MEMORIES OF MECH.–MATH. IN THE 'SIXTIES,
INSPIRED BY USPENSKY'S JSL ARTICLE ON KOLMOGOROV'S WORK IN LOGIC*

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> There is no remembrance of former things; neither shall there be any remembrance of things that are to come with those that shall come after. — *Ecclesiastes*, 1:11.

(IN MEMORY OF A.V. KUZNETSOV, F.A. MEDVEDEV, AND A.P. YUSHKEVICH)

§1. The recent article of Uspensky [*1992*] about Kolmogorov's works in mathematical logic should be considered, in my opinion, as a significant event. A.N. Kolmogorov (1903–1987) was undoubtedly one of the most prominent mathematicians of our time; his spiritual heritage is tremendous. It would take efforts of a large team of authors to shape at any detail his contributions to the most various areas of pure and applied mathematics. Such a team could be composed from Kolmogorov's pupils since he left his touch and his school in every area of mathematics in which he worked, even episodically. Mathematical logic was not an exception. Though Kolmogorov's works in this area are not numerous, they are marked by his genius. Time increasingly confirms their permanent significance. However, as I discovered through my personal experience, those works are still not well-known in the West.

Hardly an author more qualified than Vladimir Andreevich Uspensky could be found to write an article under the title "Kolmogorov and Mathematical Logic." As Kolmogorov's closest student and colleague, an excellent mathematician, one of the creators of the

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modern theory of numerations, the author of the first Soviet monograph on recursive functions, the author of a number of other monographs, editor of a collection of Kolmogorov's papers [Kolmogorov *1991*], and a person highly gifted in the humanities, Uspensky possesses the highest professional and personal qualifications to write such an article. If I still miss something in the excellent survey created by Uspensky, it is specifically his personal reminiscences. I can only try to guess how rich they are. But some hint may be given in his [*1991*] estimation of Kolmogorov as "our great contemporary."[1] Such memories could hardly find a place in the framework of the rather formal style of *The Journal of Symbolic Logic* but their absence is still regrettable. V.A. Uspensky had for many years been at the center of a mathematical life of unusual intensity, and I have no doubt that he could present historians of mathematics with lively images of such scientists as A.N. Kolmogorov, P.S. Aleksandrov, P.S. Novikov....

If these lines inspire V.A. to undertake this difficult task I will feel that my work is not done in vain.[2]

§2. I met Uspensky for the first time in 1960 or 1961 when I was an undergraduate student in the School of Mechanics and Mathematics («Мех.-Мат.» = "Mech.-Math.") of Moscow State University. Those were really the "golden years" of Soviet mathematics. A young person like me could in the course of a minute, while walking along one of the Mech.-Math. corridors, meet A.N. Kolmogorov, P.S. Aleksandrov, A.A. Markov, I.G. Petrovsky, S.L. Sobolev, A.N. Tikhonov, L.A. Lyusternik, D.E. Men'shov, I.M. Gel'fand, A.G. Kurosh....

When I entered Mech.-Math. in 1959 people there were still excited by the excellent achievement of V.I. Arnol'd (then a student of Kolmogorov) who had solved one of the

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[1]Kolmogorov wrote an "Autobiography" which was published in Uspensky's collection of [Kolmogorov *1992*], 21–23. Uspensky also wrote a [*1991a*] biography of Kolmogorov, which was published in [Kolmogorov *1991*], 216–221. In March 1981, P.S. Aleksandrov wrote "Some Words on A.N. Kolmogorov," which first appeared in [*1983*] and was likewise included in [Kolmogorov *1991*], 7–10. Uspensky's [*1983*] interview has been published. In addition, there is a brief unsigned piece "Andrei Nikolaevich Kolmogorov (1903–1987)" in the collection of Kolmogorov's non-technical and pedagogical papers, [Kolmogorov *1988a*, 3], presumably by the collection's editor G.A. Halperin. English translations of reminiscences of Kolmogorov by his students V.I. Arnol'd [*1993*] and A.N. Tikhomirov [*1993*] have been published in the collection *Golden Years of Moscow Mathematics* edited by Smilka Zdravkovska and Peter L. Duren [*1993*] which include the memoirs in English of Soviet mathematicians of mathematics in Moscow, especially Moscow University's Mechanics and Mathematics Faculty throughout the Soviet period. The history of logic at Moscow State University's Mech.-Math. is discussed by [Kolmogorov, Adian, Dragalin, Kuzichev, Nogina, Semenov & Uspensky *1992*].

[2]While this work was in the process of publication the author became aware that V.A. Uspensky did write his reminiscences of Kolmogorov. This paper is part of a volume of reminiscences to be published in Moscow. Uspensky has also published his eulogy "Our Great Contemporary Kolmogorov" [*1991*] and, as mentioned in the previous note, a sketch of Kolmogorov's life [*1991a*] in the collection of papers of [Kolmogorov *1991*] which he edited.

Hilbert problems. We were soon to hear such names as A.A. Kirillov, Ya.G. Sinai, Yu.I. Manin, S.P. Novikov....

The very air of Mech.-Math. was intellectually electrifying. The combination of vibrant, readily available classics with the wild (sometimes too wild) energy of the youth was unique. In any case, I never experienced anything similar to that since. I believe today that it was a gleam of the long-past *Luzitania* (several members of Luzitania left very interesting memories).[3] As it had been in those days, student folklore in my student years was very much alive. Long poems about Mech.-Math., written in the inevitable "Eugene Onegin" meter, were passed from hand to hand. A song about a student who died under the unbearable burden of exams was a permanent part of students' parties. The tune and the plot ideas of the song were picked from the very popular folklore song "A sea was spread wide..." (this song seems to have originated in the time of the Russo-Japanese war 1904–1905; our parents' generation associated the song with the popular singer L.O. Utyosov). It is hard to determine the time when this student masterpiece was created. We would sing something like this: "It is impossible to try to pass an exam counting on 'perhaps'. Tumarkin is not pleased with you.... You are to prove Cauchy's Theorem or you will be expelled from Mech.-Math." But I happened to hear this very song with Efimov's name instead of Tumarkin.[4] It seems that a number of Mech.-Math. deans once found their place in this song. The song was concluded with the following impressive line, that had been used as an epigraph by G.E. Shilov in his book about graphs: "And the graph of the sine runs away, wave after wave, along the x-axis...." It was the exciting time of Khrushchev's "thaw." The first man was launched into the cosmos. Professor G.E. Shilov invited me to

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[3] The name "Luzitania" is derived from the name of N.N Luzin (1883–1950), who established an outstanding mathematical school in Moscow in earlier post-revolutionary years. Discussions in English on Luzin, the Luzitania, and the Moscow school of function theory and topology are presented by [Arboleda *1979*], [Cameron *1982*], [Cameron *1986*], [Demidov *1988*], and [Phillips *1978*].

L.A. Lyusternik [*1967*] left his recollections of the Luzitania, P.S. Aleksandrov [*1955*] left his recollections of mathematics in Moscow University during the first quarter of the twentieth century, and Men'shov [*1983*] provided his recollections of the Luzitania years and the work of the Moscow school of function theory. Other histories of mathematics of the period, especially at Moscow University, include [Bogolyubov *1968*], [Lapko & Lyusternik *1967*] and [Rybnikov *1992*].

A rich source for readers of English of reminiscences and remembrances of mathematics and mathematicians focussing on Moscow University from the 1920's to the end of the Soviet period is to be found in the collection on *The Golden Years of Moscow Mathemaztics* edited by Smilka Zdravkovska and Peter L. Duren [*1993*]. It includes, for example, A.P. Yushkevich's account of his "Encounters with Mathematicians" [*1993*], among them Luzin, P.S. Aleksandrov, and Kolmogorov; it also includes important bibliographies of sources in English [*1993*] and Russian [*1993a*] on the development of mathematics in the USSR by S.S. Demidov.

[4] L.A. Tumarkin, an outstanding topologist, was once the dean of Mech.-Math. (long before I entered the school); N.V. Efimov, an outstanding geometer, was the dean when I entered the university in 1959. Efimov's calculus classes that I was privileged to attend were superb.

stop by at his place at the day of Gagarin's flight. He had just composed the tune for a song about Gagarin and wanted me to write verses. The song was performed at a festival concert on that very evening. Certainly it was rather a naive production, but hardly worse than any of the numerous professional productions on this subject released in those days. There are dangerous, "unmelted" places during every thaw. One can slip. One of my classmates talked carelessly at a dormitory, another classmate reported immediately to Party authorities. A turmoil with numerous *Komsomol*[5] meetings followed. The "criminal" was finally expelled from the *Komsomol* and consequently from the university. I especially remember an incident that had happened at one of the above meetings. A young *Komsomol* leader told a large audience that his father was once repressed. "So what?!" he challenged his college classmates. We were silent.... Evidently, the relevant Commandment was declared on Mount Sinai with good reason. This Commandment is not as evident as many people believe. The ghost of *Pavlik Morozov*[6] continued to wander across the country and the grip of the Party machine was as strong as ever. Many years later, an older colleague of mine told me: "I joined the Party at that time to make things better." Really, human naïveté is limitless....

