bulk of Russell's writings. And if history is more than a mere list of names, dates and published theorems, but also includes an understanding of the various intellectual and general non-intellectual factors surrounding, coloring, and influencing the work of those who publish the theorems and those who prepare the way for them, then we owe our gratitude to the authors of the introductions and the headnotes for the BREP volumes for helping us understand the biographical, social, and historical background of Russell's work and thought and for giving us a glimpse of Russell at work and of Russell "talking" about his work.

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А.Г. Барабашев, *Будущее математики: методологические аспекты прогнозирования*, Москва, Издательство Московского университета, 1991.

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I should like to preface my remarks by suggesting that much of what I am about to say concerning the history of mathematics within the context of this review of Alexei G. Barabashev's book *The Future of Mathematics: Methodological Aspects of Prognostication*, doubtlessly applies, *mutatis mutandis*, with equal force to the history of logic as well.

It seems that there has always been an awareness that mathematics has a history. We can see this, for example, even from the extant writings on *History of Arithmetic* and *History of Geometry* of Aristotle's student Eudemus of Rhodes (fl. ca. 320 B.C.). The importance of history of mathematics for contemporary mathematics was understood by Proclus Diadochus (410 – 485), a geometer and historian of geometry whose *Commentary on Book I of the Elements of Euclid* includes the *Eudemian Summary*, a fragment from

Eudemus's *History of Geometry*. Much of our knowledge of the work of Eudemus is due to Proclus.

Despite this, the discipline of history of mathematics, Detlef Spalt [*1994*, 3] recently told us, "seems to have little prestige in mathematics as a whole." The reason for this, in Spalt's opinion, is that the history of mathematics has not yet clearly defined itself, has not, that is, standardized, or even articulated, its methods, its standards of scholarship, its purposes or its purview. It is well known that there are mathematicians who are interested in the work of their predecessors only to the extent that they can incorporate into and utilize past results for their own on-going research, and others whose interest is even more severely limited to avoidance of repetition of previously proven theorems. What is not clear quite clear, then, is what history of mathematics is. What is even less clear is that the history of mathematics has any substantial or significant influence for the "working" mathematician (despite frequent reminders that many of the best mathematicians have always learned directly from the past masters, by studying directly the work of the best of their predecessors). Questions of the intellectual legitimacy and academic "place" of history of mathematics have no doubt had a long *sub rosa* existence, but recently were explicitly and forcefully brought to the fore by Ivor Grattan-Guinness (for example, in [*1990*] and especially [*1993*]).

For an age in which "publish or perish" — or its mathematical equivalent, "a theorem a day means promotion and pay" — is both a necessity and a way of life, not to say the very standard of academic success, this neglectful attitude is easily comprehensible. But if history of mathematics is to have any validity and viability at all as an intellectual and academic discipline, it must do so on its own terms and on its own cognizance, providing its own standards and justification, just as is demanded of any other legitimate discipline. There is, however, a crucial distinction to be made between the academic self-justification and respectability of a discipline for its own sake and its intellectual utility as a by-product of its production. In the utilitarian case, the history of mathematics ought to — and properly done, can — help mathematicians to understand their intellectual heritage. Whether it can help with the prognostication of the future of mathematics, and if so, how, is another matter. This is the question raised and dealt with in the book under review, Barabashev's book *The Future of Mathematics*.

An awareness of the importance of writing histories to preserve the knowledge of the contributions of the past to the present seems to be especially stimulated by sustained periods of rapid growth. Thus, an increasing number of scholars undertook to write histories of mathematics in the waning years of the nineteenth century. As we have come to increasingly appreciate the importance of the history of mathematics, we have also begun to think increasingly about the future of mathematics. The connective tissue between history of mathematics and prognostication on the future of mathematics has been philosophy of mathematics, for while historians of mathematics have been content to trace and describe the history of mathematics (either as an end in itself or from the perspective of contemporary mathematics), philosophers of mathematics have attempted to discern and identify patterns, processes, laws or regularities of the development of mathematics.

If indeed "the past is prologue," then our knowledge of the history of mathematics, coupled with what we learn about the tendencies and regularities of mathematics history and what we know about the present state of research should give us a clue to the directions that mathematics will take in the future. Thus, Hilbert introduced his famous list of mathematical problems at the International Congress of Mathematicians in Paris in 1900 by asking (see [Hilbert *1976*, 1]):

> Who of us would not be glad to lift the veil behind which the future lies hidden; to cast a glance at the next advances of our science and at the secrets of its development during future centuries? What particular goals will there be toward which the leading mathematical spirits of coming generations will strive? What new methods and new facts in the wide and rich field of mathematical thought will the new centuries disclose?

Hilbert's fundamental assumption [Hilbert *1976*, 1] was that "history teaches the continuity of the development of science." The unstated conception behind this supposition is that the progress of mathematics is linear, that there are no detours, or only minor detours and occasional bumpy roads and only rarely are wrong turns made. Thus, foundational philosophies of mathematics, especially Formalism and Logicism, tend to look at mathematics as a finished product, in which historical development is replaced by logical construction.

