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Bart Kosko, *Fuzzy Thinking: The New Science of Fuzzy Logic* (New York, Hyperion, 1993), xviii +318pp., and Daniel McNeill and Paul Freiberger, *Fuzzy Logic* (New York/London/Toronto/Sydney/Tokyo/Singapore, Touchstone Book, Simon & Schuster, 1993), 319pp.

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Fuzzy logic has achieved much noteriety recently, attracting the critical attention not only of logicians and engineers, but has also captured the popular imagination because of its brilliant applications in the tools of everyday life, from cameras to washing machines to high-speed railway systems. The goal of the two books under review is to satisfy the curiosity of those who seek an explanation of the new science of fuzzy logic and to appeal to its noteriety. Kosko's book has received acclaim

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from his colleagues in fuzzy theory; McNeill and Freiberger's book is particularly appropriate for the interested layperson.

Exactly what is fuzzy logic, and exactly what is its connection to multiple-valued logics and probability logics? What are the characteristics of fuzzy logic which distinguish it, if at all, from these other non-classical logics? How, in particular, does fuzzy logic differ from many-valued logic? And what is the history of fuzzy logic?

Let us begin with a bivalent logic, in particular the Boolean-valued logic whose only values are 0 and 1, the logic which fuzzy logicians call "crisp" in order to distinguish it from fuzzy logic. 'Fuzzy logic,' Kosko says (p. 13), 'is reasoning with fuzzy sets.' McNeill and Freiberger (p. 12) say that 'fuzzy logic is not logic that is fuzzy, but logic that describes and tames fuzziness. . . . It is a theory of fuzzy sets, sets that calibrate vagueness.' Let us unpack these statements.

Fuzzy logic is a superset of Boolean-valued logic that has been extended to handle partial truth-values between "completely true" and "completely false". Fuzzy logic deals with fuzzy subsets of truth-values between "completely true" and "completely false".

Let us now consider the relationship between fuzzy logic and fuzzy sets. For crisp logic, we define Boolean a subset U of a set S as a set of ordered pairs, each having a first element that is an element of S and a second element that is an element of the set $\{0, 1\}$, with exactly one ordered pair present for each element of S. This defines a mapping between elements of S and elements of the set $\{0, 1\}$. The value 0 represents non-membership and 1 represents membership. The truth or falsity of the statement '$x \in U$' is determined by identifying the ordered pair whose first element is x. '$x \in U$' is true if the second element of the ordered pair is 1, and is false if the second element of the ordered pair is 0. For fuzzy logic, we similarlly define a fuzzy subset F of a set S as a set of ordered pairs, each having a first element that is an element of S and a second element that is a value in the interval [0,1], with exactly one ordered pair present for each element of S. This defines a mapping between elements of S and values in the interval [0, 1]. The value 0 represents complete non-membership, the value 1 represents complete membership, and values in between in the interval [0, 1] represent intermediate degrees of membership. Here, S is the universe of discourse for F. The degree to which '$x \in F$' is true, then, is determined by identifying the ordered pair whose first element is x, and the degree of truth of '$x \in F$' is the second element of the ordered pair. Thus, fuzzy sets include "crisp" sets as subsets. A "crisp set" is thus defined by McNeill and Freiberger (p. 36) as 'just a fuzzy one with membership values of 1 and 0 . . . . If an item is in a crisp set, it must have a value of 1; if it's "in" a fuzzy set, it can have any value except 0.'

McNeill and Freiberger (p. 72) and Kosko (pp. 123–125) characterize fuzzy truth-values, and operations of fuzzy arithmetic, as approximations. But neither book specifically or explicitly addresses the question of the relationship between fuzzy logic and multiple-valued logic. We may assume that in an $(m + 1)$–ary multiple-valued logic, by comparison with a fuzzy logic in which each truth-value corresponds to an element on the continuum in the interval [0, 1], each truth-value corresponds to a point in the set $M = \{0, \frac{1}{m}, ..., \frac{m-1}{m}, 1\}$ of [0, 1], where each element of the M is "crisp", that is, to a discrete and discontinuous element of M (and which is presumably expressible as a rational number, as opposed to being a rational approximation). For Zadeh, "fuzzy truth-values," McNeill and Freiberger say (p. 72), "are words, not numbers. They include such terms as *very true*, *rather true*, and *not very false*. Each of these truth-values is a fuzzy set along the line from 0 to 1." For Kosko (pp. 123–125), a fuzzy truth-value would correspond to a fuzzy number, that is, to a set of numbers in the neighborhood of the point p on the interval [0, 1] which comes closest to the truth-value p selected. That the truth-value p is represented by the set of points in the neighborhood of p means, for Kosko, that $p - 2\varepsilon$, $p - \varepsilon$, p, $p + \varepsilon$, $p + 2\varepsilon$ are all representatives of the truth-value p, although $p - \varepsilon$ and $p + \varepsilon$ are closer than either $p - 2\varepsilon$ or $p + 2\varepsilon$. For multiple-valued logics, on the contrary, the only possible number corresponding to the appropriate truth-value would be p itself.

