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THE IMAGINARY GEOMETRY OF N.I. LOBACHEVSKY AND THE IMAGINARY LOGIC OF N.A. VASILIEV

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Abstract. This article deals with conceptual parallels between N.I. Lobachevsky's and N.A. Vasiliev's ideas. The emphasis is placed on heuristic prompts connected with Lobachevsky's geometry which promoted construction of imaginary logic by N.A. Vasiliev.

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The great heuristic significance of the discovery of non-Euclidian geometry is in undermining the conceptual basis of the idea of Absolutism. Thus the perspectives for the ideas of relativity, and of a plurality of theoretical systems were cleared up and consolidated. The possibility of Lobachevsky's geometry enables us to reason according to analogy and along with classical systems (the icon of which is Euclidian geometry) assume the fact of the existence of non-classical systems, to claim, that is, the fact of their presence. This fact inspired scholars in the quest for non-classical theories.

Surely the path to constructing such theories was rocky, as it was for non-classical logic.

In describing the landmarks of this path, one should take into account that the shifts within classical systems leading to the appearance of non-classical versions have been accomplished gradually. These changes may occur steadily and sometimes take long periods of development to loosen the foundation of the classical system, exerting contention with the latter and, hence, open up the possibility of a breakthrough to non-classical systems.

The fact that *in principle* alternative systems complementary to classical systems are admissible — the realization of this fact, — acted as a powerful stimulus in the search for these systems.

Until about the 1880's there was a widely spread conviction that classical, Aristotelian, logic is unique and contained an absolutely complete formulation of the laws of logic. This standpoint is clearly expressed by I. Kant and predominated through almost all of the twentieth century (before mathematical logic was created).

In the "Preface" to the second edition of the *Critique of Pure Reason*, Kant argues:

> That logic from the earliest times has followed this sure path [i.e of a science] may be seen from the fact that since Aristotle it has not had to retrace a single step, unless we choose to consider as improvements the removal of some unnecessary substleties or the clearer exposition of its doctrine, both of which refer to the elegance rather than to the solidity of the science. It is remarkable, also, that to the present day it has not been able to advance a step, and is thus to all appearance completed and perfect. If some of the moderns have thought to enlarge [it], ...this could only arise from their ignorance of the peculiar nature of logical science.... The limits of logic are quite precisely determined [Kant *1787*, B VIII].

The same point of view was expressed by the philosophers of science in the early twentieth century (e.g., Duhem in 1915).

The irony of history rather often displays itself in the fact that soon after a certain prominent scholar's judgement of some domain of science as "completed" and "perfect," movements begin the result of which is a demonstration of the openness and incompleteness of this domain of science. That happened with Kant's appraisal of the state of Aristotelian logic.

The person who did so much to refute the millennial Kantian conviction was N.A. Vasiliev (1880 – 1940), the Russian logician and philosopher now considered to be the forerunner of multi-valued and paraconsistent logics [Bazhanov *1988*, *1990*]. In one his works, Vasiliev stressed that "Kant himself did his best to refute his own view point concerning logic" [Vasiliev *1913*, 79].

Soon — at least on a historical scale — after Kant wrote in the "Preface" to the *Critique of Pure Reason* on Aristotelian logic a powerful movement emerged which eventually resulted in drastic changes in logic. Assessing this movement, Vasiliev names its following landmarks: Hegel's dialectical logic, Mill's inductive logic and his critical approach towards Aristotelian syllogistic, Sigwart's critique of the classical doctrine of modal judgements and, finally, the development of mathematical logic by Boole, Schröder, Poretskii, Peano, Frege, Russell [Vasiliev *1912a*; *1913*; *1924*]. It is worth noting that Vasiliev especially stressed the "subjective" character of his choice. (Incidentally, Vasiliev's works did not merely mention, but thoroughly analysed, the works by Poincaré and Couturat, Hilbert and De Morgan, Jevons, Venn, C.S. Peirce and W. Hamilton.)

The breakthrough beyond the horizon of traditional logic took place in several places. First of all, one of the cornestones of Aristotelian logic — the law of contradiction, according to which the simultaneous existence of the judgement (*A*) and its negation (non-*A*) is intolerable, i.e. a system should be consistent — was severely critized by those philosophers who belonged to dialectical trend. They were seeking the realization in the world of contradiction and its reflection in human consciousness (N. Cusanus, Hamann, Hegel, Bahnsen, Meinong) [Vasiliev *1913*, 57, 70].