By contrast, whatever one may think of Marxism as a philosophy and dialectical materialism (diamat) as its methodology, it is undeniable that one of Marxism's chief benefits to Soviet philosophers is that it provided them with the conceptual tool of historical materialism (histomat) required to allow them to think about philosophy of mathematics in particular (and philosophy of science in general) in historical terms, and to see the history of mathematics, therefore, as dynamic rather than static. (Barabashev [*1986a*] has given an overview of the history and ideas of philosophy of mathematics in the USSR.)

So far, Spalt would certainly appear to agree. He writes [Spalt *1994*, 4]:

> Historians in general are first of all the chroniclers of the transitoriness of things, and historians of mathematics are (or should be!) the chroniclers of the transitoriness of mathematical things. If well done, history of mathematics first of all focusses on the changes of mathematical knowledge. At its best, it guides us to a former way of mathematical thinking — and to an understanding of this thinking in such a way that we are able to practise it ourselves. This former way of mathemati-

cal thinking is different from ours today but valid in its own concepts and categories. Only by grasping the difference between this sleeping art and our current way of handling the subjects are we able to realise the true development of mathematical thinking.

Spalt's conception of the nature and role of history of mathematics, however, appears, at least on the surface, to preclude, if not actually exclude, the utility of the study of history of mathematics to trace any "laws" there may be that underlay or direct the growth and development of mathematical knowledge. I say "apparently" because Spalt does not specifically address, or even raise, this as an issue; his concern for history of mathematics is with the subject for its own sake.

Certainly if the development of mathematics really is dynamic, then history of mathematics is more than a collection of proven theorems. Whether it is instead a process whose trends and tendencies one can search for, identify, and study is another question. Barabashev believes that it is. This is a view which is in turn predicated upon the expectation and belief that the history of mathematics is progressive, in the sense that each new result is based upon, and in an important sense incorporates, or generalizes, previous results. This is a view indubitably shared by most mathematicians and which is clearly reflected in most texts on history of mathematics.* Let us accept, at least for the sake of argument, that the development of mathematics indeed is dynamic and that in examining that history a process can be detected whose trends and tendencies can be identified and studied. If so, then as these trends and tendencies emerge through historical development and are identified by students of the history and philosophy of mathematics, one can also ask whether there are any laws and regularities that underlie these trends and tendencies, and then begin to study those laws and regularities.

Our author has devoted much of his career to examining the history of mathematics with the critical eye of a philosopher in search of discovering the tendencies and regularities of mathematical development in such books as *Dialectics of the Development of Mathematical Knowledge* [*1983*] and papers such as "On the Problem of the Origin of Theoretical Mathematics" [*1985*], "Basic Trends in the Philosophico-methodological Analysis of the History of Mathematization" [*1986*], "Methodolgical Problems of the Establishment of Mathematics in Modern Times: An Analytical Review" [*1987*], "Regularities and Modern

* In [Anellis *1989*], it was suggested on the basis of several examples from different eras and different topics in the history of mathematics that mathematical development is not always necessarily continuous or even strictly linear. We cannot therefore conclude, however, just because the history of mathematics sometimes moves by fits and starts, that occasionally the development may be retrogressive rather than progressive, or that blind alleys are sometimes pursued, i.e., that its development is sometimes locally discontinuous, that this development is therefore necessarily globally discontinuous, or that the concept of "progress" in the history of mathematics is either unjustified or unrealistic.

Tendencies of the Development of Mathematics" [*1987b*], and joint studies on "Philosophical Problems of Mathematics" [*1981*] written with V. Ya. Perminov, "On the Evolution of the Structure of Mathematical Knowledge" [*1983*] written with S.S. Glushkov, and "Actual Problems of the History and Philosophy of Mathematics" [*1987a*] written with S.S. Demidov and M.I. Panov. He combines the historicism of the best and most sophisticated version of Marxism with the careful and meticulous training of a historian and philosopher of mathematics, and combines that with the critical acumen and erudition of a philosopher well-versed in the history of mathematics. He sees mathematics as a socio-cultural phenomenon, so that he understands that the development of mathematics thus corresponds to the development of society itself. Once these laws are understood, they provide a predictive anchor for examining the future of mathematics. In the book under review, Barabashev first examines the stages in the development of mathematics and identifies the trends and tendencies throughout its history. This enables him in turn to identify the laws or regularities that underlie the historical development of mathematics and our developing conception of mathematics and its history.