Nevertheless, as in the days of *Luzitania*, vague and naive hopes were in the air. Just as then, mathematics had a halo and its creators were the subjects of a remarkable folklore. N.E. Zhukovsky (1847–1921), in his character of an exemplary absent-minded mathematician, was replaced by the excellent member of Luzitania D.E. Men'shov (1892–1988). I will retell here only two of many legends.

Once D.E. took a walk outside of the city. Being deeply engrossed in his thoughts, he managed somehow to avoid all the sentries and finally found himself in the middle of a restricted area. He was arrested and taken to the security office. To understand the following scene properly, a reader should have in mind that D.E. was a remarkably tall and quite thin man. He had a short but matted beard. To put it mildly, he did not pay particular attention to his clothes. And, besides that, D.E.'s manner of speaking was somehow unusual: hoarse and staccato.

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[5]*Komsomol* is the abbreviation for the All-Union Communist Union of the Youth. It acted under the supervision and the authority of the Communist Party.

[6]*Pavlik Morozov* is one of the symbols of communist propaganda, a half-legendary teenager who in 1932 discovered (allegedly) that his father, a *kulak* (in plain words a rather well-to-do farmer) had hidden some grain from communist requisitioning. Pavlik reported his father to the authorities, the GPU [an earlier name for the KGB.] arrested the father. (The following fate of the father was not mentioned by the legend but one could easily guess...). Pavlik was killed by vengeful kulaks. The fame of Pavlik was limitless: poets wrote verses about him, children sang about him, ships got his name, etc. Thousands of towns had monuments of Pavlik Morozov (almost every children's park had such a monument). The message was clear: loyalty to the Communist Party and its cause goes ahead of every other human obligation.

— "Who are you?"

— "I am a mathematician."

Laughter.

— "Perhaps you are even a professor?"

— "Yes, I am a Professor at Moscow University."

Loud laughter.

— "Perhaps, you are an Academician, aren't you?"

— "No, I am an associate member of the Academy of Sciences."

The rank and file cries of laughter...

Fortunately, the commanding officer finally telephoned the University....

Here is another story. Once, during World War II, D.E. taught a class, supposedly in Tashkent. Class space was short and the weather was hot. Under these circumstances students were placed in the open air, in a courtyard, and D.E. lectured to them from a small balcony. As usual, he became animated and began to gesticulate. It is not known how the seasoned students reacted to his inspiration but Muslim passers-by knelt down. They believed that an esteemed, highly-learned *mullah* arrived to deliver his sermon....

A dramatic breakthrough into infinity performed by G. Cantor was still at the center of discussions in mathematical workshops for devoted high school children in these years. I am afraid that young people are supplied with something more utilitarian and transient, like Prologue or Unix, nowadays.

Even a traditional old "fermatist" with a shabby violin case and a pile of pages filled up in a fancy handwriting (just another proof of Fermat's Last Theorem that was immediately offered on the spot for reading to every interested person around) seemed to be an integral part of this extraordinary world.[7]

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[7]By "fermatist" I have in mind a word usage customary among Russian mathematicians. A "fermatist" is a dilettante who tries to prove Fermat's Last Theorem (we call it Fermat's Big Theorem, as well, in contrast with the nice number-theoretic theorem called Fermat's Little Theorem).

During winters the violin case contained "proofs" of Fermat's Last Theorem. Legends (which were not denied by its hero) stated that in the summer time D. used to travel on riverboats earning money for living and contemplating Fermat's Last Theorem by playing the violin for passengers. I suppose that the productivity of D. was 1.5–2 "proofs" of Fermat's Last Theorem per season. The mathematical public was presented in my time with proofs under numbers 16*ca.* (variants of "proofs" were marked by letters, say 16E). D. knew by heart all the leading universities and mathematical institutions and all the leading mathematicians in the USSR. His personal relations with the latter were sometimes tense. He even took somebody in court (at least D. stated it). A legend said that shortly after the phototelegraph was introduced in the public service, D. sent a New Year's congratu-

Pavel Sergeevich Aleksandrov, who already in my time wore glasses with huge lenses, was always surrounded by a crowd of disciples. Being nearsighted, he would confuse his pupils with "outsider" students. So a classmate of mine was once pleasantly surprised when P.S. offered him his hand in an elevator and asked friendly: "Hello, how are you doing?" At the end of the short trip to the thirteenth floor my friend admitted that he was a freshman, after all. "And I promoted you into graduate dignity," laughed P.S.

The memory of P.S. Urysohn, who had died tragically in France in 1924 (he drowned while swimming in the ocean), was still as painful as if the disaster happened just recently. Swimming was a customary element of famous "topological walks" (out of town escapes of Aleksandrov and his pupils). Once P.S. was almost killed by a careless motorboat driver in Dnester river. The friendship of two "P.S." bore a romantic halo and Aleksandrov's disciples used to tell a touching story that P.S. Aleksandrov had once presented a book to P.S. Urysohn with the signature "to PSU from PSA."[8]

P.S. Aleksandrov, one of the founding fathers of modern topology, was an exceptional person. For example, he was able in a matter of course to utter a long citation from, say, "Faust" (in the original, of course) in the middle of a Scientific Council meeting. Sure enough, such meetings made one recall both Faust and Mephistopheles quite often. Once

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lations phototelegram to the Steklov Institute. One could see on the blank an impressive set of donkey heads. The last names of famous mathematicians from the Steklov Institute were calligraphically written under the heads. But I should admit that I myself never saw D. in an aggressive mood. He would usually sit in a corner on a bench with students crowded around him and present his work to everyone interested. After concluding his presentation he usually asked for a receipt of a quite moderate nature: "I, so and so, a student of such and such year of Mech.-Math. came through the proof 16E of the Fermat's Last Theorem performed by D. No evident fault was discovered during my superficial examination of the proof." It was difficult to judge whether D. was himself confident in his proofs. I had once heard him saying with proud: "I had shown this proof to Mikhail Mikhailovich Postnikov. He told me that my mistakes grew more and more flowery." Besides Fermat's Last Theorem, D. was involved in his early years in constructing *perpetuum mobile*, too. He loved to tell a story about that time. There was an Associate Professor who used to drive him out at the beginning of their acquaintance. Than he began to qualify D.'s ideas as those of a genius. But precisely at the time when cooperation was about to begin, the Professor was taken away to a madhouse. Every interested person was allowed to look into a movie plot "Mathematical Stalingrad," as well. The planned movie presented the unfortunate fate of Mathematicians (all were mentioned by name) who denied the ideas of D. [Editor's note: the sketch of a plausably correct proof of Fermat's Last Theorem was given at a series of lectures at the Newton Institute at Cambridge University by Andrew Wiles of Princeton University on 23 June 1993. Wiles's tactic is to prove the Taniyama-Weil conjecture for elliptic curves, which, by results of Frey and Ribet, implies Fermat's Last Theorem. See Kenneth A. Ribet [*1993*] for a sketch of Wiles's proof.. The details of the proof, now taking some 200 manuscript pages, are still being worked out and checked.]

[8]A play on words that can not easily be translated. "псу от пса [*psu ot psa*]" means in Russian "to a dog from a dog."

Aleksandrov's [*1925*] obituary of Urysohn was published in French. Yushkevich [*1985*] wrote on Aleksandrov's work in history of mathematics.

in the mid-sixties, I attended a public lecture of P.S. about geometry. Evidently, P.S. treated geometry in a very broad, rather French way. A large hall on the first floor of the main building of Moscow University was filled with mathematicians and other university people. The lecture proceeded in an excellent way till a noise in the entrance interrupted it just in its middle. After a momentary confusion, a whole army of photographers, television and newspaper reporters invaded the hall. President De Gaulle and the rector of the university I.G. Petrovskii, surrounded by a group of people of unknown professions and in civilian clothes, emerged behind them (the President of France was then in Moscow on an official visit). Both De Gaulle and Petrovskii made a sign in Aleksandrov's direction. The meaning of it would be the same in every language: "For God's sake, please, forgive the interruption and continue, do not pay attention to us!" P.S. switched immediately to French and continued his inspired account about dimension theory. The guests listened with attention from their places in the first row. But in about ten minutes Petrovskii excused himself and interrupted the lecture again. P.S. let De Gaulle take the floor. The President, in turn, excused himself and delivered a short speech. He told that he was sorry that due to the shortage of time he was not able to enjoy the excellent lecture of Academician Aleksandrov in full. He was extremely grateful to both the lecturer and the rector for this exceptional opportunity to speak in the walls of such an illustrious institution etc, etc ... Then De Gaulle and the rector walked to the exit. And all the crowd disappeared after them exactly as quickly as it had emerged earlier. I should admit that it occurred to me at that moment that politics and politicians had been changed with time: Napoleon would definitely listen to such a lecture until its very end.[9]

It seems that P.S. could be quite sharp from time to time. A big painting, performed in the best tradition of "socialist realism,"[10] hung on a wall of Mech.-Math. It pictured a meeting of the university faculty with the "All-Russian Elder" M.I.Kalinin in the 30's.[11]

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[9]It is widely known that Napoleon had a special interest in geometry. There is even an elegant theorem about triangles that is attributed to him (the so-called Napoleon's Theorem).