The law of contradiction was subjected to subtle critique by the outstanding Polish logician J. Łukasiewicz in 1910. Łukasiewicz argued that the law of contradiction cannot be treated as putatively proved by direct evidence for the evidence cannot be a criterion of truth. Moreover the law, Łukasiewicz claimed, was never considered as self-evident in the history of science; it is highly doubtful that the law of contradiction may be viewed as a natural law, determined by the physical organization of human beings, or that it could be proved, either by a certain definition of affirmation and negation or by a definition of false judgements. Profoundly and keenly critizing the law of contradiction, Łukasiewicz in 1910 did not make an attempt to propose a logic free of this law. His castigation of the law was not reenforced by building up a system to replace Aristotelian logic. The law of excluded middle, was, in 1910, in fact beyond his analysis. Only in 1920 did Łukasiewicz put forward a three-valued logic, which in certain sense really superceded Aristotelian logic.

The deductive method of Aristotelian logic was opposed by the inductive doctrine in the works of F. Bacon and J.S. Mill. However, the most vigorous onslaught that traditional logic experienced came from mathematical logic, which had been developing intensively since the middle of the nineteenth century. The work of Boole, Peano, and especially Frege initiated a revolution in logic, and resulted in the flourishing of mathematical logic.

The emergence of non-classical logic was initiated by all the trends mentioned, but the the most noticable contribution was made by supporters of mathematical logic. Moreover the notion of "non-Aristotelian logic" most likely emerged within the scope ideas of mathematical logic. Nevertheless the idea of non-Aristotelian logic was still pretty vague and uncertain even in the early twentieth century.

The following sample from the article by P. Carus [Carus *1910*, 44–46] where the vistas of the creation of non-Aristotelian logic were discussed is rather typical for the turn of twentieth century (by the way this work was known to Vasiliev soon after its appearance):

> Aristotelian logic is incomplete and insufficient. It treats only the most simple relations and does not cover the more complicated cases of thinking, but so far as it goes it is without fault...

> And why should there not as well exist a curved logic as a mathematics of curved space? A curved logic would be a very original innovation for which no patent has yet been applied for. What a splendid opportunity to acquire Riemann's fame in the domain of logic!...
>
> The world has seen many new inventions. Over the telephone we can talk at almost unlimited distances, and some of our contempararies fly like birds through the air. Radium has been discovered which is often assumed with a certain show of plausibility to upset the laws of physics, but the invention of non-Aristotelian logic would cap the climax.

Although the idea of non-Aristotelian logic at the turn of the twentieth century was very blurred, the expectations its realization seemed to be very promising; the academic climate apparently was ready to cheer the novel logic. Nonetheless, the path to discovery of non-Aristotelian logic and its social acceptance was long and bumpy, more dramatic and complicated than could be foreseen.

The real history of non-Aristotelian logic begins on May 18, 1910 when N.A. Vasiliev presented to the Kazan University faculty a lecture "On Partial Judgements, the Triangle of Opposition, the Law of Excluded Fourth" [Vasiliev *1910*] to satisfy the requirements for obtaining the title of *privat-dozent*. In this lecture Vasiliev expounded for the first time the key principles of non-Aristotelian, imaginary, logic. In this work he likewise constructed his imaginary logic free of the laws of contradiction and excluded middle in the informal, so-to-speak Aristotelian, manner (although imaginary logic is in essense non-Aristotelian).[1] Thus the birthday of new logic was exactly fixed in the annals of history. Vasiliev's reform of logic was radical, and he did his best to determine whether it was possible for the new logic with new laws and new subject to imply a new logical Universe.

Vasiliev began the modern non-classical revolution in logic, but he certainly did not complete it. Indeed,the revolution reemerged in the late 1950's and early 1960's by N. Da Costa and D. Nelson, for in 1910's the ideas of Vasiliev were still too premature to be adopted and accepted.

One of the main heuristic prompts, a sort of incentive, to the non-Aristotelian logic of Vasiliev was the discovery of non-Euclidian geometry by Lobachevsky.