In order to analyze and evaluate the historical developments of the past and understand the regularities and tendencies of mathematical evolution, Barabashev introduces the notion of *cognitive orientations* which characterize the social and cultural conditions of cognition in a given historical epoch. The basis of cognitive orientation is the comparison of unknown phenomena with ones that are known. He identifies three types of cognitive orientations: one seeks to compare objects with other objects, one seeks to compare objects with actions on these objects, and another seeks to compare objects with subjects of actions. Having identified his methodological apparatus, our author asks: "What kinds of cognitive orientations serve as the basis of mathematical knowledge in different historical epochs?" This leads to a rational reconstruction of the history of mathematics in order to answer three basic questions: (1) What are the social and cultural foundations of mathematics in different historical epochs?; (2) What mathematical theories become fundamental, and what influence do external factors have on the choice of these theories?; and (3) What are the tendencies of the development of modern mathematics.

Barabashev's analysis of ancient classical mathematics leads him to the conclusion that the socio-cultural foundation of that epoch of mathematics was the object-object cognitive orientation. This cognitive orientation gave rise to the building of formal structures. He shows that the fundamental theories of ancient mathematics (geometry, theory of natural numbers) were connected with this structure.

In modern times the cognitive orientation is based on a comparison of the objects with their changes. Formalization of this second cognitive structure leads to the structure which creates a set of new fundamental mathematical theories, e.g. group theory, topology, and, most obviously, mathematical analysis. Barabashev seeks in his book to construct and explore a formal structure of the third type of cognitive orientation (which has yet to be realized), in order to find some of the contours of future (possible) cognitive orientations and the principles of the construction of the formal structures of these cognitive orientations, and on this basis, to display new opportunities for the development of mathematics (possible fundamental mathematical structures and theories following from these structures).

Whether or not one thinks that he has succeeded in this effort depends upon whether or not one believes that there really is enough uniformity in the development of mathematics through its history to identify the kinds of general philosophical structures on which Barabashev bases his analysis (and this may well amount to a question of belief in the global, if not local, linearity of the growth of mathematics), and therefore whether the past really can serve as a guide to future potential (not necessarily, however, as a guide to future actuality). Our judgment of the success or failure of Barabashev's enterprise, and the degree of success which we attribute to it, if any, depend on what we mean when asserting that "the past is prologue." Does it mean that the work of our intellectual ancestors prepared the ground for our own work? Or that there is indeed an inevitability of present realities and future prospects that are determined by earlier experiences and achievements? Or that, not only does the work of our ancestors prepare the ground for our own work, not only is there an inevitability of present realities and future prospects that are determined by earlier experiences and achievements, but that these inevitabilities are rooted in objective, or at least theoretically determinable, laws and regularities? Another question that Barabashev's work inevitably but perhaps inadvertently raises, especially in minds which are suspicious of histomat or indeed of any sort of historical determinism, is whether the search for regularities and laws of historical development, either in history of mathematics or in political history — or any kind of history, for that matter, is legitimate or proper. In "The Problem of Hope," culled from his essay "On the Inscrutability of History," historian Arthur Schlesinger, Jr. [*1969*, 525] wrote: "Many professional historians — perhaps most — reject the idea that generalization is the goal of history. ...Indeed, it is the commitment to concrete reconstruction as against abstract generalisation — to life as opposed to laws — which distinguishes history from sociology." In dealing with intellectual history and history of science, we might only wish to replace Schlesinger's sociology with philosophy. On the contrary, there will be those who agree with Schlesinger {*1969*, 535] that "far from unveiling the secret of things to come, history bestows a different gift: it makes us — or should make us — understand the extreme difficulty, the intellectual peril, the moral arrogance of supposing that the future will yield itself so easily to us." Thus, it is important to define the task of the historian and to define the aims of history as a discipline. As Schlesinger [*1969*, 535] noted, a deep study of history can and does provide "not dogmatic certitude but diagnostic skill, not clairvoyance but insight ...[and]...a sense, at once, of short-run variables and long-run tendencies, and an instinct for the complexity of their intermingling." It is also therefore important to understand carefully exactly what Barabashev is attempting to do in his book and to be precise about what sort of "prognostication" Barabashev has in mind.

However one answers the methodological and philosophical questions which the present work raises, an appreciation for the acuity and depth of the author's perception, the philosophical profundity of his analyses, and his broad knowledge of the history of mathematics will in the end be gained by Barabashev's readers. In the process, Barabashev has arrived at philosophical insights and conclusions which will contribute significantly to discussions on the philosophy of the history of mathematics. Some of these insights will be absolutely new to philosophers of mathematics, especially those who are unable to avail themselves of the author's writings in Russian. Others of these insights will appear familiar to readers; but they will discover that the familiarity stems from a close similarity between the conclusions reached by Barabashev and those recently reached independently, but only after Barabashev had already done so, by western writers in philosophy of mathematics who have only recently begun to approach their subject historically. Those who do not read Russian may look forward to the publication by Modern Logic Publishing of his major monograph *Long Cycles in the Development of Mathematics*, which is a revised and expanded version of his doctoral dissertation, translated into English.

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