Paradoxes, Second Edition by R.M. Sainsbury. Cambridge, Cambridge University Press, 1995. x +1 65 pp, paperback.

Review by

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Unlike modern physics or computer science, logic is without a popular literature. R. M. Sainsbury's text *Paradoxes* is a partial remedy, as it is accessible to those without any familiarity with the philosophical treatment of paradox. Here Sainsbury blends analytical approach and an appreciation of the broadly engaging notion of paradox into a small book that will serve as an agreeable introduction to and a handy compendium of discussion of some important paradoxes, and, more importantly, the notion of paradox.

Perhaps more than any topic in logic, paradox is suited to a general audience. Like many interesting open problems in number theory, the paradoxes Sainsbury addresses himself to are easy to state. The essential antinomies are easy to motivate in nearly all the cases he presents. Whether due to *Alice in Wonderland*, or the popular writings of Smullyan, or riddles in the oral tradition, many readers will be familiar with puzzles like that of the Barber.

The Barber is cast by Sainsbury as: In an isolated village a barber shaves all and only those villagers who do not shave themselves. Who shaves the barber? If the barber shaves himself, then he is among those villagers who do shave themselves, and so the barber, since he shaves only those who do not shave themselves, cannot shave himself. Likewise, if he does not shave himself, then he is clearly among those who do not shave themselves, and so, since he shaves all those who do not shave themselves, he shaves himself.

This is the least interesting paradox that Sainsbury considers; and he admits that. He offers a solution to the Barber paradox, which ranks in difficulty as a one on his scale to ten (where the Liar paradox is a ten), saying that we can simply reject the premise that there is such a Barber. Thus, the paradoxical result is avoided. We can do this without cost to intuitions held dear, for no one has ever seen such a Barber, nor has reason to think that one exists.[1] In the pages that follow, Sainsbury admits that

[1] Charles Chihara discusses a paradox similar to the Barber in his useful [*1979*]. The Sec Lib club follows the rule "A person is eligible to join this club if, and only if, he is secretary of a club which he is not eligible to join." If the

most solutions are not bought so cheaply, particularly those that arise in latter sections.

Indeed, one of the book's virtues is its organization. At the outset, Sainsbury tackles the easier problems, and in this way lays the foundation for a treatment of the philosophically rich paradoxes of induction and truth. Suitably, given its mission as an introduction to paradox, the book gives a detailed treatment of even problems seen to have straightforward explanations. For instance, the first chapter, which concerns itself with Zeno's paradox and its analogs, goes to great lengths to motivate the intuition that Achilles, since he has to pass through an infinite number of half-way points as he chases the tortoise, can't in fact catch the tortoise. It isn't until four pages into the chapter that Sainsbury deploys the series $\sum 1/(2^n)$. While acknowledging that it is absurd to believe that Achilles cannot catch the tortoise, Sainsbury notes that to philosophers as different as Aristotle and Bertrand Russell, Zeno's paradox raised non-trivial issues. (Sainsbury also cites C. S. Peirce's dismissal of the paradox.)

Peirce's notion, his comment that anyone familiar with infinite series should be untroubled by Zeno, is taken up in a footnote, where the reader is asked if it isn't just a mathematical convention that the sum of an infinite series is the limit of its partial sums. *Paradoxes* is studded with about seventy-five footnotes, all are questions for the reader. Through these, Sainsbury addresses topics a bit outside the scope of the main text. There are no chapter-end questions, which is a virtue; yet with provocative footnotes, the text points to interesting issues beyond itself. (One question in the section concerned with Zeno gets at the notion of space quanta.) One could imagine the footnotes as points of departure for further study or as homework or exam questions.

In my view, these footnotes are integral to the text's utility. Sainsbury's book is not long, which helps keep it inviting to the merely curious; it is aimed, in part, at readers unfamiliar with not only the philosophical analyses of the paradoxes, but also, in some cases, with the paradoxes themselves. Thus there is less space than certain readers might like devoted to examinations of the subtleties of proposed solutions. These footnotes acknowledge that a discussion can be profitably carried on along several lines.

