Mathematics and Mind, edited by Alexander George, New York and Oxford, Oxford University Press, 1994.

Reviewed by

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There are several epistemological problems which have united the philosophy of mathematics and the philosophy of mind. One of them has been the question of the nature of mathematical truths. Kant thought that all mathematical truths were synthetic *a priori*; for Kant, that amounted to the thesis that mathematics depended on the forms of intuition, that is, on space and time. In his logicist program, Gottlob Frege tried to change the place of arithmetic in the division of sciences presented by Kant. In contribution to *Mathematics and Mind*, entitled "The Advantages of Honest Toil over Theft", on page 40, George Boolos mentions that, following Carnap, we may distinguish between two theses of logicism; the first states that the concepts of mathematics can be explicitly defined by means of logical concepts, while the second is the claim that the theorems of mathematics can be deduced from logical axioms by logical means alone. Boolos calls the first thesis the definability thesis and the second thesis the provability thesis of logicism. Frege was committed to both of those theses in his studies in the foundations of arithmetic. It may be worth noting that there was one more reading of the term 'logicism'; Theodor Ziehen remarks in his old textbook (1920) that at the turn of the century logicism meant the acknowledgement of an objective realm of ideal entities which were studied by logic and mathematics [*1920*, 173]. In Ziehen's list of names, representatives of that doctrine were Lotze, Windelband, Husserl, and Rickert, among others. Frege also defended some version of that kind of logicism.

In Frege's thought, intuition seemed to have lost its central role, as Frege placed the arithmetical truths into the group of analytic truths *a priori*. It is too much to argue that Frege relied on the same concepts of syntheticity and analyticity, or even on the same concepts of *a priori* and *a posteriori*, as Kant. However, we may say that his move did not mean a break between mathematics and mind; geometry was still regarded by Frege as synthetic *a priori*, and by trying to reduce arithmetic to logic Frege in fact tied the language of arithmetic to the formula language of pure thought, which his *Begriffsschrift* was meant to be. Hence, even if Frege wanted to get rid of all psychological considerations and any resort to intuition in his philosophy of arithmetic as well as in his logic, he was interested in what he called pure thought and "an intuitive representation of the forms of thought" ("*eine anschauliche Darstellung der Denkformen*"); that was the expression Frege used for his new logic *in "Über wissenschaftliche Berechtigung einer Begriffsschrift*" (1882) [*1964*, 113–114]. That is what Alexander George actually suggests in his "Introduction" to *Mathematics and Mind*, as he, on page 5, quotes Frege's remark that logic and mathematics could be represented as the investigation of *the* mind, even if not of individual minds.

Charles Peirce had similar ideas of the relations between logic and mathematics and mind. He wrote in his "Minute Logic" in 1902 that mathematics is observational "in so far as it makes constructions in the imagination according to abstract precepts, and then observes these imaginary objects, finding in them relations of parts not specified in the precept of construction" [*1931*, 1.240]. Peirce was interested in the study of mind, which he calls "phaneroscopy", but for him the study of mind was the study of *any* mind, not of individual minds [(1904); *1931*, 284].

It has been convincingly argued by J. Alberto Coffa that the semantic tradition from Bolzano to Carnap, hence, Frege is included, tried to get rid of every resort to intuition. He states: "The semantic tradition may be defined by its problem its enemy, its goal, and its strategy. Its problem was the *a priori*; its enemy, Kant's pure intuition; its purpose, to develop a conception of the *a priori* in which pure intuition played no role; its strategy, to base that theory on a development of semantics" ([*1991*, 22]. However, this is not the whole truth of the developments in the philosophy of mathematics in the nineteenth century. The short remarks made above show that the stories which are told about Frege and Peirce tend to become quite intricate. In addition, there was a strong trend of psychologism both in logic and in mathematics in the nineteenth century. The label of psychologism was often used as a criticism but, as Martin Kusch has argued, at the turn of the century few authors were willing to take the label [*1994*, 77]. No matter which label we choose, at least such nineteenth century logicians as Beneke [*1842*] and Fries [*1819*; *1827*]) were committed to the idea that the study of logic is closely related to the study of mind. In his philosophy of arithmetic, especially in *Grundlagen* [*1884*], Frege attacked psychologists and tried to give an analysis of the concept of number by means of what he called logical concepts, such as the concept of extension of a concept and the concept of equinumerosity. In his philosophy of arithmetic in 1891 (q.v. [*1970*]). Edmund Husserl analyzed the concept of number in terms of the concept of a mental act. Interpreters have disagreed on whether that was a psychologistic project. In any case, Husserl later attacked psychologism in the *Logische Untersuchungen* in 1900 and 1901 (q.v [*1950*; *1984*]). The debate between psychologism and antipsychologism had to do with the relation between mathematics and mind and also with the understanding of the very concept of mind. The mind which is relevant to mathematical studies could be identified with the empirical mind in which the special sciences are interested. Alternatively, it could be seen as the transcendental mind, meaning the mind as the object of purely epistemological studies, hence, rejecting the view that it is the task of the special sciences to study the foundations of mathematics. To study the transcendental mind would amount to studying the conditions which make mathematical knowledge possible without claiming that those conditions have to do with the structure of our empirically studied minds.

