editors, however, have written: "His spirit will continue to animate all our work, and we have taken his standards as our own" (p. vi). Due to his work on Russell's *Collected papers,* Gregory H. Moore declined to continue after this second volume of the series, which is regrettable, given his ability. On the philosophical side, Charles Parsons and Warren Goldfarb have been added to the team and are in charge of the edition of some of Gödel's philosophically most important manuscripts. This is good news, so that readers should no doubt look forward to reading the forthcoming volume.[6]

[6] My thanks are due to Francine Abeles and Roberto Torretti for improving the English of this paper.

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Raymond M. Smullyan, *Recursion Theory for Metamathematics*, New York, New York, Oxford University Press, 1993. xiv + 163 pp.

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This book is a sequel to *Gödel's Incompleteness Theorems* (reviewed in *Modern Logic,* Vol. 4, no. 1). Smullyan's latest could likewise have been called, with some justice, *Representation and Separation Theorems of Ehrenfeucht-Feferman, Putnam-Smullyan, and Shepherdson*. Although that would not be so catchy a title, those theorems lie at the heart of much of the book. So first, a brief description of these theorems.

In a 1960 paper, Ehrenfeucht and Feferman showed that for certain consistent axiomatizable systems, every recursively enumerable set can be represented in the system by some formula. That same year, a paper of Putnam and Smullyan proved that for such systems, a stronger separation conclusion holds: given disjoint r.e. sets A and B, there is a formula which represents A and whose negation represents B. Shepherdson in 1961 used a different kind of argument to obtain representation and separation under different assumptions about the system.

Now the purpose of R*ecursion Theory for Metamathematics* is not merely to present these results. Indeed, they already appeared, proofs and all, in *Gödel's Incompleteness Theorems*. Rather, here the theorems are examined from several angles, given multiple proofs, strengthened and generalized in various ways, and used as springboards to other techniques and results, both old and new. The book consequently does not have a single-minded linear structure but instead resembles a suite of variations on a set of interwoven themes.

In exploring the connections among different parts of this web of material, Smullyan points to the larger-scale interplay suggested in the title. Recursion theory and metamathematics inform each other repeatedly here. From one side, recursion-theoretic results and notions (the Recursion Theorem, creative sets, effective inseparability, etc.) have significant implications for the properties of first-order systems such as Peano Arithmetic. In return, the systems provide natural ways of producing sets with various recursion-theoretic properties. Smullyan explicitly draws the reader's attention to this level of interweaving as well as to the ties binding his specific themes together.

However, the framework of themes-with-many-variations may not be to everyone's taste, especially as relentlessly as this book sometimes pursues the variations. For example, the final chapter introduces the sentential recursion (SR) property, the double SR (DSR) property, semi-DSR, effectively SR, effectively DSR, and effectively semi-DSR. Then Smullyan proves a succession of parallel results relating these properties to effective, exact, effectively exact, or just plain Rosser systems for sets or binary relations. This kind of approach is typical of much of *Recursion Theory for Metamathematics*, but there are also several fish-eye lenses along the way to help keep the big picture in focus.

In fact, this book generally possesses the reader-friendliness of its predecessor and, like *Gödel's Incompleteness Theorems*, is written for a mixed audience. Several new results are included for the benefit of specialists, while, thanks to the virtual lack of prerequisites and the detailed, easy-to-follow proofs, *Recursion Theory for Metamathematics* is highly accessible to beginning logicians. Indeed Smullyan makes the book self-contained, modulo some basic logic, by devoting an unusually large portion (roughly one-sixth) of the text to a recap of material from *Gödel's Incompleteness Theorems*.

Unfortunately, the poor proofreading/copy-editing of the previous volume has, if anything, worsened in this one. There are countless minor glitches and enough unmatched parentheses to make one shudder at the thought of Oxford University Press ever publishing a book on LISP. Even more annoying is that far too often, the index listing for a topic gives a different page number than appears in the table of contents — and *neither* of those numbers is correct! Trying to track down a definition in this book thus becomes a chore at times. Likewise, many references cited in the text do not appear in the bibliography, and the two use different numbering schemes for references.

Still, the content of *Recursion Theory for Metamathematics* manages to rise above these production flaws. There is much, both in results and in methods, that will be of interest to a variety of readers. And more is in the works. Smullyan plans a "companion volume" *Diagonalization and Self-Reference*, which will bring combinatory logic into the picture — a prospect to look forward to.