In particular, the footnotes usually suggest objections not raised in the text or suggest responses to objections that do appear. As Charles Chihara writes:

> A good diagnosis should do more than merely provide a way of avoiding the contradictions; for the paradoxes can be blocked in a

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secretary of Sec Lib is secretary of no other club, is he eligible to join Sec Lib? Chihara notes that this, while of the same form, is more troublesome than the Barber because, while there is little in support of the premise that there is a Barber (as described), the rule of Sec Lib seems "to be the sort of premise that can be made true by decree: the officials of the club do seems to have the authority to make it true by simply laying down the rules of eligibility that way."

> variety of ways, and one needs special reasons for picking any one of these alternatives as the crucial one. [*1979*, 602–603]

Sainsbury seems attuned to this requirement. For example in his chapter on the paradoxes that arise from vague predicates he considers that the paradoxes instead might be the result of truth coming in degrees, where some propositions are *e.g.* 'mostly true'. This approach loads the uncertainty in the assignment of truth, rather than in the determination of the extension of the predicate, and is often aligned with a view that asserts that even seemingly vague predicates (those that admit of borderline cases, like 'is tall') do in fact have definite extension. While it seems that Sainsbury pays more attention to this possibility than is warranted, the methodological impetus is clear: We should examine all the accounts that dispel the paradox. The discussion of vagueness in *Paradoxes* has been largely and profitably reworked since the first edition.

Sainsbury discusses the Prisoner's Dilemma in the section entitled "Acting Rationally". It warrants inclusion in a book about paradoxes, because the Prisoner's Dilemma can be seen as a conflict between the course dictated by a maximin strategy — *i.e.*, one that dictates that you choose a course of action that, no matter what your opponent does, yields the best worst-case outcome—and a cooperative strategy that, if mutually pursued, results in the highest expected outcome for each participant. Consider an iterated case of the Prisoner's Dilemma. That is, think of a game with a payoff matrix like that of the Prisoner's Dilemma (PD), where each pair of decisions by the prisoners constitutes a round in the game, and where the game is played for 100 rounds.

To make it easier to envision playing this iterated game, abstract it from the prisoner's setting and use the payoff matrix given below, where accumulating points is the goal.

	X cooperates	*X* defects
Y cooperates	<6, 6>	<10, 0>
Y defects	<0, 10>	<1, 1>

(Where $<a, b>$ indicates that *X* gets *a* points and *Y* gets *b* points for that round.)

Sainsbury notes that a particularly good strategy for the iterated PD is what has come to be called Tit-for-tat. Tit-for-tat is the strategy that dictates: In the first round, cooperate; in the n^{th} round ($n > 1$), do whatever the opponent did in the $(n - 1)^{th}$ round.

Consider however, the 100^{th} round. If *X* cooperates and *Y* cooperates, they each gain moderately. However, since this is the final round, *X* can act without fear of reprisal in later rounds. Thus the 100^{th} round of the iterated PD should be played identically to the one-shot PD. That is to say, *X* should defect in round 100, for defection maximizes *X*'s expected gain regardless of *Y*'s choice. But *Y* may well realize that the final round is a special case. He too may decide to defect — either on the reasoning *X* used, or because he anticipates *X*'s reasoning and realizes that if *X* will defect, he (*Y*) should defect also to secure the moderate benefit. Thus for the final round, two rational players may well both defect, earlier cooperation notwithstanding.

If *X* reasons as above, then he will see that it is possible that he will not secure his large expected payoff in the final round. He will expect to win only 1 point. So *X* may come to think of the 99th round as the last round where there is a meaningful choice to be made. Rather than cooperate in round 99, *X* can defect and expect an advantage. But certainly *Y* can reason likewise; and so it is rational for *X* and *Y* to both defect during the 99th round. This line continues, and it seems that *X* and Y will defect on every turn. In this quest for an advantage, they assure themselves of the payoff in the lower-right corner of the matrix — a payoff less than that to be expected had they cooperated. Among game-theorists this goes by the name of the Backwards Induction Paradox (BIP).