[10]"Socialist realism" is a term invented, it seems, by the writer Maxim Gorki and widely used by Soviet propaganda. There is no room here to describe the essence of the complicated phenomenon behind it. Basically, it means the type of art that is approved by the Communist Party and that conforms to the Party's current goals.

[11]M.I. Kalinin (1875–1946) was (beginning in 1938) the Chairman of the Presidium of the Supreme Council of the USSR, a type of a puppet parliament during the communist era. In this quality he was semi-officially mentioned in the media as "the All-Russian Elder" (все-российский староста) in an approximate translation. He was the President of the USSR under diplomatic protocol; he conferred decorations etc. Readers can make a judgement about the level of the influence he enjoyed from the following inconceivable fact: after the war, while he was still in the office, his wife (the first lady, formally) was imprisoned during one of Stalin's purges (incidentally, the fate of Molotov's wife, the second or the third lady of the USSR, was the same!). Another fantasmagorical detail: Kalinin's wife was temporarily released from imprisonment to attend her husband's State funeral.

The whole scene was full of happiness; one could almost make out a halo around Kalinin's head. Young Aleksandrov was recognizable among the public. It seemed that he addressed Kalinin with a question. Long-standing faculty of Mech.-Math. used to recall this question with pleasure. The point was that restrooms were in grave shortage in the old Mech.-Math. building on Mokhovaya street. Moreover, available restrooms were in very bad condition (incidentally, Lyusternik [*1967*, part 3] mentioned this fact, too). Hence, Pavel Sergeevich asked Kalinin to help with establishing a new restroom for the faculty. Kalinin, who eagerly answered general questions about the educational system of the USSR, about the significance of sciences in the communist education etc., became angry and advised P.S. to approach the physical plant supervisor.

Musical evenings, which P.S. led on a rather regular schedule in student dormitory halls, enjoyed great popularity. He would bring some rare discs from his large record collection. Listening to the music was usually preceded by a short speech of P.S. I would like to say that I never met a person with such an astonishing eloquence.[12] P.S.'s speech was organized superbly, it developed fluently, elegantly, without any observable efforts. Subjects, images and associations arose apparently by themselves. I had once witnessed P.S. making an absolutely smooth (I would say analytical) passage from Brahms[13] to the immorality of the bacteriological weapon and back to Brahms. I had the priviledge to attend a few of Aleksandrov's public lectures based on his personal recollections. His accounts gave the impression of a miracle: such names, as Hilbert, Hausdorff, Brouwer, Noether were rising alive in front of our eyes. I can not help but to attempt to reproduce one of P.S.'s stories. He was talking about a term (in the mid-twenties, if I am not mistaken) that he spent in Göttingen. P.S. taught a class in (then young) set-theoretic topology. At the same time, N. Wiener, who was then in Göttingen too, taught another mathematical course. P.S. was an exceptional lecturer. Wiener, outstanding mathematician though he was, was not evidently the best teacher. In any case, students travelled from Wiener to Aleksandrov until almost no one remained in Wiener's class. The personal relationship between the two young mathematicians became strained, since Wiener evidently attributed all that had happened to Aleksandrov's intrigues and even filed a complaint with the Ministry of Education. By an old tradition, every mathematician who visited Göttingen had to pay visits to the Göttingen professors. When the turn of E. Noether arrived, Wiener asked her to arrange the time for his visit. "Well, stop by tomorrow, say, at 7 o'clock" answered Noether; she was not much interested in formalities. Next morning, at 7 o'clock sharp, P.S. (he resided in Noether's house) was awakened by a loud knock on the door. Having had supposed that it was some misunderstanding with the milkwoman, who every

[12]Among speakers I had heard, only I.G. Ehrenburg, V.A. Uspensky and B.V. Gnedenko approached Aleksandrov in this respect to any degree.

[13]As far as I am aware, P.S.'s preferences were rather conservative. Some of his disciples maintained that music for P.S. finished with Brahms.

morning replaced an empty bottle behind the door with a full bottle, P.S., as he was in the underwear, walked to the door, unlocked it, opened slightly and put his head outside.... At this point of the story horror appeared on Aleksandrov's face, a horror that did not lose a drop for the half a century. "Just imagine! Behind the door was Wiener in a tuxedo!"

I had once attended a Ph.D. defense as an official opponent. The subject of the thesis was the Jordan theorem for the constructive plane. I mentioned, among other things, an old work of Brouwer that treated the problem from the intuitionistic point of view. It was rather a hard task to read Brouwer's work. P.S. became excited as soon as Brouwer's name had been mentioned. He began to address questions to me. It was evident that lively memories connected him with the very name of Brouwer. "Yes, Brouwer was a great geometer, his geometrical intuition was extraordinary. It is, probably, the reason why his works are so hard to read," P.S. concluded in our exchange that I shall remember for ever.

It was during another Ph.D. defense in the mid-seventies that I heard P.S. for the last time. The thesis was in mathematical linguistics. It summarized many years of research of the author, a prominent expert in the area. At this time, something that A.A. Markov in a conversation with me once called "the Kingdom of Darkness," coalesced remarkably in Soviet mathematics. Various personalities, trends and human defects were represented in this Kingdom. It was partly an ordinary conflict between generations, but in part there were mediocre personalities who used Party and Komsomol channels to promote their careers.[14] Sometimes there were talented and very talented people who had no scruples in using the above channels, sometimes there were nationalists, etc. In this particular case the nationality of the defendant was irreproachable (exactly as the thesis itself) but on the other hand the thesis was written in the Department of Mathematical Logic chaired by A.A. Markov and, moreover, a positive reference from A.N. Kolmogorov was in the file. The last circumstance probably effected the above-mentioned public like a red rag would affect a bull. In these years the tendency to neglect any proposition, reference, etc. signed by Kolmogorov could be already felt quite clear. Numerous *mos'ki*[15] did not dare to attack the aging giant directly but they barked in plenty behind his back. Would they feel themselves ashamed one day? I would like to hope.... The thesis was attacked this time by two residents of "the Kingdom of Darkness." Both were well-known in Moscow University.[16]

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[14] I remember a remarkable student from the School of Mechanics. Getting alternating F's and D's he literally crawled to his graduation. But his *komsomol* activities were tremendous. He could be found in the 80's at the very top of the university's hierarchy.

[15] *Mos'ka* is a small dog, the hero of a very popular Russian fable of Krylov. It barked at a circus elephant. When asked why, it answered that everybody would believe that it was very strong since it barked at the elephant. Hence Mos'ka would get a reputation as a fighter without taking any risk.

[16] One of them was a classmate of mine, a very talented and unscrupulous man. His career was incredible. It included hazard card gambling, *Komsomol* and Party offices and quite professional mathematical research. Another attacker was a prominent expert in number theory.

A silent and well-controlled group of members of the Scientific Council backed the attackers. I was extremely surprised when an outstanding geometer of the older generation, P.K. Rashevskii, strongly supported the defendant. The subject of the thesis was quite outside of his own scientific interests but he could not stand to the side in the face of this insolent highway robbery.

It is worth noting that one of the attackers, let us call him, say, X, insisted in quite a demagogic way that the dissertant should explain to him complicated constructions in formal grammars for a minute, "on fingers." P.S. Aleksandrov took the floor. My heart fell down when I saw his small, bent-shouldered body, his sad eyes behind the large lenses of glasses. His speech was not long. "I happened to know Brouwer, and I can state that if X would demand of him to show "on fingers" why, say, a three-dimensional manifold can not be topologically mapped on a two-dimensional one, Brouwer, the great Brouwer would refuse to answer such a question of X." The combination of the names of Brouwer and X sounded deadly.

I can not help but to recall here words that Aleksandrov had once said beside the casket of a colleague of his. "When I die and you will attend my funeral, please, do not say that I was a 'principled man'. 'The adherence to principle' is an ersatz of lively human feelings...." Well, when the day had come, nobody uttered such banalities about him. It was a sea of flowers, music and pain. Several generations of colleagues, disciples, friends came to say farewell to him. It was Kolmogorov, who approached the coffin supported, by Arnol'd and Sinai. Fighting back a disease that impaired his speaking faculties, he tried to say farewell to the great scientist, the great personality and the friend of his life.[17]

§3. V.A. Uspensky has always made and still makes a strong impression by his artistic way of lecturing and by his exceptional personality. Once (in 1966 or 1967) a workshop for high school teachers was conducted at Moscow University. Numerous teachers arrived from all corners of the country. V.A. Uspensky gave a few lectures on mathematical logic and graduate students followed with recitation sessions. Uspensky won his unusual audience at the very first lecture. I observed an amazement and fascination on numerous faces: "Is it really possible to speak about mathematics in such an exciting way?" Explaining why it is reasonable to consider an implication with the false antecedent to be true, Uspensky argued roughly in the following way:

> Imagine that I have just said: "May I fall through this very spot if I lie!"[18] This means 'if I lie than I will fall through;' thus we deal with an implication. The

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[17]Kolmogorov's obituary of Aleksandrov, written with B.V. Gnedenko [*1983*], and his reminiscences of Aleksandrov [*1986*] were both reprinted in [Kolmogorov *1991*].

[18] A common way to swear an oath in Russia.

convincing force of such swearing rests on assuming it to be true. But look! The antecedent is false, I am not falling through!"