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[1]Modern formalized versions of Vasiliev's logic may be found in Arruda [*1980*], [Puga and Da Costa *1988*], Smirnov [Smirnov *1987*], and [Smirnov *1987a*, 161–169]. There is also the presentation Smirnov gave at the International Congress of Logic, Methodology and Philosophy of Science in Uppsala in 1991.

I plan to write a paper in English totally devoted to the description of imaginary logic.

The possibility of "another" logic, distinct from Aristotelian logic, convinces us, according to Vasiliev, of the existence of another, non-Euclidian, geometry. But it was not just the fact of the existence of another geometry that inspired the scholar. In geometry itself he found more than a mere prompt. "Imaginary logic is constructed by imaginary geometry's method.... In order to implement this method I have learned the non-Euclidian geometry.... Of all the systems of non-Euclidian geometries, I have most intently studied the geometry of Lobachevsky, which I learned from his original works," Vasiliev stated [Vasiliev *1911*, 20-21].

By analogy with the names of his logic and Lobachevsky's geometry, Vasiliev explored some internal analogies for the logical identity of their methods of creation [Vasiliev *1912a*, 208]. Just as the starting point of Lobachevsky's geometry was the rejection of attempts to prove the famous Fifth Postulate and construction instead of a geometry free of that postulate, the starting point of Vasiliev's logic was the abandonment of crucial Aristotelian logic laws, namely the laws of contradiction and excluded middle, and the construction instead of a logic free of these laws. The underlying unity of the methods lies precisely in "the striking analogies between non-Euclidian geometry and... imaginary (non-Aristotelian) logic" [Vasiliev *1912b*, 5].

Both non-Euclidian geometry and non-Aristotelian logic, Vasiliev put it, are sound systems, made possible after giving up the respective pivotal statements of Euclidean geometry and Aristotelian logic, and both are consistent, and both disturb common sense and our intuition.

In Euclidian geometry straight lines on plane surfaces either intersect or remain parallel. In Lobachevsky's geometry straight lines lying on the surface either intersect, or do not intersect, or are parallel.

In Aristotelian logic we have two types of judgements which are (with respect to their quality) different, which charactirize the subject-predicate relation, namely affirmative and negative judgements. In Vasiliev's logic there are three classes of judgements: affirmative, negative and the so-called "indifferent." Thus "the dichotomy of our ["telluric"] logic and of our geometry is transformed in the trichotomy of imaginary disciplines" [Vasiliev 1*912a*, 233, reprinted *1989*, 81; compare Vasiliev *1911*, 21].

After almost half a century of the existence of Lobachevsky's geometry, its interpretation of the surface called the pseudosphere was discovered. Imaginary logic, Vasiliev wrote, is valid not only in certain imaginary worlds with two different types of "sensations"; it may also be interpreted in the "terrestrial" world, in the logic of concepts, which not the same as the logic of "telluric" things. Vasiliev demonstrated that in the latter, the laws of contradiction and excluded middle are valid, while in logic of concepts we are to adopt the laws, as he called them, of non-selfcontradiction and of excluded fourth.

The telluric states might be described by two classes of judgements, affirmative and negative; but for the logic of concepts three classes of judgements are required — the affirmative, the negative and the so-called "accidental". The law of excluded fourth — the law of imaginary logic — is at the same time the law of the logic of concepts. To the "indifferent" class of judgements in imaginary logic there corresponds the class of accidental judgements in the logic of concepts. "Imaginary logic may be viewed as the realization of the logic of concepts, the imaginary world of realized concepts. Plato hypostasized the world of ideas; that world should live according to [the rules of] imaginary logic," Vasiliev stressed [Vasiliev *1913*, 64, reprinted *1989*, 106].

The pseudosphere is in some sense an ideal construction, but under certain physical conditions in the universe, Lobachevsky's imaginary geometry becomes the geometry of real space. "If the world or our sensory faculties are organized in a particular manner, logic must be non-Aristotelian" [Vasiliev *1912a*, 238, reprinted *1989*, 85]. Our world and sensory faculties are arranged in such a manner that all immediate sensations are positive. "Negative" sensations actually are negative; they are secondary if compared to positive sensations, and appear when one feature replaces another one that is incompatible with the first one. In a world in which living beings have two kinds of sensations, non-Aristotelian logic surely reigns. To put it another way, the logical laws and principles are determined in the first place by nature of cognitive objects and of the experiences open to the subject, i.e. they are EMPIRICAL.

