

*Development of Mathematics 1900–1950*, edited by Jean-Paul Pier, Basel-Boston-Berlin, Birkhäuser Verlag, 1994, xviii + 729 pp.

Reviewed by

ROMAN MURAWSKI

Wydział Matematyki i Informatyki  
Uniwersytet im. Adama Mickiewicza  
ul. Matejki 48/49  
60-769 Poznań, Poland

In June 1992 at Château Bourglinster (Luxembourg) a Symposium devoted to the development of mathematics in the period 1900–1950 was held. 108 persons from thirteen countries (Austria, Belgium, Canada, Denmark, France, Germany, Italy, Luxembourg, Spain, Sweden, Switzerland, the Netherlands and USA) took part in it, eleven talks were given. The book under review is the volume of the proceedings of this symposium.

The book begins with "Guidelines 1900–1950" (written by Pierre Dugac, Beno Eckmann, Jean Mawhin and Jean-Paul Pier) presenting the main events in the mathematical sciences in the years of the indicated period. Texts of the lectures follow. Of the eleven talks presented at the Symposium, nine are included in this volume. The book also contains three additional papers not presented at the meeting.

The extensive paper "Une brève histoire de la topologie" by Jean Dieudonné (119 pages) is devoted to the presentation of the development of topology. The author starts from the contribution of Riemann, then the development of the idea of metric and topological spaces is considered. The main part of the paper is devoted to the development of algebraic topology. The paper is augmented by a subject index.

Next are two rather short papers: "The Development of Rigor in Mathematical Probability, (1900–1950)" by Joseph L. Doob (13 pages) and "Vito Volterra and the Birth of Functional Analysis" by Gaetano Fichera (14 pages). The paper by Doob is a brief informal outline of the history of the introduction of rigour into mathematical probability in the first half of this century. Just at that period mathematical probability became a normal part of mathematics. The paper by Fichera discusses the birth of functional analysis. He admits that it is difficult, sometimes even impossible, to pinpoint the birth of a new science or a new branch of a science, as in the case of functional analysis. One of its important aspects, the calculus of variations, had already appeared in the work of I. Newton in the second half of the seventeenth century and successively in the works of Jacques and Jean Bernoulli, L. Euler and G. L. Lagrange. But, according to Fichera, Vito Volterra was the first mathematician who fully understood the main ideas of functional analysis and therefore he must be considered its initiator.

The development of logic in the first half of the twentieth century is the subject of the extensive paper "La logique mathématique en sa jeunesse" by Marcel Guillaume (188 pages). The author begins by discussing the situation in mathematical logic at the turn of the century, in particular at the time of the International Congress in Paris in 1900. Then the achievements of the period called by Guillaume the period of optimism are presented. In particular, these are *Principia Mathematica* of Whitehead and Russell, works of the Warsaw school, i.e., of Łukasiewicz and Leśniewski, works of Hilbert, Hilbert, Bernays and Post, contributions to the axiomatic set theory of Fraenkel, Skolem, von Neumann and Bernays as well as the idea of the metamathematics of Tarski. The year 1930 brought the first limitation results (Gödel) which showed that the hopes associated with the axiomatic method can not be fully realized. Following this, the development of proof theory, of semantics and model theory, of recursion theory as well as of intuitionistic logic and of algebraic methods in logic are presented. The paper also contains a comprehensive bibliography (about 730 items!).

In the paper "Function Theory 1900–1950" by Walter K. Hayman (15 pages) the development of the theory of functions is studied. This theory was born in the nineteenth century (Gauss, Cauchy, Riemann) but it sprang to life in the twentieth century. Three areas are covered in the paper: entire functions, meromorphic functions and functions in the unit disk (those areas reflect the personal interests of the author).

The year 1949 when the conjecture of Weil was formulated, and the year 1973 when it was solved by Deligne are the most important dates in the history of abstract algebraic geometry. In the "La préhistoire des conjectures de Weil" by Christian Houzel (29 pages) the history of the theories which preceded the formulation of the conjecture by André Weil is considered. In particular the author considers works by H. Kornblum, E. Artin, F. K. Schmidt and H. Hasse, and how they lead to the announcement of the conjecture.

The paper "Des séries de Taylor au mouvement brownien, avec un aperçu sur le retour" by Jean-Pierre Kahane (15 pages) tells the history of Taylor series. It begins with the work of Borel from 1896–1897 where he said: "une série de Taylor admet, en général, son cercle de convergence comme coupure". Contributions by H. Steinhaus, the probabilistic interpretation as well as Wiener's theory of Brownian motion are then discussed.