— At this point V.A. examined the floor with his leg in a distinctly careful gesture (the lecture was held on the sixteenth floor!).

It was not without horror, that I entered my class the next day. I felt strongly that I was only twenty-five years old. The room was filled with high school teachers, including greyhaired female teachers praised in so many songs, poems, books, movies etc. I really felt at this moment that it would not be so bad to fall through the floor. But the audience turned out to be extremely friendly. At one moment whispering in the back rows became too loud (the listeners were to take a sightseeing tour immediately after the class and they were excited). I interrupted my explanations and looked reproachfully into the audience. The noise had subsided and then everybody, those seasoned in the battles with trouble-making students soldiers of the pedagogic front, and myself, a young graduate student, exploded with laughter. I tried to continue with something from the algebra of logic but the listeners turned again and again to the same subject: Uspensky. They wanted to know how old he was, how long was he in mathematics.... Somebody even asked whether V.A. was married. So I had finally to deliver a short talk about Vladimir Andreevich. Well, I did it with pleasure.

At the beginning of the 60's I began to attend Uspensky's seminar on computable functions. I remember one of the first sessions when Uspensky, not being able to answer a question of a participant, declared straightforwardly: "I am aware that this seminar runs the risk of losing all the listeners thanks to the stupidity of its head, but nevertheless I can not answer your question!" Problems would be offered to the participants at the end of every session and the same ritual question would be asked at the beginning of every following session: "Well, who did solve the problems?" As this question was asked, we used to look simultaneously into the right back corner of the room. V.A. looked in the same direction, too. A man of a Herculean build would raise his arm high up there. He smiled broadly. "Sure, you did, Sasha," Uspensky would declare. "Well, who else? Nobody?! Impossible!" "Sasha" was the outstanding mathematician Aleksandr Vladimirovich Kuznetsov, one of the most brilliant and the most admired personalities among Soviet logicians. A self-made scholar (he did not have even a formal high school education), A.V. Kuznetsov was always surrounded by the mathematical youth. He had worked in various areas of mathematical logic, and had left behind after his premature death in 1984[19] an original and significant school. A well-disposed, placid man with an unusual smooth, tuneful manner of speaking, he could, however, explode occasionally. It happened sometimes under quite unfavorable circumstances. I recall that already after he had moved to Kishinev, in one of

[19]A.V. Kuznetsov was born on October 28, 1926 and died on July 24, 1984.

his visits to Moscow he had had a violent conflict with a militiaman (a policeman). The officer annoyed A.V. immensely by his cavils about *propiska*.[20] In a fit of anger, A.V. attacked the officer and tore off the shoulder-straps from his uniform. The victim was especially indignant since, as the officer had maintained it, it was only the day before the accident that he had received his uniform from a taylor. A formal investigation was launched right away and the case could have a very sad conclusion had not Kolmogorov and Markov intervened.

A.V. had his small somewhat pleasant weaknesses. Once he delivered a long series of talks (on intuitionistic analogues of the Sheffer stroke) on the seminar of Markov and Nagorny in the Computing Center of the Academy of Sciences of the USSR. The meetings were scheduled for 11 a.m. exactly, but A.V. would appear every time at 11:40 sharp. He followed this pattern with the precision of a good watch. When he was late for the first time, he presented the audience with wordy apologies. He blamed...the sun. It prevented him from arriving on time. Indeed, the beautiful, clear, winter sun of Moscow gleamed through windows. Really, who cares about the Sheffer stroke in a day like this? Every other time thereafter when A.V. was about to open his mouth to utter an excuse, Markov would be quicker: "It was the sun!" he would announce in a majestic voice. Everybody laughed. What a wonderful, sunny mood reigned in these talks! A.V. spoke and wrote smoothly, he would often explain again and again already covered stuff. Almost half of every session was routinely spent in bringing up topics of the previous one. But nobody objected: everybody was charmed by the harmony and deepness of his results, by the integrity of his style and his personality. It was like a good book. One reads it and by and by is getting scared observing that fewer and fewer pages are left and, hence, the separation with the miraculous world of the book is closer and closer.... But the mystery of "the quantum of the lateness" still stood. The mystery was solved by N.M. Nagorny. "But it is so simple! It takes exactly 40 minutes of walking to reach the Computing Center from Kuznetsov's apartment. Sessions start at 11, so Sasha leaves the apartment at 11, sharp."

His thoroughness and some slowness notwithstanding, A.V. had a quick reaction. He possessed a sense of humor himself and liked it in other people. During one of the above-mentioned talks he said for some reason: "And here I will argue in a constructive way!" — "Is it possible? You are still a 'classic'!" replied Markov somewhat sarcastically. Kuznetsov answered instantly: "Well, you know, who lives among wolves should howl as they do!" Everybody laughed with pleasure.

Some people confused Kuznetsov's good nature with naïveté. It was a mistake. He possessed a strong, sharp mind and he was artistic. An expository lecture was given at the First All-Union Conference (Symposium) on Mathematical Logic in Alma-Ata in June of

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[20]*Propiska* (internal passport) was an essential element of the totalitarian control system in the USSR. Every person after age 16 had to bear such a passport with a police stamp stating his residential address. Residing in a given locality without the corresponding stamp was illegal.

1969 by a leader of the then-young Soviet school of mathematical cybernetics (the term "discrete mathematics" was introduced later). This leader was beyond doubt a talented man. His career was not easy. Unfortunately, he got himself increasingly into non-mathematical intrigues, into the struggle for power.... Later his school joined almost unanymously "the Kingdom of Darkness." The lecture made a strange impression on me. The speaker discussed (if I am not mistaken) the problem of evaluation of the number of precomplete classes in multi-valued logics. The old Ph.D. thesis of the lecturer and recent outstanding results of I. Rosenberg, which were announced in the *Comptes rendus Acad. Sci. Paris*, were at the center of the consideration. After Rosenberg's publications a number of his results were obtained *independently* (as the speaker maintained) by lecturer's school. It is worth noting that A.V. Kuznetsov was one of the pioneers in the theory of multi-valued logics. He discovered the fundamental theorem about the finiteness of the number of precomplete classes in finitely-valued logics. While paying well-deserved respect to A.V., the lecturer noticed (quite justly) that Kuznetsov did not find an explicit list of all precomplete classes in the 3-valued logic. This result was obtained by the lecturer. The official conclusion of the Conference took place the next day. A lot of speeches both official and unofficial were delivered. Finally, Kuznetsov's turn came. He walked to the podium and watched for a moment the large amphitheater in front of him. The southern sun of Alma-Ata sneaked through distant, narrow windows under the ceiling. It played on his face. A.V. screwed up his eyes in the sunlight with evident pleasure. Really, he had some special, personal relationship with the sun. He began to speak in his usual smooth, good-natured and somewhat lulling manner. He continued to smile to the sun while speaking.

> The conference was of interest, of great interest. Great success. It was very interesting. I heard a number of remarkable talks. But it was Y's talk that I had heard the other day that was the most intelligible. I did not hear such an intelligible talk for a long time! Well, it is true. I did not count the precomplete classes in the 3-valued Logic. Sof'ya Aleksandrovna[21] did admonish me at that time: "Sasha, find the classes!" But I did not.

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[21]S.A. Yanovskaya was an outstanding expert in mathematical logic and philosophy of mathematics. She was one of the founders of the Department of Mathematical Logic at Moscow University. Interesting recollections about her activities in the pre-war mathematical life can be found in Lyusternik [*1967*, part 2]. Sof'ya Aleksandrovna had already suffered a dangerous sickness when I was an undergraduate and then a graduate student. Nevertheless, she continued her traditional class in mathematical logic. She continued to perform her duties as one of the heads of the department's Research Seminar, as well. Until the very last days of her life, S.A. maintained a lively interest in every new development in mathematics. On one Spring day in 1966 I saw her home. While separating, she told me that this Spring was to be the last for her, that she had already not felt the perfumes of this Spring... She died on October 25 of that same year. An English obituary of Yanovskaya was

— At this moment A.V. screwed up his eyes with ultimate pleasure and plunged his face into the warm flow of the sun: "I... was lazy!"

Evidently, it was a rather dangerous business to provoke Kuznetsov....

§4. It was the interest in philosophy and foundations of mathematics that to some extent determined my choice when I entered (in 1961 or 1962) the Department of Mathematical Logic for my graduate studies (*cf.* [Kushner *1992*]).[22] At the same time I gave a talk about intuitionism at a Seminar on History of Mathematics. A little later I gave a similar talk in the Seminar on Mathematical Logic and Constructive Mathematics (which A.A. Markov and N.M. Nagorny headed).[23] Two small books of Weyl [*1934*] and Heyting [*1936*] were the main sources of my erudition at that time. Both books were translated before the war by Professor A.P. Yushkevich, an outstanding expert in history of mathematics. Yushkevich mentions in his interesting recollections about Kolmogorov [*1990*] that it was Kolmogorov who suggested those translations be made. I found Yushkevich's work to be excellent (during my student days I did not realize, of course, what a challenge such translations offered to a translator; it is especially true in the case of an author with a remarkable literary gift, such as H. Weyl). At the same time I read two earlier ([*1925*] and [*1932*]) works of Kolmogorov on mathematical logic The essence of these works is presented in detail in the expository article of Uspensky [*1992*]. It is hard not to be amused while thinking about Kolmogorov's work of 1925. This work of a twenty-two year old student impresses one by its outstanding maturity.[24] It stands essentially ahead of its time. The creative touch of Kolmogorov's talent can be clearly recognized. There is everything in this work: formulations of problems with deep philosophical motivations, tremendous power in developing related conceptual and technical tools, and in overcoming particular mathematical difficulties. It is enough to mention that for the first time a mathematical investigation of intuitionistic logic and formulation of axiomatic systems for this logic, anticipating the far later axiomatization of intuitionistic mathematics by A. Heyting, are undertaken in this student's work. The so-

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prepared by [Bocheński *1967*]; English biographies of Yanovskaya are [Anellis *1987*; *1987a*] and [Bocheński *1973*].

