Discussion and Queries

(The section previously known as Notes and Queries)

Back-and-Forth Again

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In response to Charles Silver's query in this journal **4** (1993), 74–78, Jack Plotkin has already investigated the question of the origin of the Back-and-Forth Argument. He has given a paper on it at the Logical Methods conference held June 1 – 3, 1992 at Cornell University honoring Anil Nerode's 60^{th} birthday. The conference proceedings have just been published, and includes Plotkin's paper 'Who Put the "Back" in Back-And-Forth?' [*1993*].

There was also a reference to his research on this topic in Peter Cameron's book *Oligomorphic Permutation Groups* [*1990*]. To quote Cameron [*1990*, 124], "Back-and-forth entered the mathematical mainstream with the book of Hausdorff (1914), but had been used earlier, in an exposition of Cantor's work by Huntington (1904). It seems probable that Huntington invented it. (These remarks are based on research by Jack Plotkin[.])" Plotkin's paper does not cite any other publications by him on this topic, so I assume that when Cameron referenced Plotkin, Cameron was referring either to private communication/preprint or to Plotkin's *Journal of Symbolic Logic* abstract.

Plotkin's original announcement was apparently made at the 1989 Association for Symbolic Logic meeting in Chicago. The abstract for the talk (in the *Journal of Symbolic Logic* **55** (1990), 444–445) entitled "Who put the back in back-and-forth?," reads (*in toto*):

> Known but sometimes ignored, Cantor (1895) didn't. Hausdorff (1914) in his ***Grundzuge der Mengenlehre*** did. However, in 1908 when Hausdorff introduced η-sets he followed Cantor in using only *Forth*. Surprisingly, E.V. Huntington (1905) used *Back-and-Forth*, and by failing to say otherwise credited Cantor with its invention. We try to account for the obscurity of Huntington's contribution and we trace the evolution of Hausdorff's mastery of this technique. The η-sets led the way and countable densely ordered sets trailed along.

References

Cameron, Peter. 1990. *Oligomorphic permutation groups*, vol. 152, London Mathematical Society Lecture Notes, Cambridge University Press, Cambridge, UK.

Plotkin, J.M. 1993. *Who put the "Back" in Back-And-Forth?*, John Crossley, Jeffrey Remmel, Richard Shore, and Moss Sweedler (editors), *Logical methods: In honor of Anil Nerode's Sixtieth Birthday* (= *Progress in Computer Science and Applied Logic* **12**, Boston Birkhauser), 705–712.