Both books under review seek to present a popular account of the history, philosophical underpinnings, and commercial technical applications of fuzzy logic. Kosko's book in particular has been receiving quite a bit of attention in the fuzzy logic community since its appearance. Kosko is a scientist who has contributed significantly to the development of fuzzy logic and to its applications in Artificial Intelligence. McNeill and Freiberger seem to be primarily journalistic popularizers of science rather than scientists.

Fuzzy logic is a burgeoning field, and, although its creator was the Iranian-American Lofti Zadeh (born in Soviet Azerbaidzhan), it has attained its most significant development and commercial application in Japan (and, to a somewhat lesser extent, Korea). The most advanced high-speed railway systems in Japan, for example, apply fuzzy control systems. Japanese-produced "smart" household products such as washing machines use intelligent fuzzy control systems to automatically select the proper water temperature, amounts of soap, spin-dry cycles, etc., suited precisely for the clothes to be washed. Both books answer the question: 'Why would a system of thought whose mathematical theory was created in the West by an American and which has such promising commercial application find great acceptance in Japan and be comparatively ignored in the West?' They both provide much the same answer: oriental philosophy has a greater tolerance for *vagueness* as a guiding principle. Kosko focuses in particular on Buddhism, to the extent that McNeill and Freiberger (p. 130) quote him as saying that "My claim is the Buddha was really the world's first fuzzy theorist." In his book, Kosko actually says, however, that "the Buddha was not a fuzzy theorist in a mathematical sense . . . . But he had the shades-of-gray idea. He tolerated A and not-A" (p. 77). McNeill and Freiberger do not say *where* Kosko explicitly asserts that the Buddha was the first fuzzy theorist. The *official* debut of fuzzy logic as a mathematical theory, as everyone agrees, was in Zadeh's 1965 paper "Fuzzy Sets" (*Information and Control* **8**, 338–353).

Both books stress the cultural role which classical bivalent logic has had in the Occident and which Buddhist logic, with its willingness to forego the Law of Excluded Middle, the Principle of Non-Contradiction, or both, has had in the Orient to account for the ability of Japan to embrace fuzzy logic (and concomitantly for the United States to spurn it). They stress the Orient's adherence to Buddhism in this regard, I think, to an unwarranted degree. But they both also assert that multiple-valued logic, along with Bertrand Russell's philosophical analysis of the concept of 'vagueness' played a pivotal role in the creation of fuzzy logic as a mathematical theory. If Kosko should be interpreted as claiming Russell to have been the 'grandfather' of fuzzy logic, it must be in this particular sense only and for this particular historical reason. The reason for calling Russell the 'grandfather of fuzzy logic' rests on a mistake about the nature of the Russell paradox and its implications and about the history of mathematics. Specifically, Kosko's explicit claim (p. 98) that, because "Russell found a set that ended the certainty in math [sic] that had prevailed since before the time of Aristotle," he was "for this and for many reasons . . . the grandfather of fuzzy logic," betrays an inaccurate knowledge of the history of logic.