[22]For a discussion in English of Soviet work in philosophy of mathematics, see [Barabashev *1986*].

[23]My reflections on Markov and his work in constructive mathematics are found in [*1992*] and [*1992a*], the latter paper giving particular attention to his work in constructive analysis.

[24]Editor's note: Pioneering works on logic without the principle of *tertium non datur* was carried out as early as 1910–1912 by the Kazan logician N.A. Vasil'ev (1880–1940). He developed the so-called "imaginary" (non-Aristotelian) logic based upon rejection of the Law of Excluded Middle, in analogy with the "imaginary" (non-Euclidean) geometry based upon rejection of the Parallel Postulate (see Bazhanov's article on Vasil'ev, THIS ISSUE). This work was known to the teacher of Kolmogorov N.N. Luzin. Luzin's review of Vasil'ev's work has only recently been published (see, [Luzin *1988*]).

called "minimal calculus" is quite recognizable in this work, too. This calculus in its full modern form was introduced by Johansson in [*1937*] (the term itself was introduced by Johansson, as well). I consider even more important the idea of embedding of classical mathematics into intuitionistic mathematics that was (for the first time) developed by Kolmogorov in his work of 1925. As a result, a proof of the relative consistency of classical mathematics with respect of intuitionistic mathematics came to be possible. For the realization of the above idea the first of the known embedding operations was invented. (This operation was based on a deep understanding of the mathematical usage of the negation). At that time it would take a prophet to voice the idea that intuitionistic mathematics is only on appearance narrower than classical mathematics! This idea was rediscovered by K. Gödel only in 1933. All the problems under consideration were suggested by important philosophical problems related to the Law of the Excluded Middle. After Brouwer's criticism, the unreliability of this logical principle with respect of infinite sets was perceived by a number of mathematical thinkers, especially by D. Hilbert and H. Weyl. Those types of doubts were familiar to Kolmogorov as well. But unlike many of his older colleagues, the twenty-two year old student was completely aware of the challenge behind the following question. Why had the unreliability or even illegality of unrestricted usage of the principle of the excluded middle stood unnoticed for such a long time? Why did such an unrestricted usage not manifest itself in contradictions?[25] The answer Kolmogorov gave to the above challenge was, in a few words, as follows. First, the use of the law of the excluded middle is fully justified in the case of finite collections, i.e. for finitary sentences. Second, there is a far more important argument: had a contradiction appeared in a classical theory that admitted the unrestricted use of the principle of the excluded middle, a contradiction would necessarily have appeared in the corresponding intuitionistic theory, where the use of the principle was allowed only in safe finitary cases. In other words, the principle of the excluded middle does not introduce *new* contradictions. A considerable impact from Hilbert's ideas can be felt in the first thesis. But the second one (embedding classical mathematics into intuitionistic mathematics) is strikingly new. Technical tools for the realization of this embedding is given both by Hilbert's concept of formalization of mathematical theories and by Kolmogorov's idea of an embedding operation. At the same time, besides the justification of the usage of the law of the excluded middle (an extremely important mathematical tool from ancient times[26]), Kolmogorov's approach evidently assures some justification for our remarkable (but as everything remarkable not always safe) ability to operate with actual infinity. Classical mathematics

[25]It is well-known that the principle of excluded middle is not responsible for paradoxes of the set theory. However, I met many good mathematicians who were not aware of this fact. In some manuals Russell's paradox is presented in a rather careless way, so that it seems that it has to do with the principle of *tertium non datur*.

[26]Aristotle expressed it, e.g., in the *Metaphysics* B, 2 (996^b26–30) and Γ, 7 (1011^b23–24).

with its actual infinite sets is being embedded into a mathematical universe where infinity is admitted in its much milder, potential form, only.

A.G. Dragalin and I were asked in 1974 to write an article on intuitionism for the third edition of the *Great Soviet Encyclopedia.* The article [Dragalin & Kushner *1974*] was sent to Kolmogorov for a reference. When I kept in my possession the copy of the manuscript with Kolmogorov's remarks, I was again amazed by the freshness of his grasp of the philosophical and mathematical area he had left so many years ago.

It is interesting to ask the question: Why should a young student become at all interested in such peripheral questions that were apparently so distant from the interests of his mathematical environment. Certainly, a strong influence of D. Hilbert and the heated discussions on the foundations of mathematics between him and, Brouwer, the leader of intuitionists, should not be dismissed. On the other hand, the remark about the mathematical environment to which Kolmogorov was subjected is completely unjustified! Thanks to the coincidence of a number of various reasons, problems of foundations of mathematics, and of intuitionistic mathematics in particular, were at the center of lively discussions in Moscow during the 20's (*cf.* the recollections of Yushkevich [*1990*]). A.Ya. Khinchin gave public talks on intuitionism in these years. He published in 1926 an article on intuitionism. Some echoes of this Khinchin's involvement can be felt in some of his books, as well. And finally, it has to be mentioned that N.N. Luzin, who was the founder of Luzitania, the teacher of Kolmogorov, of Aleksandrov and a number of other outstanding mathematicians, was not only a mathematician of a great practical power. He was a deep mathematical thinker, too. It is enough to mention that he took part in the famous correspondence-discussion at the beginning of the century on set theory (especially, the Axiom of Choice) among leading French mathematicians (see [Borel *1928*]). The prophecy of the latest independence results in set theory, made by Luzin, is fascinating, too. Thus, it is not surprising that Luzin's disciples perceived mathematics rather as a living organism, the very functioning of which presented an exciting problem, rather than as mere technical manipulations with formulas and puzzles. The Marxist enthusiasm that had been so characteristic in earlier post-revolutionary years was, paradoxically, favorable for this attitude of mathematics as a live and growing organism, as well. I can not evaluate the level to which this enthusiasm was already poisoned in the 20's by sheer careerism, demagogy and the complete dogmatization of philosophy that I was to observe later.[27] But it is difficult to avoid the impression that numerous "hot heads" assumed (or better to say, believed) quite sincerely in that post-revolutionary era that the "philosopher's stone," the final scientific answer to all problems of Existence and the Universe, is found in Marxism.

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[27]With the onset of *glasnost'*, the story of the vissicitudes faced at Mech.-Math. by Florenskii, Luzin, and Egorov in the late 1920's have become well-known in the West; in English, see e.g. [Demidov *1988*], [Ford *1990*; *1991*], [Levin *1990*], and [Shields *1987–1989*], and, in Russian, Yushkevich [*1989*].

Some understanding of this interesting psychological phenomenon could probably be obtained by reading Lenin's works. They are filled with inexhaustible, purely religious confidence that he does possessed the only correct, ultimate methodology capable of explaining everything and anything. This confidence entails that this man with the exceptionally sharp, critical mind intervenes without a shadow of doubt and humor into areas of science in which he is completely incompetent. He lectures Poincaré, Mach, Einstein, *et al.* The same attitude is apparently responsible for Lenin's famous aphorisms like "an electron is as inexhaustible as an atom," "Marx's teaching is omnipotent because it is correct," etc, etc.... Those aphorisms were literally hammered (along with other less innocent dogmas) by *bol'shevik* propaganda into the conscious (and subconscious!) of the subjects of the former Soviet Empire.[28] Perhaps, the following, illustrious declaration of Lenin constitutes one of the ultimate expressions of this ridiculous, aggressive incompetence: "...One needs imagination EVEN in mathematics; EVEN to discover the differential and integral calculus imagination was needed." (These immortal "even" are stressed by me). Later, in Stalin's case this initial confidence in possessing of the absolute truth was evidently overshadowed by possessing of the absolute power and by the feeling of ultimate impunity. But something of this confidence is still recognizable, say, in the notorious research of "the Leader of All Peoples" in linguistics. It seems that the attitude, in what unforgettable A.I. Zhdanov lectured (allegedly, even behind a piano) Shostakovich, Prokofiev and Khachaturian how to compose good, melodic music, was of the same nature.

It is not surprising that the temptation to apply Marx's miraculous medicine to cure mathematics was quite considerable in the 20's. Discussions on foundations of mathematics were welcomed and, of course, a lot of interesting ideas occurred in those discussions (along with tons of wordy garbage) in those distant years of coldness and hunger. Yushkevich [*1990*] had mentioned such discussions in his recollections of Kolmogorov. Kolmogorov's speech was somewhat unsophisticated in its manner but it made a strong impression on Yushkevich, especially, thanks to Kolmogorov's remark that intuitionistic mathematics is only on appearance narrower than classical mathematics. I

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[28]Correspondingly, I am here citing Lenin from memory. I believe that such a way of "citing" is quite natural in this context.