André Lichnerowicz in his paper "Géométrie et relativité" (11 pages) considers the interactions between differential geometry and theoretical physics, in particular relativity theory, in the first half of the twentieth century.

Two papers from the book under review are devoted to the development of differential equations: "Boundary Value Problems for Nonlinear Ordinary Differential Equations: From Successive Approximations to Topology" by Jean Mawhin (34 pages) and "Partial Differential Equations in the First Half of the Century" by Louis Nirenberg (37 pages). The first one discusses the nonlinear boundary value problems concentrating on results about the existence of solutions and leaving aside the important questions of uniqueness, multiplicity, bifurcation and approximation. In the second one the history of partial differential equations is presented. In particular, problems related to

the existence of solutions under various boundary conditions or initial conditions, the problem of the uniqueness of solutions, estimates and regularity of solutions are discussed. Some interrelations between the theoretical material and the calculus of variations are also considered.

The paper "Intégration et mesure 1900–1950" by Jean-Paul Pier, the editor of the volume, (48 pages) is about the history of integration theory and measure theory in the first half of the twentieth century. Works and contributions of Cantor, Peano and Borel (as founders of measure theory), of Riemann, Darboux and Stieltjes (as main contributors to the integral calculus at the end of the nineteenth century) and of Lebesgue (the founder of the modern integral calculus) are discussed.

The volume closes with the paper "Some Remarks on the History of the Prime Number Theorem from 1896 to 1960" by Wolfgang Schwarz (52 pages). He presents a short discussion of mathematical achievements culminating in the proof of the Prime Number Theorem by J. Hadamard and independently by Ch.-J. de la Vallée-Poussin. Also, the impact of this result on subsequent investigations in the field of analytic number theory is considered. In particular, the development of the sieve method, the Hardy-Littlewood method in additive prime number theory, the Page-Siegel-Walfisz prime number theorem, investigations on the zeros and order of the zeta-function, the Erdős-Selberg elementary proof of the prime number theorem as well as the Bombieri-Vinogradov prime number theorem are discussed.

The volume also contains an extensive bibliography (edited by Pierre Dugac) of about 1700 items as well as an index of names. It is augmented also by portraits of many of the mathematicians whose works are discussed in the papers.

The book *Development of Mathematics 1900–1950* is the first book — as far as the reviewer knows — presenting such an extensive history of mathematics of the first half of the twentieth century. There are, of course, books and papers devoted to the development of particular branches of mathematics in this period (as, e.g., the book *A History of Algebraic and Differential Topology 1900–1960* by J. Dieudonné [1989], and the new chapter of the fourth revised edition [1987] to the classical book by D. Struik *A Concise History of Mathematics* [1948]). The reasons are quite clear and obvious — the subject is still being developed and it is rather difficult (if even possible) to present the recent history of a discipline if one is not both a historian and a specialist in the particular field. Hence the best way to do this is to invite specialists from various fields and ask them to describe the development of their domains — the method chosen in the book under review. The result is this volume presenting the recent history of some parts of modern mathematics. It is an excellent complement to books describing the development of mathematics in earlier centuries, in particular, the classical textbook by D. Struik [1948; 1987] and the books by C. Boyer [1968], D. E. Smith [1951], H. Eves [1983] and J. Dieudonné [1978] (the latter covers the history of mathematics in the period 1700–1900).

I would like to end this review by expressing the hope that future conferences devoted to the recent history of mathematics will be organized and that they will bring forth new volumes of proceedings that will provide a foundation for writing a history of mathematics of the twentieth century.

### References

BOYER, C. B. 1968. *A history of mathematics*, New York, Wiley.

DIEUDONNÉ, J., 1978. *Abrégé d'histoire des mathématiques 1700–1900*, Paris, Hermann.

—. 1989. *A history of algebraic and differential topology 1900–1960*, Boston, Birkhäuser.

EVES, H. 1983. *An introduction to the history of mathematics*, New York, Holt, Rinehart & Winston.

SMITH, D. E., 1951. *History of mathematics*, New York, Dover Publications, Inc.

STRIJK, D. 1948. *A concise history of mathematics*, New York, Dover Publications, Inc.

—. 1987. *A concise history of mathematics*, New York, Dover Publications, Inc., 4th revised edition.