possible determinations.) Every singularity introduced into the space of logical relations by a new abductive hypothesis is, by the dynamics of the logical process, turned into a part of some unidimensional continuum or other. Its temporary logical status as a topological singularity has its only justification in the unattainable limit of all logical sequences — "the very reality itelf" [*Ibid.*, 134]. But reality is an unattainable limit of our inquiry because it is the only possible last consequent of all logical sequences which is the only "true singularity of the logical continuum" [*Ibid.*] we can logically admit. To this singularity all abductions refer as long as they have the status of logical discontinuities which they all have to lose by being interpreted by all the logical sequences to which they belong. For this reason, it holds true for all points in time that what we can produce as hypotheses are only arbitrary and temporary singular abductions that future logical processes of interpretations will have to understand as structuring some qualitative continua. Still, their temporary status as singularities, the surprise and insistence of the newly coined hypothesis, reminds us of the unattainable limit every hypothesis aims at.

Peirce's insistence that forms are captured adequately only in the iconic or diagrammatic format, that all logical and argumentative sequences structure a realm of possible quality is at the heart of his philosophy of mathematics and its epistemological and especially abductive dynamics in its theoretical development. In a letter to William James from Dec. 25, 1909, Peirce tried to make his friend aware that the usefulness of the sciences which use formal reasoning consists just in discovering, of course abductively, what a realm of possible qualities of constructions may contain: ". . . you do not fully appreciate possibility. Mathematics, for example, deals with nothing but hypothetical states of things, which far more often than not are either known to be false or dubious. And the foundation, mother, and essence of possibility is subjective, in us, dreams" [NEM III, 875].

References

BURCH, Robert W. 1990. *A Peircean reduction thesis and the foundations of topological logic*, Lubbock, Texas Tech University Press.

HERZBERGER, Hans G. 1981. *Peirce's remarkable theorem*, in L. W. Sumner, J. G. Slater, F. Wilson (editors), *Pragmatism and purpose: Essays presented to Thomas A. Goudge* (Toronto, University of Toronto Press), 41-58.

JOSEPHON, John R. & JOSEPHON, Susan G. *1994*. editors, *Abductive inference —Computation, philosophy, technology*, Cambridge, Mass., Harvard University Press.

KAPITAN, Tomis 1992. *Peirce and the autonomy of abductive reasoning*, Erkenntnis **37**, 1–26.

LISTING, Benedict. 1862. *Der Census räumlicher Complexe*, Göttingen, Teubner Verlag.

LORENZEN, Paul & LORENZ, Kuno. 1978. *Dialogische Logik*, Darmstadt, Wissenschaftliche Buchgesellschaft.

PEIRCE, Charles Sanders.
The following editions of Peirce's writings are cited in abbreviated form:

—. 1982–1993. Max Fisch, Christian J.W. Kloesel *et al.* (editors), *Writings of Charles S. Peirce: A chronological edition*, Vols. I–V, Bloomington, Indiana University Press. ("W 1" refers to the first volume of the *Writings*.)

—. 1931-35. *Collected papers of Charles Sanders Peirce*, Vols. I–VI, ed. by Charles Hartshorne and Paul Weiss, Vols. VII and VIII, ed. by Arthur W. Burks, Cambridge, Mass. Harvard University Press; 2nd edit., Cambridge, Mass.: The Belknap Press of Harvard University Press, 1958. (The decimal notation, e.g. "**1.289**", stands for the first volume and the 289. section of the *Collected papers*)

—. 1976. *The new elements of mathematics by Charles S. Peirce*, Carolyn Eisele (editor), four volumes in five, The Hague/Paris, Mouton Publishers. (Referred to as **NEM I-IV**.)

—. All numbers prefixed by "MS" refer to the microfilm edition of the manuscripts of C. S. Peirce as listed in Richard S. Robin, *Annotated Catalogue of the Papers of Charles S. Peirce*, Worcester, Mass., University of Massachusetts Press, 1967.

PRIOR, Arthur N. 1976. *What is logic?*, P. T. Geach and A. J. P. Kenny (editors), *Papers in logic and ethics* (London, Duckworth & Co.), 122–139.

STEPHENS, G. Lynn. 1985. *Noumenal qualia: C. S. Peirce on our epistemic access to feelings*, Transactions of the Charles Sanders Peirce Society **21**, 95–108.