Lenin's major "contributions" to philosophy along these lines were his «Матеріализмъ и эмпиріокритицизмъ» (*Materialism and Empirio-criticism* [*1909*]) first and foremost and his posthumously published «Философские тетради» (*Philosophical Notebooks*), first published in his collected works and then published as a separate volume in 1933.

"Quotationism" or "citationism" and praise of dialectical materialism did not escape mathematics, but were often a matter of self-preservation. Loren Graham [*1991*, 277], for example, wrote: 'Kolmogorov also praised dialectical materialism at official occasions, such as a 1949 celebration at Moscow University of Stalin's seventieth birthday. A. N. Kolmogorov, "Matematika Stalinskoi epokhi," Archiv MGU, fond 2, op. 4, ed. khr., esp. pp. 11–12.'

believe that this remark was some fifty years ahead of its time. In any case, I began to hear such remarks only in the early 80's. But then they were already based on the enormous experience accumulated by a few generations of researchers.

The second pre-war work of Kolmogorov on logic [*1932*] is of the same level of originality. This work was published seven years later than [*1925*], in German. The subject of the work is an interpretation of intuitionistic logic. While the situation with the semantic of classical logic was more or less satisfactory, the substance of intuitionistic logic was the subject of numerous discussions. Speaking quite roughly, the classical, set-theoretic conception of mathematics, that is going back to G. Cantor, suggests a platonistic, ideal, completed Universe. Mathematical objects exist in this Universe independently of our creative mind in the same sense as stars exist in the sky. The completed, actual infinity is a quite natural idea for this Universe (thus, e.g., the natural series of this completed Universe should be completed, actual infinite, as well; otherwise one should admit the existence of the greatest natural number, and that is, at least, a strange thing to do). Mathematical sentences speak about the order of things in this Universe. Hence, they are true or false independently of our mind, the level of our knowledge, etc, too. Not only the existential status of mathematical objects is admitted in its utmost absolute sense, but the very cognitive process is considered here in the same absolute sense. Mathematical theorems are not being invented by researchers but rather are being discovered by mathematicians in the same way as new islands used to be discovered by navigators. It is evident that the law of the excluded middle is quite in place in this "black-and-white" Universe and that correspondingly classical logic may be considered the logic of theoretical truths, i.e. the logic of completely idealized mathematical existence.

In contrast, the intuitionistic mathematical Universe is by its very nature incomplete. It develops as a result of the creative activity of a creative individual. Speaking poetically, the act of the Creation is attributed here not to God, but to a human being, more exactly to an idealized human being who lives and creates in Time. The nature of the resulting mathematical world depends on the activities and skills of this individual. What, in this case, is the essence of intuitionistic logic, this "constitution" of intuitionistic mathematics? The concept developed by Kolmogorov assumes that the objects of intuitionistic mathematics (and, hence, of intuitionistic logic) are not absolute truths (as in the classical case) but problems. Logical operators form new problems from already obtained problems. Formulas of intuitionist logic express abilities to solve related composed problems. Thus, intuitionistic logic is considered to be a logic of skills, abilities, etc. Under this approach the law of the excluded middle loses its universal nature. To assume this law would mean to postulate the ability to solve every problem at every moment of time. Such a postulate hardly looks natural. Kolmogorov's interpretation is interesting also for its neutrality. From now on, intuitionistic logic can be explained to a person who either does not understand the complicated philosophy of intuitionism or who simply is not interested in it. Being

deprived of its esoteric status, intuitionistic logic arises as an attractive research area for "ordinary" mathematicians. I think that the essential progress in the development of intuitionistic logic achieved after World War II is essentially due to the a new approach, just sketched, that goes back to Kolmogorov. It is worth noting that, among other things, this progress discovered practical applications of intuitionistic logic in computer science.

Kolmogorov's research on the interpretation of intuitionistic logic was developing alongside with efforts of A. Heyting, an outstanding Dutch logician, a student and a disciple of Brouwer. Many of the ideas of Kolmogorov and Heyting turned out to be similar. Nevertheless, Kolmogorov was practically almost never mentioned in this respect in the logical literature in the West. It seems very important to me that, to restore historical fairness, two outstanding members of the Dutch School, Heyting's pupils D. van Dalen and A. Troelstra have introduced the new term "Brouwer-Heyting-Kolmogorov interpretation" in their recent excellent two-volume monograph [*1990*]. Thanks to Troelstra, recent publications ([Troelstra *1990*] and [Kolmogorov *1988*]) of letters from Kolmogorov to Heyting became available. The letters were discovered by Troelstra in the Heyting archives. Professor Troelstra, with whom I kept a friendly correspondence for many years, kindly sent to me copies of these priceless historical documents (belonging to 30's). It would have been just great to find letters from Heyting to Kolmogorov among Kolmogorov's papers. To the best of my knowledge, it unfortunately proved to be impossible. In the meantime V.A. Uspensky suggested publishing Kolmogorov's letters in a Russian translation (the originals were in French and German) in *Uspekhi Mathematichekikh Nauk*. This idea was realized thanks to the kind permission of Professor Troelstra. The Kolmogorov-Heyting correspondence, even only partly available, sheds new light on the early history of intuitionism and on the personalities of these two outstanding scientists.

Exactly as was the case with the article of 1925, this new work of Kolmogorov did not become widely known. It seems that Kleene was not aware of this work at the time when he was writing his famous [*1945*] paper on realizability. The semantic of realizability, that turned out to be so fruitful, has something in common with earlier ideas of Kolmogorov.

In general, there is a mystery in the fate of these two works. In spite of the worldwide fame of their author they were practically unknown outside of Russia. As was already mentioned, a number of related results were rediscovered by other researchers. As I found out myself after emigrating to the US, the significance and the very existence of these works are not known to numerous first-class experts in the West even now. One can hope that Uspensky's articles published in English and in one of the most prestigious journals on logic will change this unfortunate situation.[29]

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[29]The language barrier is often mentioned in connection with such problems. I am afraid that the situation is rather more complicated. Firstly, it is not any easier, say, for Kolmogorov to read English than for any of his English-speaking colleagues to read Russian. Secondly, the article of 1932 is

§5. The next section of Uspensky's article talks about Kolmogorov's works on general theory of algorithms and on algorithmic foundations of probability theory. I would like to stress that Uspensky took an intimate part in these efforts of Kolmogorov. The general concept of algorithm that was conceived and developed by Kolmogorov in cooperation with Uspensky and that came to be widely known constitutes probably the most general

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written in German, and the article of 1925 had long ago been published by Professor van Heijenoort in an English translation. Thirdly, I can not help but to remember the similar fate of the excellent work of P.S. Novikov [*1943*], that was published in 1943 IN ENGLISH. Even this did not help. To this very day the work is practically unknown outside of the (former) USSR. I leave it to the reader to look for the reason of this phenomenon. (Editor: It should be borne in mind that Soviet mathematicians worked in virtual isolation from their Western colleagues for about half of the 20th century, ca. 1925–1975. Reflecting this situation in the late 1920's, Beeson [*1985*, 432-433] wrote: "It does not seem that either Heyting or Glivenko was aware of the work of Kolmogorov; but Glivenko wrote in French, and was in touch with Heyting...." It may also be noted that Soviet journals were sometimes inaccessible in the West during much of the Soviet period, and that Glivenko, in contrast with Kolmogorov, frequently published in Western journals, especially Belgian scientific journals. On the other hand, Heyting quickly learned of Kolmogorov's [*1932*] paper and lists it in the bibliography of his *Intuitionism: An introduction*; cf. p. 130 of [Heyting *1980*]. One may also cite the hostility with which a portion of the Western mathematical community regards constructivism of any variety; witness, for example, the assertion by Keisler [*1977*] that Bishop, being a constructivist, was *ipso facto*, unqualified to review his calculus textbook [*1976*].)

In connection with the above-mentioned English translation of Kolmogorov's article of 1925 I would like to present a short but remarkable letter of Kolmogorov (in my English translation; the original is in Russian; the copy of the letter was obtained thanks to the kindness of Professor I.H. Anellis from the Jean van Heijenoort papers, 1946-1983, Archives of American Mathematics, University Archives, University of Texas at Austin).

Professor John van Heijenoort
100 Washington Square Section L,
New York 3 N.Y. USA

Moscow V 234
University
apt. 10

A.N. Kolmogorov

Dear Colleague!

My work published in 1925 can be considered as a general property of experts on mathematical logic and I have nothing against its translation. I count still on your kindness in sending me a copy of the volume you are editing, upon its publication.

With a sincere respect,

November 12, 1963

Yours, A. Kolmogorov.

