A History of Logic Trees

Editor's Note

The use of graphical methods in logic can be traced back, in one form or another, to the "tree of Porphyry", in the third century A.D. Martin Gardner, in his history *Logic machines and diagrams* (Brighton, Harvester Press, 2nd ed., 1983, p. 29), has even suggested that Aristotle used tree diagrams to represent successive subdivisions of genera and species. For further discussions of some of the graphical methods in logic, and in particular of Euler diagrams and Venn diagrams, see also H.G. Hubbeling, *A diagram–method in propositional logic*, Logique et Analyse 8 (1965), 277-288, A.S. Kuzichev, *Diagrammy Venna*, in *Istoriia i Primeneniia* (Moscow, Nauka, 1968), and Z.A. Kuzicheva, *Graficheskie metodu logiki klassov*, Istoriia i Metodologiia Estestvennyh Nauk 29 (1982), 75-85, the latter paper dealing not only with Venn diagrams and Euler diagrams, but also with Lambert diagrams and De Morgan diagrams.

In the two papers that follow, the authors are concerned with the modern development of trees as mechanical decision procedures and as tests of the validity of logical deductions. The article by Francine Abeles focuses on the nineteenth-century development by Charles Dodgson (Lewis Carroll) of trees for syllogisms, while the article by Irving Anellis focuses on the recent evolution of truth trees and falsifiability trees for first-order quantification theory from Gentzen's natural deduction sequents (N-sequents) by Beth, Hintikka, Smullyan, and van Heijenoort. Both of these developments of trees are shown to have close associations with the semantic tableaux of E.W. Beth.

The trees developed by Carroll in 1894, which anticipate concepts later articulated by Beth in his development of deductive and semantic tableaux, have their roots in the work of Charles Peirce, Peirce's students and colleagues, and in particular in Peirce's own existential graphs. The trees developed by Hintikka, Smullyan, and van Heijenoort in the 1950s and 1960s are simplifications of Beth's semantic tableaux. Although Beth showed in his booklet *La crise de la raison de la logique* (pp. 24-28; Paris, Gauthier-Villars & Louvain, Nauwelaerts, 1957) that his method of semantic tableaux could be applied directly to the assertoric syllogisms of classical logic, it is still not clear that this line of development from Peirce and Carroll to Beth and from Beth to

Hintikka, Smullyan and van Heijenoort is direct, with Beth's work forming the crucial linchpin which historically and conceptually binds Carroll's trees with those of Hintikka, Smullyan, and van Heijenoort. There is, unfortunately, no evidence that Beth knew the work of Carroll, all the more so since Carroll's work on trees was discovered only after Beth, Hintikka, Smullyan, and van Heijenoort had completed their work on trees. It is therefore more likely that the historical roots for Beth's work, like Carroll's, are to be found in Peirce's none too numerous published works on existential graphs, particularly those found in the 1932-1934 edition of Peirce's *Collected papers, vols. 2-4* (P. Weiss & C. Hartshorne, editors; Cambridge, Mass., Harvard University Press), since Beth was quite familiar with Peirce's published work. (Indeed, Beth was one of the few logicians outside of the algebraic tradition of the immediate post-*Principia mathematica* period who gave serious consideration to the tradition of Boole, Peirce, and Schröder and its historical role in the development of modern logic.) We cannot determine precisely, however, the extent of Peirce's influence on Beth, or in particular the extent to which Peirce's work on existential graphs directly influenced Beth's own work on semantic tableaux, since Beth did not address this particular issue.

In his historical remarks to *Semantic entailment and formal derivability* (Mededlingen van de Koninklijke Nederlandse Akademie van Wetenschappen (n.r.) 18, no. 13 (1955), p. 332), Beth mentions only the tree of Porphyry; otherwise, his primary concern is to relate the tableau method which he develops with Gentzen's method of natural deduction. Nor did he raise this issue in any of his other historical remarks detailing the background to his tableau method. The question of the development of trees by Carroll was first presented by William W. Bartley in his *Editor's introduction* to Lewis Carroll's *Symbolic logic* (New York, Clarkson N. Potter, Inc., 1977; pp. 31-32), in which it is said that Carroll's is strikingly similar to the tree method based upon Beth's semantic tableaux and as developed by Kurt Schütte in *Ein System des verknüpfenden Schliessens* (Archiv für math. Logik und Grundl. 2, nos. 2 / 4 (1956), 55-67) and Stig Kanger in *Provability in logic* (Stockholm, Almqvist and Wiksell, 1957). Bartley did not, however, either clarify or justify his claims. It is the task of clarification and justification which Abeles undertakes in her paper.

Studies of Peirce's graphs and of their relationship to natural deduction are given in D.D. Roberts, *The existential graphs of C.S. Peirce* (Ph.D. thesis, Univ. of Illinois,

1963), D.D. Roberts, *The existential graphs and natural deduction* (in E.C. Moore and R.S. Robin (editors), *Studies in the philosophy of Charles Sanders Peirce*, Amherst, University of Massachusetts Press, 1964), 109-121), which considers Peirce's logic diagrams for extensional syllogistic of 1896, D.D. Roberts, *The existential graphs of Charles S. Peirce* (Hague/Paris, Mouton, 1973), J.J. Zeman, *The graphical logic of C.S. Peirce* (Ph.D. thesis, Univ. of Chicago, 1964), J.J. Zeman, *Peirce's graphs, the continuity interpretation*, Transactions of the Charles S. Peirce Society **4** (1968), 144-154, and J.J. Zeman, *Peirce's logical graphs*, Semiotica **12** (1974), 239-256.

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