Jean-Louis Lassez and Gordon Plotkin (editors), *Computational Logic: Essays in Honor of Alan Robinson*, Cambridge, Massachusetts and London, MIT Press, 1991.

Reviewed by

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Computational Logic is a *Festschrift* dedicated to John Alan Robinson on the occasion of his sixtieth birthday. The papers in the volume include both surveys and original research of many of the areas in computational logic to which Robinson contributed, including automated theorem proving, unification theory, term-rewriting systems, functional programming, and logic programming.

The contributions to *Computational Logic* are divided into three subject groups, under the headings "Inference," "Equality," and "Logic Programming":

INFERENCE
- *Subsumption, a Sometimes Under-valued Procedure*
  - Larry Wos, Ross Overbeek, and Ewing Lusk
- *The Markgraf Karl Refutation Procedure*
  - Hans Jürgen Ohlbach and Jörg H. Siekmann
- *Modal Logic Should Say More Than It Does*
  - Melvin Fitting
- *Interactive Proof Presentation*
  - W. W. Bledsoe
- *Intelligent Backtracking Revisited*
  - Maurice Bruynooghe
- *A Science of Reasoning*
  - Alan Bundy
- *Inductive Inference of Theories from Facts*
  - Ehud Shapiro

EQUALITY
- *Solving Equations in Abstract Algebras: A Rule-Based Survey of Unification*
  - Jean-Pierre Jouannaud and Claude Kirchner

The historical roots of much of this work can be found in Robinson's work on the resolution method of theorem proving, and indeed to five central papers of Robinson's career, "A Machine-oriented Logic Based on the Resolution Principle" [*1965*], "Automatic Deduction with Hyper-resolution" [*1965a*], "The Generalized Resolution Principle" [*1968*], "Mechanizing Higher-order Logic" [1969], and "Computational Logic: The Unification Computation" [*1971*], especially the first two. Robinson's work was the inspiration for all of the subjects of computational logic treated in the *Festschrift*, even for those which did not originate directly out of Robinson's work.

In their "Preface", the editors define computational logic as comprising those parts of logic which provide a foundation for computer science, and they attribute to Robinson the majority of the credit for the conception and birth of the endeavor to explore the relationships between logic and computer science. They regard his contribution to computational logic as "fundamental" and note that his work has "exerted a profound influence in major fields of computer science, and in particular to those areas already mentioned, from automated theorem proving to logic programming.

If we go beyond the broad generalities and vague platitudes offered by the editors, we see that Robinson's influence in computational logic goes back to and centers on his work on resolution theory, which applies the tree method to *clausal sequents*, that is to Gentzen sequents that are in clausal form.

Robinson's [*1979*] *Logic: Form and Function* was written as an introduction for computer scientists, and especially for artificial intelligence researchers, in the practical use of the resolution method. It is an informal textbook, in the sense that it was meant to instruct, but had no exercises and was written in a leisurely, almost fireside chat manner, and readers are required to already know something about logic. The resolution method was first announced by Robinson in [*1963*], in an abstract in the *Journal of Symbolic Logic*. Resolution is described by Jean-Pierre Jouannaud and Claude Kirchner in their paper for the Robinson *Festschrift* as the "first really effective mechanization of first-order logic" (p. 257). They add the historical remark that solving equations on first-order terms emerged with Herbrand's work on proof theory" in his doctoral thesis *Recherches sur la théorie de la démonstration* (1930) "and was coined unification by Alan Robinson" in his first major paper, "A Machine-oriented Logic Based on the Resolution Principle" [*1965*].

Robinson developed the method itself in detail in a series of pioneering papers, the most important and better-known of which is the [*1965*] article "A Machine-oriented Logic based on the Resolution Principle." The origin of the resolution method is roughly contemporaneous with the early development in the mid-1960's by Smullyan of analytic tableaux which grew out of Beth's development of semantic tableaux from Gentzen's natural sequences, and Smullyan's analytic tableaux and Robinson's resolution method share some common features.