As for the incredible life story of van Heijenoort himself, one can find it in an excellent biography written by Anita Feferman [*1993*]. Van Heijenoort's work in logic and its history is discussed in detail by [Anellis *1994*].

precise description of intuitive algorithms. Algorithms in this specific sense are usually called algorithms of Kolmogorov-Uspensky. I stress this nuance specially since it is not mentioned by Uspensky himself for obvious reasons. The definition of Kolmogorov-Uspensky turned out to be very fruitful both for applications (e.g., complexity theory) and foundations of mathematics. In contrast with other classical precise definitions of algorithms (Turing machines, recursive functions, normal algorithms of Markov, etc), that intend to reproduce the performance of an arbitrary intuitive mathematical algorithm by means of an algorithm from the given precise class (Church's thesis, the Turing thesis, the principle of normalization, etc., are all declarations of the universal possibility of reaching this goal), the definition of Kolmogorov-Uspensky tries to present the most general conceivable mathematical algorithms explicitly.[30] The analysis of the nature of finitary processes that precedes that definition is of great interest from the methodological point of view. Some authors go so far as to think that this analysis gives a legitimate proof of Church's thesis (cf. the interesting work of E. Mendelson [*1990*]).

In 1953/1954 Kolmogorov invited Uspensky to share with him leadership of the seminar on recursive arithmetic. Uspensky's remarks about this seminar is definitely of historical interest. Historians of mathematics will be interested to follow the connection between work on descriptive set theory in the Moscow School of Luzin and researches on recursively enumerable sets in this seminar on recursive arithmetic.[31] (If I am not mistaken, similar developments took place at about the same time in the seminars of P.S. Novikov as well.) It was during the recursive arithmetic seminar that Kolmogorov suggested the main ideas of the future theory of numerations. These ideas were developed in a precise form for the first time by V.A. Uspensky.

Uspensky's article gives an impressive account of Kolmogorov's last feat, namely the creation (by him and by yet another generation of his students) of a basis for the algorithmic theory of information and probability. These works of Kolmogorov take us directly to the present. Related theories are not completely formed as yet. The sharpening of intuition, the search for fundamental concepts and for their perfection is still in progress. The dramatic initial steps of this process are vividly presented by Uspensky. I would only supplement his descriptions by a few personal observations and recollections since I too have been a witness to these events.

Since I was not an immediate student of Kolmogorov, my personal meetings with him were not numerous. But I shall forever remember each such meeting. The first meeting took place in the mid-60's when I was a graduate student of the Department of Mathematical Logic. S.A. Yanovskaya intended to organize a meeting of the Moscow Mathe-

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[30]See [Kolmogorov & Uspensky *1958*].

[31]A connection between these two theories is especially recognizable in the hierarchies of sets in the theory of recursive functions (Kleene-Mostowski heirarchy, etc).

matical Society on programmed methods in education. Leading experts in mathematics, education and psychology were to be invited. S.A. asked me to deliver her letter to Kolmogorov who had been staying then in his *dacha* in Bolshevo-Komarovka near Moscow. This *dacha*, that he had shared this dacha for many years with P.S. Aleksandrov, was, of course, well-known in mathematical circles. It was not without trouble that I finally found the unremarkable house on that cold and dark winter evening. Kolmogorov came to greet me in a ski suit. As always, his head was a bent forward a little. Every visitor, whatever age and rank he/she had been, was always treated by him extremely correctly. Thus he (seeing me for the first time) shook my hand and invited me to sit down and make myself warm. Reading through Yanovskaya's letter he told me that unfortunately he would not be able to give the talk that S.A. had asked him to deliver. He did not feel that he was an expert in the area. Kolmogorov suggested that we approach B.V. Gnedenko instead, who (if my memory does not fail me) gave the required talk. As for the meeting of the Mathematical Society, I remember only one rather comic episode. One of the speakers, a psychologist-enthusiast presented in a very animated way his unique and beyond any doubt final solution of the problem of teaching mathematics to children. "For example, how should one teach addition?" he asked quite rhetorically. "Only a few people know what addition is all about." Then he looked into the hall that was filled with mathematicians and added: "You do not know what the addition is about!" At this point, A.G. Kurosh exploded and objected with indignation: "WE know what the addition is all about!"

Generally, activities for preparation of this meeting turned out to be very beneficial for me. I got to know S.A. Yanovskaya more closely and I met one of her disciples, B.V. Biryukov, a philosopher. It was from him, that I had heard the name of Aksel' Ivanovich Berg for the first time. A.I. Berg was an academician, an admiral, an outstanding scientist and personality. Many years later, when my monograph was stacked in the depths of the Editorial Council of the Nauka Publishing House,[32] it was Berg's vigorous interference that saved the situation. During those days I happened to spend a few hours in the apartment of Biryukov's mother. Her residence was at one of the side streets of the *Taganka*.[33] This world disappeared long ago. I am so sorry that I did not immediately put her stories on paper. What a tragic and authentic document about life in the communist state might have been obtained! With her recent passing another living witness of tragic events, capable of describing them, had gone forever.

My subsequent personal meetings with Kolmogorov almost always took place on occasions of presentations of my articles to *Doklady Akademii Nauk SSSR* (Proceedings of the Academy of Sciences of USSR). I remember the following episode in particular. I obtained some results that I considered interesting on some rather exotic systems of

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[32]Nauka was the official publisher of the Academy of Sciences of the USSR. The "Kingdom of Darkness" mentioned earlier exercised a remarkable influence on its editorial board.

[33]The *Taganka* was a very colorful neighborhood in old Moscow.

computable real numbers. As always in such cases, A.A. Markov telephoned A.N. Kolmogorov and Kolmogorov asked me to deliver the article to his private apartment, situated in one of the wings of the main building of Moscow University. A.N. met me at the doors, found a vacant corner on his desk (the desk was piled with papers), wrote his report and asked for a copy of the work. I thanked A.N., handled him a copy of the manuscript and was about to leave. But he stopped me and made a few remarks about my work (quite positive to my extreme delight). It was not simply a courtesy. From the nature of these remarks I concluded that A.N. was aware of the essence of my previous work and had a clear perception of the results I obtained this time. It should be noticed that at the time the majority of my colleagues, who devoted themselves exclusively to mathematical logic, had no understanding of the subject on which I had then been working.

The universality of Kolmogorov's talent, his capacity to grasp the whole body of mathematics (and not only mathematics), to grasp it immediately, at a single glance, was amazing. Already in my student years I had heard stories, similar to legends, about the ease with which Kolmogorov read mathematical works. Once a Professor of mechanics told me a story about a Ph.D. defense of a friend of his. A multi-level formula for some probability appeared on the blackboard. The defender did not perform complicated calculations using this formula (there were no computers then), so the probability under consideration stayed, so-to-speak "a thing in itself." But the defender caught a quiet remark of Kolmogorov who declared (though not insisting) that this probability should be a $1/3$ (or something to this effect). This remark astonished the dissertant and after returning to his place he took the pain to calculate the related number. Kolmogorov's prognosis was completely confirmed. I remember too my wife's amazement after a seminar on turbulence. A talk of the Academician Millionshchikov attracted numerous listeners. Kolmogorov was among them. It is known that A.N. obtained outstanding results about turbulence and even created a scientific school of his own in the area. But in the 70's turbulence was hardly at the center of his interest. Nevertheless, from a few remarks he made during the talk, it was evident that he understood what was happening more quickly and clearly than had the rest of the audience. This despite the fact that a number of first-rate experts on turbulence were present.

A humorous principle in the "formalization of "Female Logic" which Kolmogorov discovered, meant as a good-natured joke, became a part of Russian mathematical folklore. The "principle" says: "If B follows from A, and B is nice, then A is true."

Kolmogorov's public lectures always turned out to be remarkable events. In the 60's A.N. delivered a few lectures on the theory of automata. A huge hall on the first floor of the university could not hold all the public; part of the audience used to stand in the lobby and to listen to the lecture through loudspeakers. In those years discussions about cybernetics were still not forgotten. Out-and-out Marxist philosophers still managed to curse it as "bourgeois pseudo-science." It was perhaps one of the last manifestations of the

Marxist enthusiasm that I have mentioned previously. Demagogy is a wide-spread and common disease of public conscious. But this flaw takes an especially malignant turn in a totalitarian society. I believe that it would be extremely useful to translate into numerous languages the stenographic report on the discussions on biology that took place in VASKHNIL in 1948. (VASKHNIL is the abbreviation for the Lenin All-Union Academy of Agricultural Sciences. Lenin was evidently a great innovator in agriculture, too.) The war, launched with the blessing of the Leader of All Peoples by the charlatan Lysenko and his henchmen against the "bourgeois theory of weismanism-morganism-mendelism," ended in the destruction of the Soviet biological science.[34] As in any other war, this one had its victims, victims in the literal sense of the word. Some heroes of this war, like the notorious Academician Mitin, distinguished themselves on other battlefields. Their talents seemed to be universal. They assisted in dismantling (in the course of a chemical discussion) the bourgeois quantum theory of molecules of (it seems) hydrogen. This theory was malignantly developed by the capitalist obscurantist "Poling" to undermine the advanced, domestic theory of Butlerov-Markovnikov. (If I am not mistaken, this very scientist was almost at the same time or a little later praised to the skies by Soviet propaganda, under a little different Russian spelling of his name — Pauling — as a progressive figure, a friend of the USSR, a fighter for peace, etc.)[35]

In one of his public lectures Kolmogorov talked about a tour around the world he undertook on a research ship of the Academy of Sciences. A discussion arose among members of the crew provoked by a popular-science wireless radio program. Various opinions were raised. The discussion was settled only after an Academician X. gave his explanations in the next program. Kolmogorov was amazed: "But I told them exactly the same.... But to no avail.... The Academician of their own, here aboard, is not a real Academician...." Well, no one is a prophet in his own village....