The above mentioned themes are touched upon, but notdiscussed, in *Mathematics and Mind*. The volume begins with the observation that the relation between mathematics and mind has not received the attention it deserves even if the history of mathematics and the philosophy of mathematics suggest its importance. In his "Introduction" Alexander George takes up the distinction between Platonism, formalism and intuitionism, which is a clear example of the connection between the questions of foundations of mathematics and the questions discussed in the philosophy of mind. What the reader of the volume expects to find is a deep analysis of that connection, which would show familiarity both with the philosophy of mathematics and with the philosophy of mind. Such a study would be especially welcome for several reasons. For example, contemporary theories of constructions are related to the tradition of geometry and with the very concept of intuition in that tradition. Moreover, cognitive science seeks to bring the questions of logical and mathematical reasoning into the field of cognitive psychology and, as Bechtel [*1994*], among others, argues, asks questions concerning actual human reasoning rather than questions con-cerning ideal reasoning.

The contributions to the volume are descendants of presentations held at a conference on the philosophy of mathematics. In the "Introduction", on page 3, Alexander George asks if the study of mathematics might be important for an inquiry into mind, or vice versa. On page 4, he answers that "through the study of the conceptual basis of mathematics we learn about the powers of the mind, for it is just by the grace of these powers that mathematics is accessible to us", and "conversely, a deeper understanding of the mind should clarify the foundations and development of mathematics". On page 7, he writes: "The preceding remarks are intended merely to introduce and render plausible the *and* in *Mathematics and Mind*. Each of the contributors to this volume has focused on a different facet of the conjunction".

In the first article entitled "What Is Mathematics About?", Michael Dummett discusses the logicist thesis. On page 13, he formulates it as the claim that "mathematics is not about *anything in particular*: it consists, rather, of the systematic construction of complex deductive arguments". Dummett argues that the logicist answer is closer to the truth than any other that has been put forward. He pays special attention to Frege's logicism and Frege's defence of abstract objects, and raises criticism against the use of classical logic in mathematical proofs. In his paper "The Advantages of Honest Toil over Theft", George Boolos continues the discussion of logicism by investigating Frege, Russell and Whitehead. Boolos seeks to show that the number principle, or Humes's principle, cannot be thought to be *the* foundation of arithmetic. He also shows some strengths of Russell's logicism in comparison with that of Frege's. He concludes that the question whether Russell after all was a logicist cannot be given a direct answer.

In "The Law of Excluded Middle and the Axiom of Choice", W. W. Tait defends constructivism and the type theoretic point of view. Here it would have been especially interesting to bring in the idea of constructions of proofs of some proposition in connection with the tradition of geometry as well as with views of the activities of the mind.

In his "Mechanical Procedures and Mathematical Experience" Wilfried Sieg asks what an effectively calculable function is, which is an important question for mathematical logic as well as for cognitive science and artificial intelligence. He gives a short but perceptive survey of the history of the idea of calculus from Leibniz to Frege and Gödel and then presents the decision problem as Hilbert's and Ackermann's formulation. He concentrates on comparisons between Church and Turing and argues that Turing deepened Church's step-by-step argument by focusing on the mechanical operations underlying the steps. He also touches upon the problem of how the human mind "transcends" the limits of mechanical computers.

The paper by Daniel Isaacson, "Mathematical Intuition and Objectivity", is a defence of the thesis that the philosophy of mathematics must respect our sense of the objective reality of mathematics. Isaacson argues for a doctrine which he calls concept Platonism. He states on page 125 that the "genesis of our mathematical concepts reflects both constitutive features of mind and elements abstracted from experience in the world around us". He admits that many mathematical concepts arise from mathematical experience itself. He remarks that Popper's doctrine of the "third world" is close to what he means by concept Platonism. One may wonder why Isaacson uses the label of Platonism at all. In his paper "Intuition and Number" Charles Parsons defends the idea that natural numbers are given in intuition. He also discusses the very concept of intuition and connects his own view with that of Edmund Husserl. That comparison would have been illuminating also for Isaacson's position.

The volume closes with the paper entitled "Hilbert's Axiomatic Method and the Laws of Thought" by Michael Hallett, which is a study of different views of the laws of thought. The point of view chosen by Hallett manages to throw some new light on the controversy between Frege and Hilbert on the foundations of mathematics.

The volume contains extremely interesting papers of high quality on the philosophy of mathematics. Many of the papers touch upon the questions of the role of mind in mathematics. The book is compact, and a number of central philosophers of mathematics are discussed in detail. However, what is still needed is a volume of mathematics and mind, of both with equal emphasis. It might be wise to look deeper into the writings of the classics, such as Descartes, Kant, Peirce, and Husserl, in order to see the various ways in which the philosophy of mathematics and the philosophy of mind have been, and are, intertwined. For example, the intricate concept of intuition discussed and used in the philosophical tradition is one of those links. One might even want to say a few words about mathematics and the life-world, following the themes raised by Husserl in his later philosophy. The possible effects of current naturalistic trends on the philosophy of mathematics would deserve special attention. It might also be useful to think of the classical AI and the connectionist challenge and to ponder upon their impacts on our views of mathematical thinking.

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