Literals are predictions or negations of predictions. A sequent is in *clausal form* if it is an ordinary sentence in prenex form whose quantifiers are universal and whose matrix is a disjunction of literals (in which case it is a universal clause), or if it is an ordinary sentence in prenex form whose quantifiers are existential and whose matrix is a conjunction of literals (in which case it is an existential clause). Resolutions are carried out on clauses which are sets of quantifier-free formulae in normal form.

Quantified formulae are rewritten in "quad" notation, within which quantified clauses are translated as quantifier-free sequences of formulae in disjunctive normal form, with the "quad" (□) replacing the quantifiers and connectives of the original quantified sequence. For a finite set S of expressions, the *resolution* of S is the pattern of inferences applicable to the finite clausal sequents. Let R be the *resolvent* of the sequence S. A resolution proof is valid if the sequent S, R is derivable from the sequent S. Thus, for the series of resolution steps, s_0, s_1, ..., s_n, where s_n is a terminal sequent containing a quad-clause, the series s_0, s_1, ..., s_n is the resolution proof of its initial sequent s_0.

The final goal of Robinson's efforts was to provide a LISP program that gives a computer-based process of resolution proofs. Because Robinson's method uses Gentzen sequences rather than the Deduction Theorem to justify logical inference, he can use Gentzen's cut-elimination rather than the tree decomposition rules found in Jeffrey's *Formal Logic*, to obtain inference trees. He was therefore able to establish a close connection between the computer language LISP and Gentzen's **LK** quantification system on the one hand and to give a proof-theoretic foundation to his discussion of Resolution theorem proving. As I wrote when discussing his [(*1979*)] book *Logic: Form and Function*, [Anellis *1981*, 192], the resolution method was "indeed his main contribution to the literature of computer logic and proof theory."

The relationships between resolution and other automatic theorem-proving methods based upon Gentzen sequences, and in particular Maslov's inverse method, was pointed out by Kuehner [*1971*], who introduced the English version [Maslov *1971*] of Maslov's detailed [*1969*] comparison of resolution with his own inverse method and by Vladimir Lifschitz [*1986*, 18-19], who gave a detailed characterization of Maslov's method and its applications [*1986*, 78-97]. It was shown by Maslov that there is a one-to-one correspondence between his inverse method and Robinson's resolution. Davydov [*1971*], in fact, was able to obtain a synthesis of the two methods.

The application of the resolution method to computerized proof-finding was enhanced by Robinson's development in [*1965a*] of a type of resolution called *hyper-resolution*. This is a resolution whose search space is considerably more sparse than the search space of general resolution. Here, we have a sequence of connected search trees which which converge to a ground resolution; these converging trees connect several levels of resolutions.

None of the authors in this *Festschrift* touch on the relations of the resolution method with Maslov's inverse method, despite the wide variety of contributions in this massive volume of more than 720 pages; and few of the authors have occasion for more than the sketchiest discussion of the historical background of either Robinson's work or of the role which Robinson's work had in their own research. Despite the occasional historical remarks made in passing, there is much, however, that the historian of logic can learn if he or she already knows something about Robinson's original work. Because nearly every paper in this *Festschrift* owes something, either directly or indirectly, to Robinson's pioneering work on resolution, what can be learned is the diversity to which that work has given rise and the extent and depth of new developments in the ever-widening and ever-deepening field of computational logic which has grown in just a few years less than three brief decades from the work begun by Robinson. The questions for future historians will be "How much of the work found in the Robinson *Festschrift* has proven to have had lasting importance?", "Have the results obtained in the wake of Robinson's work proven to be of equal importance with Robinson's original work?" and even "What role did Robinson's work play in the subsequent development of logic in general and of computational logic in particular?"

The last question is necessitated by the absence of an analysis of Robinson's work, or indeed of any detailed treatment of this question, in the Robinson *Festschrift*. That, together with the wide diversity of subjects handled by the *Festschrift*'s contributors, is why I have focused in this review on the resolution method as one central aspect of Robinson's work and on the connections of the resolution method with the related work of Maslov and his colleagues on the inverse method.

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