Various activities of Kolmogorov included educational problems, as well. One might mention in this respect a boarding-school for mathematically gifted children that was organized in Moscow with his support, a reform of the secondary school mathematics curriculum, and many other things. In 1972 for the first time Kolmogorov taught an obligatory undergraduate course in mathematical logic for mathematics students of Moscow University. I already wrote (in [*1992*]) about the unusual atmosphere and events surrounding this course. I believe that mathematical logic owes to Kolmogorov the survival of its scientific center in Moscow University. When A.A. Markov died in 1979, the Department of Mathematical Logic was in real danger of being swallowed by the

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[34]For general discussions in English of the history of Soviet science, see, e.g., [Graham *1972*], [Joravsky *1961*] and [Medvedev *1979*]. For a specific discussion of Lysenkoism, see [Joravsky *1970*] and [Medvedev *1969*]. For Kolmogorov on Lysenko, see [Arnol'd *1993*, 133].

[35]Editor's note: Both references are indeed to chemistry Nobel laureate and peace activist Linus Pauling.

school in "discrete mathematics" I mentioned previously. This school enjoyed essential administrative influence at the time. Quite probably the Department of Mathematical Logic would have been absorbed into the school of discrete mathematics if Kolmogorov had not intervened. His already poor health notwithstanding, he chaired the Department. For a number of years thereafter he led the research Seminar of the Department, too, the seminar connected with the names of P.S. Novikov, A.A. Markov, S.A. Yanovskaya. In his later years it was evident that his very presence at sessions was very difficult for him. Still he was almost always in his place in the first row.[36] Thanks to him...!

At the beginning of the 60's Kolmogorov began to develop a new concept for the theory of information and probability, based on the notion of algorithmic complexity of a constructive object which he had introduced. It is hard to overestimate the boldness and unexpectedness of this undertaking. It is known that probability theory retained a touch of mysticism as late as the beginning of our century. Attempts to place this theory on a reliable mathematical basis were only partially successful. This theory was still waiting for its Weierstrass. It was Kolmogorov who had succeeded at the beginning of the 30's in developing the rigorous axiomatic system for probability theory that reduced probability theory to measure theory. Kolmogorov's axiomatic system is universally accepted today. Thus Kolmogorov can for every reason be considered one of the founding fathers of the mathematical science of probability. It makes it the more surprising to see that, the great achievements, the safety and comfort today's probability theory enjoys notwithstanding, its own creator returned to the very beginning, to the mystery of randomness and suggested a completely new approach to the problem. A thorough mathematical account of this approach can be found in the excellent article of Uspensky. I would like only to add that, at about the same time, A.A. Markov also became interested in problems of the complexity of algorithms.

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[36]I remember a talk given by N.A. Shanin about quantifiers of limited existence. Talks of Nikolai Aleksandrovich were always remarkable events. They impressed one by the significance of the problems under consideration, by the temperament and the human charm of the lecturer, and by his uncompromising search for the truth in mathematics. I did not ordinarily share the philosophical standpoints of N.A., so heated discussions between us were a usual occurrence. Some other participants of the seminar would follow the same pattern. It is worth noting that Shanin definitely liked those discussion and in those exceptional cases when everybody was quiet, he looked quite disappointed. I was especially interested in the above-mentioned talk since I worked with systems of computable real numbers for which such types of quantifiers was natural. My interest in such computable numbers (and in related types of quantification) was repeatedly and sharply condemned by N.A. Hence I anticipated a kind of revenge. Nevertheless, no discussion occurred. Kolmogorov, who was sitting in the first row, looked so unwell, that nobody could think about something else. Nikolai Aleksandrovich delivered rather quickly his talk. His sorrow and anxiety were evident. Nevertheless, Kolmogorov found the strength to half-raise himself and to thank N.A. at the end of the talk. I think that was the last time that I heard Kolmogorov.

In contrast with the complexity of computations where some definite successes were already obtained,[37] problems of the complexity of descriptions of algorithms were completely unexplored in the earlier 60's. The pioneering work of A.A. Markov carried out during the period 1962–1964 ([*1964*; *1967*]) created a basis for the corresponding theory. In particular, it became possible to find a new quantitative representation of the complexity of unsolvable algorithmic problems through so-called estimates of the complexity of the solution. I will explain this topic in short: Suppose we would like to find an algorithm that recognizes, for every natural number n, whether or not n belongs to a given set M. It is well known that in numerous cases such an algorithms does not exist. At the same time, the problem under consideration, say P, can be approximated by finitary problems P_k where the problem P_k demands that an algorithm be found which recognizes the membership with respect to the set M for all natural numbers not greater than k. One can, for every particular k, try to estimate the complexity of the description of the arbitrary algorithm that solves P_k. It is clear that, if this estimate grows to infinity while k increases, then initial problem P is algorithmically unsolvable.

Markov's ideas and results were significantly developed in the work of his students. On the other hand, thanks to a similarity between problems in Kolmogorov's theory of complexity of constructive objects and in the theory of complexity of algorithms in Markov's sense, an crucial cooperation developed between Markov's and Kolmogorov's schools in the 60's. As had happened once with Uspensky, the young mathematician N.V. Petri was invited by Kolmogorov to co-chair for the newly established seminar on complexity theory. I should mention on this occasion the tact with which Kolmogorov handled the case. Since Petri was a student of Markov, Kolmogorov telephoned Markov and asked him if he had any objections. Markov told me about this phone call. "Sure, I answered that I have no objections at all. Quite the contrary." I saw that Markov was very pleased.

On the other hand, Kolmogorov's students began to appear in Markov's seminars. I remember especially the excellent, forceful and eccentric L. L. (now a professor at an American University). L's unpredictability[38] would upset Markov from time to time. But,

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[37] Pioneering results in the estimates of the complexity of algorithmic computations were already obtained in the 50's by Markov's student G.S. Tseitin. The book of B.A. Trakhtenbrot [*1967*] gives an excellent introduction to the area.

[38] A.G. Dragalin once complained to me: "You see, I asked Lenya to give a talk on information theory in my seminar. Not only was he considerably late, but he began his talk like this: 'Consider some senseless sequence of words, say 'Glory, to the KPSS!'" (KPSS is the Russian abbreviation for the Communist Party of the USSR. This slogan could be seen almost on every street throughout the USSR.) I remember the following funny episode, as well. At one of the meetings of our seminar the question about the quantity of information that one object contains about another was discussed. L. stood at the blackboard and Markov asked him a tricky question: "Well, for God's sake, what information does a phone book contain about Eugene Onegin?"—"The phone number of Eugene

beyond doubt, Markov considered the great mathematical talent of Levin very highly and later took a lively part in his fate, especially in 1971, when the "Kingdom of Darkness" dealt with Levin's Ph.D. thesis (the defense took place in Novosibirsk). All the reasons for the harsh treatment of the thesis were present: the defendant was of the outrageous nationality and, moreover, Kolmogorov was his adviser.

§6. On a gloomy day of October 1987 Moscow mathematicians bid farewell to Kolmogorov. Trees, under the guard of iron gates, old brick walls and militiamen, were still in the smooth yellow colors of Moscow's autumn.[39] It was warm and quiet. Only crows cried about something of theirs, something eternal.... The silhouette of the university could be vaguely guessed far away, on the hill behind the river. When, by an old tradition, I dropped a handful of earth into the open grave, I finally felt with pain something that my mind already knew: a whole epoch was gone forever with Kolmogorov. Then people dispersed on the cemetery. Everybody had somebody among the buried here, if not a relative or a friend then Chekhov or Shostakovich.... I paid respect to the tomb of P.S. Novikov and L.V. Keldysh, contemplated near the niche where the ashes of S.A. Yanovskaya were resting and walked to the gates.... It was getting dark, the year 1987 was approaching its end. My separation with Russia lay ahead.

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Onegin"—suggested somebody from the audience. (Eugene Onegin is the main personage of Pushkin's novel in verse "Eugene Onegin;" the novel and the hero's name are greatly popular in Russia).

[39]A.N. Kolmogorov was buried in the cemetery of the Novodevich'e Convent in Moscow. The convent was founded in 1524 by Grand Prince Vasilii Ioaanovich (son of Tsar Ioaan Vasil'evich, i.e. of Ivan the Terrible) as a fortress-convent to honor the recapture of Smolensk from Lithuania and played an important political role in Russian history from its founding; many important figures from the history of Russian political, scientific and artisitic life are buried in the section of the cemetary known as the "Old Cemetery." Novodevich'e was built in a bend in the Moskva River, southwest of the Kremlin on an ancient road to Smolensk, on the north bank of the river; a short distance across the river, in the Sparrow Hills (during the Soviet period called the "Lenin Hills") overlooking the convent, is Moscow State University.

Yushkevich [*1983*] wrote in English on Kolmogorov's work in history and philosophy of mathematics; an obituary of Kolmogorov was prepared by the American Mathematical Society [*1988*].

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