Arguing that the origin of laws of logic depend on some sort of imaginary reality, Vasiliev persistently stressed the primacy of an ontological aspect of logic, the thought that material conditions determine various kinds of logic. By changing the ontology, combining the features of reality, we can get different imaginary logics, since the method of imaginary logic method opens up the possibility of experimentation in logic, of giving up certain logical principles and seeing what comes of this rejection. This method resembles the "experimental methods of the natural sciences" [Vasiliev *1913*, 78].

Despite the apparent differences of logics which could be constructed by this Lobachevskian method, all of these logics have a common feature, invariant for any logic and responsible for the possibility of their construction. This common feature manifest is in METALOGIC, which contains some logical minimum, independent of the diversity of the contents of thought, but vital for sound reasoning.

Non-Euclidian geometry teaches one more lesson crucial for both non-Aristotelian logic and for logic in the broad sense. It is that non-Euclidian geometry not only greatly influenced the flux of mathematical ideas, but exhibited the importance of foundational studies. D. Hilbert axiomatized geometry and, hence, clarified its foundations, the premises of geometrical knowledge that had been implicitly assumed. Vasiliev highly appreciated Hilbert's axiomatics and stressed his primacy in the foundational problems: "Hilbert showed remarkable accuracy in his treatment of the matter, which can serve as a standard for logic" [Vasiliev *1911*, 22; *1912a*, 245].

The development of logic reached the stage, said Vasiliev, when the problems of its foundations and axiomatization should be put to the fore. Every logician feels the "chaotic" state of the study of the laws and principles of thought, of the axioms and postulates of logic, is the most fundamental problem.

According to Vasiliev, the methods of imaginary logic allow determing which axioms are fundamental for logic and belong to its foundations; giving the the axioms accurate definitions; studying the problem of the independence of the axioms; to ascertain which logical statements and operations depend on which specific axioms; and, finally, classifying logical axioms. As a result, logic might be put in a "strongly provable form, similar to that of mathematics" and "logical formulas could be generalized and used in the most general style" [Vasiliev *1913*, 78].

In seeing in mathematics an undisputable standard for logic, Vasiliev was not thinking of the external similarity between them. He was quite knowledgeable about developments in contemporary mathematics (thanks largely to his father, eminent Professor of Mathematics A.V. Vasiliev [Bazhanov & Yushkevich *1992*]). N.A. Vasiliev was informed about the achievements in mathematical logic which "have been influencing informal [i.e. Aristotelian - V.B.] logic in a desicive, even crucial way." Mathematical logic, Vasiliev claimed, can demonstrate the tightest connection between logic and mathematics and to be a powerful tool in foundationa studies [Vasiliev *1913*, 9].

Logic is based, according to Vasiliev, on geometrical intuition. The basic logical relation, as in geometry, is the relationship between whole and the parts of the whole, reduced to the relation between foundation and its consequences. Foundation is a whole and consequences are its parts. This relation in essence should be assessed as mathematical and it lies at the basis of the syllogistic principle.

Logic and mathematics enrich each other. That is why "non-Aristotelian logic is not merely an application to logic of non-Euclidian geometry method; we may argue that non-Euclidian geometry is a special case of the application of the non-Aristotelian method of logic" [Vasiliev*1911*, 21].

Vasiliev seriously discussed the problem of the relation between logic and mathematics with several mathematicians, first of all with the mathematician and geometer N.N. Parfentiev, who was well-known in Russia. The result of this discussion was their joint course on "Problems at the Boundary of Logic and Philosophy of Mathematics" which took place at Kazan University in 1914.

The relationship between logic and mathematics was viewed differently by logicians. Vasiliev distinguished at least two groups: one — the "mathematical" — was in favor of a close connection between logic and mathematics, another — the "gnoseological" (which included, e.g., B. Croce and W. Windelband) — was in favor of a close connection between logic and theory of knowledge and attacked "formal" (mathematical) logic.

