# SOF'YA ALEKSANDROVNA YANOVSKAYA'S WORK IN THE FIELD OF MATHEMATICAL LOGIC*

A. A. MARKOV
A. S. KUZICHEV
Z. A. KUZICHEVA

Sof'ya Aleksandrovna Yanovskaya devoted considerable vigor and energy to the study of the foundations of mathematics and mathematical logic. She wrote articles on foundations of mathematics and mathematical logic, composed commentaries to translations on their problematic, conducted special seminars, delivered lectures. In her papers "Foundations of Mathematics and Mathematical Logic" (written so clearly and charmingly that it was recommeneded that it be included in a collection of papers surveying the state of mathematics in the USSR during the 30 years since the 1917 revolution, all the more that she put a completely accessible face on the subject, not calling for any special mathematical expertise) and "Mathematical Logic and Foundations of Mathematics"[1] in which she gave an outline and analysis of the work of Soviet students of the field of mathematical logic and foundations of mathematics.

Here we stop to consider in detail two of Sof'ya Aleksandrovna's works devoted to the subject of definition by abstraction. The first of

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* English translation by Irving H. Anellis of A. A. Марков, A. C. Кузичев, З. А. Кузичева, «Работы С. А. Яновской в области математической логики», Ф. П. Дашевская, А. П. Непароков, Х. С. Топоровская, И Э. Южный-Горенюк, состав., И. И. Минц & А. П. Непароков, ред., *Женщины-революционеры и ученые* (М, Издат. «Наука», 1982), с. 96-99.

[1] *Osnovaniya matematiki i matematicheskaya logika,* in *Matematiki v SSSR za tridtsat let, 1917-1947* (A. G. Kurosh, A. I. Markushevich & P. V. Rashevskii, eds.; Moscow & Leningrad, GITTL, 1948), 9–50, and *Matematicheskaya logiki i osnovani matematiki,* in *Matematika v SSSR za sorok let, 1917-1957* , vol. 1 (Moscow, Fizmatgiz., 1959), 13–120.

these, "On So-called 'Definition by Abstraction'," was published in a collection of papers on philosophy of mathematics.[2] In this paper it is shown that what the process of definition means in mathematics is an analog of what organization means in other sciences, in particular in political economics. In a surprisingly accessible, but at the same time strict and precise manner, she analyzed the process of forming the concept of numbers as properties of collections of objects, carrying out a comaprison between the concepts of number and economic value to study the character in particular of definition by abstraction, and the point of departure was layng out the basis of these definitions. Here, for example, is her explanation of the concept of equinumerity: "In order to approach this definition, we attempt to elucidate what in general is the connection between the collection of letters in the word "letter" [«буква»] and the corresponding collection of letters in the word "number" [«число»]. It is easy to see that in writing the words one atop the other,

буква
↕↕↕↕↕
число

that each letter of the collection on top may be set in correlation to letters of the collection below, and *vise versa*, and moreover, that distinct letters of the collection on top correspond to distinct letters of the collection below, and distinct letter below to distinct letters above. Such a correlation is called in mathematics a one-to-one correspondence. For its establishment, it is not required to know the number of objects in each collection, but only that we be able to set their constituents side-by-side. However, the establishment of such a correspondence gives us the possibility of asserting the equinumerity of the two sets. Thus, if we know that in a theater performance is sold out, and that there are no people without seats, then the fact that we do not know the number of seats in this theater or the number of tickets sold, we may affirm that the number of spectators on this day is equal to the number of seats in the theater. It is possible to establish the equality of these numbers in this way without knowing what that number itself is:

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[2] *Sbornik statei po filosofii matematiki* [Collected Papers on Philosophy of Mathematics], Moscow, 1936.

indeed, we now have the possibility of defining the concept of equinumerity of two sets, although not being able to determine the character of their numbers. Namely: we can say that two sets are equinumerous — at times say equipollent — if they can be brought into a one-to-one correspondence with one another.[3]

Sof'ya Aleksandrovna delivered a lecture on the theme of defining abstract concepts and their role in mathematical logic in a symposium in Warsaw (September 1961), a lecture published in 1965. She said: "In order that science be able to solve practical societal problems of people, it must conform to general rules and laws, to reveal itself to relate to different objects and appearances of concrete (invariant) substantive things. But in order to expose these substantive things, their hard kernel, core, we need to abstract from their inessential details. The result of such an abstraction exposes the abstract concepts and objects; and without introducing these it is impossible to formulate any one of their general rules of laws."[4]

And further: ". . . introduction of abstraction always by itself always a certain roughness, a simplification, a certain idealization of reality."[5]

Sof'ya Aleksandrovna explained to students the process of idealization, abstracting from nonessential details, by various expressive examples. Here is one of these examples. Suppose that we see the edge of the bay of a sea standing on the shore. We see the shoreline to be winding and strange. That then we see a part of the coast from the window of a house, inclining some distance from the bay, the coastline becoming smooth, we no longer observe all of the details and bends. The shoreline appears even smoother viewed from an airplane flying in the direction of the bay.

In the works devoted to problems of the development of mathematical logic, Sof'ya Aleksandrovna analyzed the connections of mathematical logic with its technical applications, claiming that these are the cause of the acceleration of the tempo of the development of mathematical logic in the first half of our century, which proved to be one of rapid development of computational techniques. Sof'ya Aleksandrovna then turned to an analysis of philosophical problems of mathematical logic.

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[3]*Методологические проблемы науки* Methodological Problems of Science] (Moscow, 1972), p. 37.

[4]*Ibid.*

[5]*Ibid.*, p. 240.

Mathematical logic and foundations of mathematics occupied the greatest place in Sof'ya Aleksandrovna's lecture courses. Yanovskaya conducted over 40 general and specialized courses in different areas of mathematical logic. Here are the names of some of them: "Selected Problems in Mathematical Logic", "Gentzen's Natural Calculus",[**] "Mathematical Logic and Beth Tableaux", "Combinatory Logic", and many more.

Sof'ya Aleksandrovna was an excellent lecturer, spirited and erudite. A profound attitude was always illustrated by the diversity of her examples, at time surprising. In a lecture devoted to the theory of relations, she proposed to explain to the audience these are sets, saying: "all that's frequentl is all that's ascendant". Or she would say: every concept is like a tree. But is it easy to count the trees on some plot, even if not too big? At one stroke she showed that in reality we then bump up against the problem that we must first decide where to begin to count. Firstly, as with the trees, do we consider the shade at the edge of the plot or not? This problem is solved by special agreement. Further, the century-old oak — indubitably, is a tree, and we count it, without a moment's thought. And what about the sapplings,[†] some of which it is possible to count as trees? And is the birch of two years a tree or not? And so on. In this way she explained that even ordinary prose and familiar concepts, when subjected to careful consideration can prove themselves to be utterly vague and require study and refinements. And one of the important problems of mathematical logic appears in this instance to be the clarification of mathematical concepts.

All of S. A. Yanovskaya's activity in propaganda, popularization, and instruction in mathematical logic prepared the ground for creating the department of mathematical logic.

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[**] I.e. Gentzen's calculus of Natural Deduction.

[†] The Russian says "very young plants" , literally "minimally aged plants".