The Surprise Examination Paradox (SEP) looks similar to the BIP (which Sainsbury doesn't discuss). In the SEP a teacher announces to his class that there will be a quiz during the next week (taken to be Monday through Friday) and the students will not expect the quiz. The students reason that the quiz cannot occur on Friday, for if it did they would be expecting it, knowing that it hadn't occurred on any other day that week. Since it cannot take place on Friday, the students conclude that it won't occur on Thursday either: For if on Wednesday they hadn't yet been given the quiz they can conclude that it must be given on Thursday. In the end they conclude that there can be no such surprise quiz. Yet, on Tuesday the quiz is given and the students are surprised.

Given that such a surprise examination can take place (a matter of empirical record), the tendency is to find fault with the students' argument. As stated, the argument is subject to several criticisms; Sainsbury notes them. He then strengthens the problem and considers more devious versions of the SEP, which he analyzes formally, in a natural deductive system. (These are the only formal proofs in the volume.)

The problem of rational belief about the future is at the heart of classical epistemology. David Hume presented an argument against inductive knowledge — a slightly stronger attitude than belief — in his *Enquiry*, and that argument grounds philosophical skepticism. Hume's account rules out inductive knowledge, but is a positive account insofar as it offers a mechanism by which we come to hold beliefs about certain sorts of future events. Hume says that we observe long-running correlations between, *e.g.*, something's being a raven and that something's being black, and so we come to expect that the next raven will be black. (I've chosen ravens, but of course ravens are typically associated with Carl Hempel's Paradox of the Ravens, and not with Hume, who was more likely to expect that the sun will rise tomorrow. Sainsbury discusses Hempel's Raven Paradox in this chapter.)

Nelson Goodman [*1955*] showed that Hume's account of how we come to hold those certain beliefs isn't sufficient. Goodman introduces the predicate 'grue'. Something is grue iff (it is observed before 1 January 1997 and is green) or (is blue). Note that every observation heretofore of a green emerald confirms the hypothesis 'Emeralds are green' to the same extent as it confirms 'Emeralds are grue'. Our experience of green emeralds is no more frequent than our experience of grue emeralds. Says Goodman, Hume's account isn't enough; it doesn't account for our choosing to believe 'Emeralds are green' instead of 'Emeralds are grue'. (The paradox can be cashed out as: For an emerald examined in 1998, we expect it to be green, and of that emerald, we expect it to be blue.) Sainsbury doesn't link this to inductive logics — he is concerned with the contradiction *per se* — but Skyrms [*1966*] contains a discussion.

The penultimate chapter will be the most familiar to mathematically-minded readers. It is entitled "Classes and Truth". Sainsbury here gives a clear, if short, introduction to Russell's Paradox and the Liar Paradox. Russell's Vicious Circle Principle is mentioned. The discussion of the Liar ranges over familiar ground: indexicals, Tarski's truth hierarchy, and grounding, but, to Sainsbury's credit, also incorporates, at a level suitable for the intended audience, the discussion of nonwellfounded sets that takes up so much of Barwise and Etchemendy's [*1987*]. As in all the chapters, endnotes provide a selective bibliography.

The brief final chapter considers truth gaps and dialethism, the thesis that some propositions are both true and false. The interested reader will have to investigate Sainsbury's citations to gain anything more than the barest introduction to the notion.

Paradoxes, because they remind us of the need for care and caution in thought and reasoning, are worth our consideration. But, especially in the case of the Liar, paradox can serve a more fundamental role in logical studies. Elsewhere [*1995*, 79], Sainsbury quotes Russell:

> A logical theory may be tested by its capacity for dealing with puzzles, and it is a wholesome plan, in thinking about logic, to stock the mind with as many puzzles as possible, since these serve much the same purpose as is served by experiments in physical science. [*1905*]

The Liar is the great puzzle (whether for the insights into formalism that it has sparked or for the reflections on semantics that it engenders). Other problems are worthwhile, to be certain, but it is the Liar that most captures the imagination. Sainsbury has written, and updated, a work that makes the intricacies of this paradox accessible. *Paradoxes* is an excellent starting point for the study of paradoxes in general, the Liar in particular.

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