While it is certain that the Russell paradox led some mathematicians to take non-classical logics, especially multiple-valued logics, seriously, and even led D. A. Bochvar, in 1943 ("*K voprosu o neprotivorechivosti odnogo trekhznachnogo ischisleniya*", *Mat. Sbornik* **12** (54), nr. 3, 353–369) to show that the Russell paradox cannot be formulated in his [Bochvar's] trivalent logic, the primary result was to cause mathematicians to reject Cantor's intuitive set theory in which such pernicious sets as the Russell set arise and to send them on a search to develop axiomatic set theories which excluded pathological sets. Non-classical logics existed prior to any questions raised by Russell's paradox. Charles Peirce raised the question of the possibility of developing non-classical logics as early as 1895, when he entertained the idea of toying with and altering the laws of logic (see Paul Carus, quoting a letter of Peirce, in *The Monist* of 1910, pp. 44–45), a year before Russell even began studying the work of Cantor. Kosko's argument is also based in part on the assertion from Russell's paper "Vagueness," [*Australian Journal of Philosophy* **1** (1923), 84–92] that "All traditional logic habitually assumes that precise symbols are being employed. It is therefore not applicable to this terrestrial life, but only to an imagined celestial one. The law of excluded middle is true when precise symbols are employed but is not true when symbols are vague, as, in fact, all symbols are" (Kosko, p. 92). But basing the claim that Russell first initiated questions about the reliability of classical, bivalent logic by raising questions about the Law of Excluded Middle (LEM) is again false history, since the possibility of developing logics without LEM date back at least to Peirce and N. A. Vasiliev. In fact, several non-classical logics were developed before Russell ever wrote the "Vagueness" paper. Specifically, Peirce (as we just noted) began kicking around the possibility of playing around with and altering the laws of logic (possibly as early as 1895, inspired by non-Euclidean geometry's rejection of Euclid's parallel postuate after reading and reviewing for *The Nation* Halsted's 1894 English translation of A. V Vasiliev's (i.e. N. A. Vasiliev's father's) book on Lobachevsky. And N. A. Vasiliev also began thinking about these possibilities at about the same time. And in 1910, partly triggered by reading the Carus article and partly inspired by Lobachevsky's geometry, explicitly developed a logical system without LEM [and also without the Law of Non-Contradiction], which he called "imaginary (nonaristotelian) logic" in analogy with Lobachevsky's "imaginary" (non-Euclidean) geometry. N. A. Vasiliev began working out the technical and philosophical details of his system from 1910-1913. For this episode, see, e.g. V. A. Bazhanov, "C. S. Peirce's Influence on the Logical Work of N. A. Vasiliev," *Modern Logic* **3** (1992), 45–51, and Note for 166.6-10 of C. J. Kloesel, *et al.* (editors), *Writings of Charles S. Peirce: A Chronological Edition*, Volume 5: *1884–1886* (Bloomington/Indianapolis, Indiana University Press, 1993), 439. We also know that Emil Post published on *m*–valued logics in 1920 and that Jan Łukasiewicz also published his paper on 3-valued logic that same year. Hence, multiple-valued logics, which Kosko takes to be one of the roots, in combination with Russell's evaluation of the concept of "vagueness" of fuzzy logic, existed before Russell entered the field with his "Vagueness" paper.

We might ask, then when and how Bertrand Russell came to express his doubts about LEM. The most likely prospect is that Russell heard about N.A .Vasiliev's non-Aristotelian logic from A. V. Vasiliev, since, as V. A. Bazhanov points out (personal communication, 23 March 1994), Russell met A. V. Vasiliev in Saint-Petersburg in 1920 and had lengthy discussions with him on numerous topics. A. V. Vasiliev's book *Space, Time, Motion* is dedicated to his son N. A. Vasiliev and was published in London with Russell's "Preface."

The assertion by Kosko that Russell is the 'grandfather' of fuzzy logic comes down in the end merely to Kosko's recognition (p. 137) that 'Max Black used the term "vague" because Charles Peirce and Bertrand Russell and other logicians used it to describe what we now call "fuzzy".' This led to assertions by Kosko (p. 298) that Russell carried out work on multi-valued logic 'in the early part of the twentieth century.'

