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Richard L. Epstein and Walter A. Carnielli
Computability. Computable Functions, Logic, and The Foundations of Mathematics 2nd edition, with
Computability and Undecidability—A Timeline. The Story of the Development of Computable Functions and the Undecidability of Arithmetic to 1970
by Richard L. Epstein
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REVIEW

ROMAN MURAWSKI

The book under review is based on courses on recursive function theory given by the first author for the Philosophy Department of the Victoria University of Wellington in 1975-1977. It is primarily an introduction to the theory of recursive functions and their applications to logic. The book consists of four parts. Part I has an introductory character—one finds here some considerations about paradoxes, whole numbers, functions, proofs as well as infinite collections. They provide only basic information and background of what will follow. Part I ends with some fragments of a famous and influential paper by David Hilbert, "On the Infinite" (1927).

Part II and Part III form the main part of the book. Part II is devoted to the theory of recursive functions. The following subjects are considered there: computability (in general), Turing machines, Church's thesis, primitive recursive functions, the Grzegorczyk hierarchy, multiple recursion, the least search operator, partial recursive functions, numbering the partial recursive functions, listability and the problem: Turing machine computability vs. partial recursiveness.

Part III deals with applications of the theory of recursive functions to logic and the foundations of mathematics. It begins with an introduction to propositional logic. An overview of first-order logic and Gödel's theorems are also given. Next the system of first- order arithmetic is described in detail. In later sections one finds considerations of: functions representable in formal arithmetic, the undecidability of arithmetic and the unprovability of consistency.

The main theme of Part IV closing the book, is a discussion of the significance of the technical work presented earlier, *i.e.*, Church's thesis and constructivist views of mathematics. In the discussion of Church's thesis one finds information on its history, arguments for it and against it—supplemented by quotations from the original papers of the classical authors Church, Gödel, Kalmár, Post, Kleene and others. In the part devoted to constructivism one reads about intuitionism (a quotation from Brouwer and remarks on modern intuitionism), recursive analysis, Bishop's constructivism, Paul Bernays's and Nicolas Goodman's criticism of intuitionism and constructivism as well as about strict finitism (D. van Dantzing, D. Isles).

The characteristic feature of the book that makes it a very good and useful manual and handbook for students is the fact that many exercises have been included, beginning gently in Part I and progressing to graduate level in the final chapter. The most difficult ones, marked with a dagger, may be skipped, although all are intended to be read. Solutions to the exercises can be found in the Instructor's Manual, which also contains suggestions for course outlines. Some sections have been marked "Optional"—they are not essential for the technical development of chapters which follow (but they often provide important motivations).

Another important and characteristic feature is that all the problems discussed in the book are philosophically well motivated because in Part I the philosophical background for discussions about the foundations of mathematics is provided and, in the next chapters, appropriate quotations from the works of the classical authors are given.

The second edition of this book includes a number of technical corrections as well as a new *Timeline on Computability and Undecidability.* In the latter one finds a schematic story of the development of the theory of computable functions and the undecidability of arithmetic from 1834 (when Charles Babbage designed his Analytical Engine) to 1970 (the negative solution of Hilbert's 10th problem by Y. Matiyasevich). A description of the main events is supplemented by quotations from appropriate books or papers (the quotations have been taken from recent English translations).

There is one more change from the first edition—this change reflects a recent development mathematics: the replacement of Fermat's Last Theorem by Goldbach's Conjecture as an example of an unsolved arithmetic problem used in several examples because the former has been solved by A. Wiles.

The book under review is a nice introduction to the theory of computable functions and to incompleteness theorems and undecidability problems. It can serve as a handbook in courses in logic for philosophy students as well as in introductory courses in logic and foundations of mathematics for students of mathematics. Probably the latter group will not be fully satisfied and will need more technical information. On the other hand, it is amazing how many items have been covered by the authors—even items usually not included in such introductory courses. One should also add that the book has been published in a very nice way and supplemented by indices and an extensive bibliography. Finally, I would like to stress again the very important feature of the book, namely the unity of technical details and philosophical motivations—it is a rather seldom found but very much needed feature!

Faculty of Mathematics and Computer Science, Adam Mickiewicz University, ul. Umultowska 87, 61-614 Poznań, Poland

E-mail address: rmur@amu.edu.pl