"What path should logic chose?" Vasiliev asked. Will logic be intensively enriched by mathematical methods or will it continue to ignore the success of mathematics? This is the Herculean threshold of logic. Vasiliev was greatly in favor of the first alternative. In the mathematization of logic he saw the guarantee of a bright future for logic. "Who could neglect the connection between logic and geometry manifested, for instance, by the geometrical diagrams of logic? The possibility of the algebraization of logic clearly shows this relationship" [Vasiliev *1912c*, 389].

Vasiliev zealously studied mathematics. Moreover, he carefully studied mathematical logic for "the mathematical logic can provide the special proof of the conceivability of imaginary logic" [Vasiliev *1911*, 24]. Finally, it is worth noting that in the1920's Vasiliev attempted to construct a "mathematical logic of intension" in opposition to the logic of extension. But this work did not survive.

Thus Lobachevsky's method as implemented in the creation of imaginary logic has deep roots in Vasiliev's position on the foundations of logic.

REFERENCES

ARRUDA, A.I. 1980. *A survey of paraconsistent logic*, in A.I. Arruda, R. Chuaqui and N.C.A. Da Costa (editors) Mathematical logic in Latin America (Amsterdam, North-Holland), 1–41.

BAZHANOV, V.A. 1988. *Nicolai Alexandrovich Vasiliev (1880 – 1940)*, Moscow, Nauka. (In Russian).

—. 1990. *The fate of one forgotten idea: N.A. Vasiliev and his imaginary logic*, Studies in Soviet Thought **39**, 333–344.

BAZHANOV, V.A. & YUSHKEVICH, A.P. 1992. *A.V. Vasiliev as a scientist and a public figure*, in A.V. Vasiliev, *Nikolai Ivanovich Lobachevskii (1792 – 1856)* (Moscow, Nauka), 221–228. (In Russian).

CARUS, P. 1910. *The nature of logical and mathematical thought*, The Monist **20**, 33–75, 158–159.

KANT, I. 1787. *Critik der reinen Vernunft. Zweyte hin und wieder verbesserte Auflage*, Riga, Johann Friedrich Hartnoch.

PUGA, L., & DA COSTA, N.C.A. 1988. *On imaginary logic of N.A. Vasiliev*, Zeitschrift für mathematischen Logik und Grundlagen der Mathematik **34**, 205–211. Russian translation by V.A. Bazhanov in M.I. Panov (editor), *Methodological analysis of the foundations of mathematics* (Moscow, Nauka, 1988), 135–142.

SMIRNOV, V.A. 1987. *Axiomatization of the logical systems of N.A. Vasiliev*, in *Contemporary logic and methodology of science* (Moscow, Nauka), 143–151. (In Russian).

—. 1987a. *The logical method of analysis in scientific knowledge*, Moscow, Nauka. (In Russian).

VASILIEV, N.A. 1910. *On partial judgements, the triangle of opposition, the law of excluded forth*, Record of Studies of Kazan University (October), 1–47. (in Russian). Reprinted in (*1989*), 12–53.

—. 1911. *Report on academic activities in 1911 – 1912*. Manuscript. (In Russian). Printed: (*1989*), 149–169.

—. 1912a. *Imaginary (non-Aristotelian) Logic, Journal of the Ministry of People's Education* (August), 207–246. (In Russian). Reprinted in (*1989*), 53–94.

—. 1912b. *Imaginary logic. Abstract of lectures.* (In Russian). Reprinted in (*1989*), 126–130.

—. 1912c. *Review of: Encyclopädie der philosophischen Wissenschaften in Verbindung mit W. Windelband herausgeben von A. Ruge. I Band: Logic. Verlag von I.C. Mohr. Tübingen*, Logos, Book 1/2 (1912-13), 387–389. (In Russian). Reprinted in (*1989*), 131–134.

—. 1913. *Logic and metalogic*, Logos, Book 1/2 (1912-13), 53–81. (In Russian). Reprinted in (*1989*), 94–123.

—. 1924. *Imaginary (non-Aristotelian) logic*, G. della Volle (editor), *Atti V Congresso Internazionale di Filosofia, Napoli, 5-9 maggio 1924* (Naples, 1925; reprinted: Nedeln/Liechtenstein, Kraus Reprint, Ltd., 1968), 107–109. (In English). Russian translation in (*1989*), 124–126.

—. 1989. *Imaginary logic: Selected works* (V.A. Smirnov, editor), Moscow, Nauka. (In Russian).