I cannot testify to the degree of accuracy or inaccuracy of Kosko's account of the history of philosophy, but we can readily conclude, from his remarks about the role which Russell played in the history of the development of fuzzy logic, that his account of the history of logic is superficial and incomplete. We also see the superficiality of his treatment of the history of logic, perhaps firstly and foremostly, because he gives the impression that the entire history of logic in the West is Aristotelian. To be fair, however, "Aristotelian" as Kosko uses it in characterizing logic, really means classical, bivalent logic, not syllogistics. McNeill and Freiberger also use "Aristotelian" in much the same sense. More seriously, both books trace the roots of fuzzy logic to many-valued logic, but give (as we have shown in challenging Kosko's account) an incomplete and inaccurate portrait of the history of many-valued logics. Both books note that the "pre-history" of fuzzy logic has its joint origins in the analysis of the concept of "vagueness" by Charles Peirce and Bertrand Russell and in many-valued logics. As McNeill and Freiberger express it (p. 33), Max Black "outlined his proto-fuzzy sets in a 1937 article ["Vagueness: An Exercise in Logical Analysis", *Philosophy of Science* **4** (1937), 427–455]. He agreed with Peirce that vagueness stems from a continuum and with Russell that it has degrees." Kosko gives much less attention and credit to Peirce. On the other hand, Kosko gives much greater attention and credit to Max Black than do McNeill and Freiberger. "Black," says Kosko (p. 139), "extended multiplevalued [*sic*] logic to sets." Both Kosko and McNeill and Freiberger name Łukasiewicz as the creator of multiple-valued logic (Kosko, p. 19) McNeill and Freiberger (pp. 30–31), and Kosko (pp. 19, 103) states unequivocally that multiple-valued logic arose directly in response to the challenge to Boolean-valued logic posed by Heisenberg's Uncertainty Principle. (Presumably, then, the Russell paradox, by virtue of which, according to Kosko's explicit claim (p. 98) that, because "Russell found a set that ended the certainty in math [sic] that had prevailed since before the time of Aristotle," he was "for this and for many reasons...the grandfather of fuzzy logic," was only historically secondary after all as a step in the development of fuzzy logic.) Kosko's book mentions no multiple-valued logics other than Łukasiewicz's; McNeill and Freiberger (p. 32) mention Post's three-valued logic and the multiple-valued logics of Gödel, von Neumann, Kleene, Bochvar, Zawirski, and Reichenbach.

From this historical perspective, the origin of fuzzy logic is owed to the application of the concept of *vagueness* to classical multiple-valued logics. Thus, the relationship between fuzzy logic and multiple-valued logic is that, while fuzzy logic indeed has its origins in multiple-valued logic, many of the ideas that make fuzzy logic effective as the logic of approximate reasoning are not present in traditional m–valued logics, e.g. the concept of a linguistic variable, the fuzzy if-then rule, fuzzy graph, and the fuzzy quantifiers.

If one is searching for a popular account and impressionistic presentation of fuzzy logic along with an overview of its basic ideas and history, either one of these two books would suffice. The McNeill and Freiberger book is, to my taste, too episodic, especially in regard to history, but it has the advantage of giving a more concise definition of its subject than does Kosko's book. It also has an enduring, but not necessarily endearing "gee-whiz" quality. Kosko's book, by contrast, gives a more systematic, but I would not say more technical account of the history and philosophy of the concepts that play a crucial role in the development and application — and certainly not an historically complete or accurate account — of fuzzy logic. There is less emphasis in Kosko's book on the commercial technological breakthroughs to which fuzzy logic has contributed. My personal preference, bad history aside, is thus for Kosko's book. If one deletes his Buddhist framework and his effort in the later chapters to build an entire philosophical system around *fuzziness*, and ignores the historical lapses and infelicities, it gives a much more systematic, if at times relatively informal but still technically sound, presentation of its subject, and is much more concerned with helping the novice understand and appreciate the subject for its own sake, much less concerned, certainly far less concerned than the McNeill and Freiberger book, with whiz-bang gadgetry.

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Alan Ross Anderson, Nuel D. Belnap, Jr., and J. Michael Dunn. *Entailment*, Volume II. Princeton University Press, 1992. xxvii + 749$ pp.

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This is an enormous work, in many senses of that word. It is *long* at over 700 pages. It is blessed with an exhaustive bibliography (of 146 pages, provided by Robert G. Wolf). It has an extensive cast of contributors: the three main authors — Alan Anderson, Nuel Belnap and J. Michael Dunn, together with the contributors of smaller sections — Kit Fine, Alasdair Urquhart, Daniel Cohen, Glen Helman, Steve Giambrone, Errol Martin, Dorothy Grover, Michael McRobbie, Anil Gupta and Stuart Shapiro. It covers a wide range of topics, and it had an unusually large period of gestation, the acknowledgements state that the book had been in preparation since 1959. There were thirty-three years between the book's inception and its completion. But most importantly, the book contains a wealth of insights given only through the many collective years of hard work. In this review I will attempt to give the prospective reader an idea of the range of its contents, and then I will cast a (friendly) critical eye over the work as a whole and just some of the detail.