Patrick Grim. *The Incomplete Universe. Totality, Knowledge and Truth.* Cambridge, MIT Press, 1991. 165 pp.

Reviewed by

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This is a book whose single argument is the impossibility of totalities of knowledge and truth. Grim uses Liar-like arguments, one of the Gödel theorems and Cantorian diagonalization to show that we cannot speak consistently of a set of all truths or any kind of related totality. This fact has several philosophical consequences which we should not pass over. The notion of truth is deeply related to the notions of proposition, fact and knowledge and for this reason Grim's argument has semantical, epistemological and ontological versions. Using the Cantorian diagonal method, Grim shows that the familiar account of possible worlds as maximal sets of sentences or propositions is inconsistent, that there is no set of all facts (so that the *Tractarian* characterization of the world gives rise to contradictions) and that omniscience (as an attribute of God or of any other being) is a contradictory notion. Grim's argument pervades most of the familiar semantical, epistemological or set theoretical systems which we use in the formal treatment of subjects in which the basic notions are set, knowledge, truth, world or fact. The problem Grim points to cannot be solved by the usual minor changes in theories such as substituting the notion of set for class or any other kind of totality, talking about propositions instead of sentences, adopting a redundancy theory of truth or following any of the available set theories.

In the extended first chapter Grim comes back to the most significant proposals in order to escape the Liar paradox—Russell's theory of types, Tarski's hierarchy and Kripke's, Barwise's and Etchemendy's contributions—and shows that all of them amount to or imply a forbidding of these totalities which is also the ultimate moral—learned from the Liar—of his book. The second chapter is devoted to Kaplan and Montague's Paradox of the Knower, a new version of the paradox of the surprise examination. Here, as throughout the book, Grim's strategy is to apply old arguments to new subjects and to analyse the consequences. On p. 49 Grim says, "The full impact of the Knower, however, has not yet been realized—or so I will argue. For what the Knower offers is a surprisingly powerful argument against the coherence of a broad range of common notions if taken in full generality [...], it offers an intriguing argument against any notion of a totality of truth or of total knowledge." In the third chapter, he argues against the ideas of a complete and final knowledge and an omniscient being. An omniscient being must possess some kind of self-reflectiveness in the sense that she must know everything including herself. Nevertheless, any suitable notion of self-reflectiveness is forbidden by one of Gödel's theorems which states that every system powerful enough to contain a deductive system like the arithmetic is essentially incomplete.

All Grim's results should be considered as facets of the same problem, which boils down to: there can be no set of all truths. In chapter four, this result is analysed against the background of some of the best-known set theories. None of our usual set theories can

avoid the consequence that there cannot be a set of all truths. In Zermelo-Fraenkel (ZF) set theory, this negative statement is proved using the Power Set theorem. Alternatives to ZF are the Quinean systems contained in his works "New Foundations" (NF) and *Mathematical Logic* (ML), as are the set theories of von Neumann-Bernays (VNB), of Kelley-Morse (KM) and of Ackermann (A). The system contained in NF has basically two axioms: Extensionality and a principle of comprehension subjected to stratification. The theory in ML consists in an enlargement of NF to include ultimate classes. The other systems Grim considers—VNB, KM and A—also introduce a distinction between sets and proper classes. In von Neumann-Bernays set theory there is a principle of predicative comprehension for classes that Grim formulates as follows, "There is a class C that consists of precisely those sets x that satisfy the condition $C(x)$, where $C(x)$ is a condition that does not contain quantifiers over classes, i.e., $C(x)$ does not contain expressions of the form 'for every class X' or 'there exists a class X'." (p. 104). In KM this principle is replaced by one of impredicative comprehension: "There is a class C that consists of precisely those sets x that satisfy condition $C(x)$, where $C(x)$ is any condition." (p.106). But in spite of the differences, Grim's argument may be rebuilt in all of them. NF stops Grim's conclusion at the very high price of admitting what has been called "Non-Cantorian sets" which are sets that have more elements than their singleton subsets. The Power Set theorem, therefore, cannot be proved within this system. One is left with the choice of two alternatives - either accepting Grim's argument or embracing very anti-intuitive set theoretical systems. In Grim's words, "The lesson of alternative set theories considered so far—NF, ML, VNB and KM—seems to be uncompromisingly negative regarding prospects for any collection of all truths. Stratified and predicative comprehension principles, as in NF and VNB, do seem to cripple the basic Cantorian argument at issue, and in that sense they do seem to offer a tantalizing prospect for some global class of all truths. But in both cases the cost of such restricted comprehension seems to be a corresponding and strongly counterintuitive crippling of mathematical induction as well.[...].

The alternative seems to be an *un*restricted principle of comprehension on the pattern of comprehension for classes within ML and KM. Expanded to classes of *truths*, however, such a principle gives us a full-blown Cantorian argument against a class of all truths and a none too subtle threat of inconsistency as a result." (P. 107).

Grim relativizes his view with care and stresses that it is not always correct to draw philosophical consequences from formal results. On p. 127 we read, "*within any logic we have*, it appears, there can be no coherent notion of total knowledge or of a totality of truths." This cautiousness is sensible most of the time although in this particular case I think that Grim is being too modest. Set theories together with Cantor's and Gödel's views form the theoretical basis of our present mathematical knowledge and we cannot coherently harbour any serious doubt about them. Grim uses diagonal and reflexive strategies against the ideas of a closed world and omniscience and his arguments are as reliable as the very foundations of the paradigm we live in. In a nutshell, only an incomplete universe is compatible with our most widely-accepted formal theories.

Grim's book is highly commendable and its conclusion should be taken into account in epistemology, semantics and ontology. Possible-world theorists would be well advised to think seriously about Grim's results as would theologians and philosophers. This a deep, solid philosophical work.