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REVIEW

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The Cambridge Companion to Bertrand Russell is one of the latest in a very popular series of anthologies dedicated to influential figures in the history of philosophy. Readers of this journal most likely best know Bertrand Russell as the discoverer of Russell's paradox, and as co-author of *Principia Mathematica* (hereafter *PM*). However, in addition to these accomplishments, Russell contributed significantly to almost every area of philosophical thought. In addition to a lengthy introductory and biographical piece by the editor, the *Companion* contains fifteen chapters, each dealing with a different aspect of Russell's philosophy. Two chapters, by Nicholas Griffin and Richard Cartwright, deal with Russell's inculcation into the tradition of British idealism as a student, and the break he made away from it along with his peer at Cambridge, G. E. Moore. Three pieces, by Bernard Linsky, William Demopoulos, and R. E. Tully, address various aspects of Russell's metaphysics. Two chapters are dedicated to Russell's epistemology, written by A.C. Grayling and Thomas Baldwin. The *Companion* also contains chapters dedicated to Russell's theory of descriptions (by Peter Hylton), Russell's analytic philosophical methodology (by Paul Hager), and Russell's contributions to ethics (by Charles Pigden). In addition to these, the *Companion* contains five pieces of significant interest to scholars of logic and the history of logic worth discussing in full.

The first of these is a piece entitled "Mathematics in and behind Russell's Logicism, and its Reception," by I. Grattan-Guinness. Here we find a detailed treatment of the main mathematical influences on Russell's work in mathematical logic, beginning with the early work in mathematical analysis by Cauchy and Weierstrass. Grattan-Guinness identifies Cantor and Peano as the two principal influences on Russell's

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work: Cantor supplied inspiration regarding set theory and the nature of cardinal and ordinal numbers, and Peano supplied the core predicate logic, symbolism and axiomatic method. The article then moves on to a quite detailed examination of the writing of both Russell's *Principles of Mathematics* (1903), and the three volumes of Whitehead and Russell's *Principia Mathematica* (1910, 1912, 1913). Grattan-Guinness does an admirable job sketching the make-up of each, what portions of mathematics are covered, and the pitfalls Whitehead and Russell encountered with their approach, especially those stemming from the adoption of a theory of types to avoid the set-theoretic and related paradoxes. The piece also discusses the reception of Russell's work, especially among German and Polish logicians, mathematicians and philosophers, such as the members of the Vienna Circle, and ends with a discussion of the importance of Gödel's results and their corollaries, and Russell's reaction to, and understanding of, these findings.

Interestingly, in his discussion of Russell's influences, Grattan-Guinness downplays the influence of Gottlob Frege, claiming that "some commentators grossly exaggerate the extent of Frege's influence on Russell," although noting that Russell's reading of Frege prompted the former to modify certain passages of the *Principles*, and influenced the treatment of ordinal numbers in *PM*. It is somewhat interesting then that later in the *Companion* we find a lengthy article solely devoted to "Russell and Frege," penned by Michael Beaney. Nevertheless, we do not find there much by way of countervailing argumentation to the effect that Frege did in fact have substantial influence on Russell. While Beaney discusses Russell's repeated claims to the effect that he was the first person to appreciate, bring to light, or even read Frege's writings in detail, Beaney admits, as with Grattan-Guinness, that Russell independently rediscovered many of Frege's insights, such as the logicist definition of number in terms of equinumerousity defined by one-one functions. Beaney's contribution continues with a discussion of Frege's analysis of mathematical induction in terms of ancestrals of relations, a comparison of Frege and Russell with regard to the analysis of number predications as statements about concepts or their extensions, and a discussion of Frege's response to Russell's paradox. The article closes with a lengthy discussion of three different conceptions or modes of *analysis*: a *regressive* mode in which one aims to identify ultimate premises or fundamental assumptions of a given domain of inquiry, a *resolutive* mode in which one hopes to identify the components or constituents of something and how they relate, and an *interpretive* mode in which something in one language or framework is translated, paraphrased or explicated in terms of another. Beaney contends that it was in failing to distinguish these modes of analysis that lead Frege and Russell into certain errors, such as those bound up with Frege's Basic V of his *Grundgesetze* (leading to Russell's paradox), and early Russell's postulation of denoting concepts such as that corresponding to the phrase "all men" as independent constituents of a proposition.

It is somewhat disappointing then that Beaney does not tackle head on the question as to what extent Frege influenced Russell. There is certainly some truth to Grattan-Guinness's complaint. In particular, it is often thought that Russell simply took the logicist program over from Frege, as well as took over Frege's invention of predicate logic or a logic of (propositional) functions, definition of number, and conception of philosophy. However, in truth, Russell's early logic owed more to Peano than to Frege, and Russell independently came to the Fregean definition of number and to adopt logicism. However, while these well known points of overlap were not things Russell inherited from Frege, Frege did significantly influence the development of Russell's work in mathematical logic during the period immediately after writing *The Principles of Mathematics* (from late 1902-1905). During these years, Russell's chief occupation was the attempt to discover a philosophically plausible solution to Russell's paradox which would nevertheless allow the logicist program to succeed. He began by following up certain suggestions of Frege's, as he admits in a 1906 letter to Jourdain (see [3], p. 78). In mid-1903, inspired by Frege's theory of the value-ranges of functions, Russell came to the view that classes were superfluous in logic and mathematics, and that the work of classes could be done using functions (see his letter to Frege dated 24 May 1903, in [2], pp. 158-59; see also [6]). By a somewhat torturous process, this evolved into the "no-classes" theories one finds in Russell's mature logical writings such as *PM*. It is unlikely that Russell's views would have developed as they did without Frege's influence.

The development of Russell's type-theory and mature logical views is a fascinating subject. Two pieces in the *Companion* address it in some detail. One finds an overview of the development in Alasdair Urquhart's contribution, "The Theory of Types". Urquhart begins his exposition by sketching the rather unusual early theory of types found in Appendix B of *The Principles of Mathematics*, which put individuals, classes (as many) of individuals, classes (as many) of such classes as many, *etc.*, in different types, but also allowed infinite types and collective types. Urquhart notes that this theory was abandoned due to a paradox involving propositions; by Cantor's theorem, there must be more classes of propositions than propositions. But since a different proposition can be constructed for each class of propositions, *e.g.*, the proposition that all members of the class are true, the assumption that propositions all fall into the same type leads back to contradiction. Urquhart then briefly mentions the various type-free strategies Russell attempted between 1903-1907 for the avoidance of logical paradoxes, noting, quite rightly, that Frege was a considerable inspiration here. Urquhart then turns his attention to the ramified theory of types.

However, since the exposition of type-theory given by Whitehead and Russell in the first edition of *PM* is so severely lacking in precision by modern standards, Urquhart bases most of his exposition on the formulations of ramified type-theory given by later logicians such as Alonzo Church and John Myhill. This is followed by a discussion of Russell's "Axiom of Reducibility", which postulates that, within the ramified hierarchy, for every propositional function of a given type, there exists a co-extensive propositional function of the lowest order for that type. Urquhart notes that for most mathematical purposes, this undoes the effects of ramification, but nevertheless still allows for the solution of various intensional or semantic paradoxes. This leads naturally into a discussion of the origins of the simple theory of types, as found in the work of Ramsey, and arguably, in the second edition of *PM*. Urquhart concludes with a discussion of the recent influence of the theory of types in mathematical logic and computer science, noting that while iterative set theories such as ZF and NBG are far preferred for regular mathematical practice, the theory of types has had a role to play in inspiring certain recent mathematical proofs, and in the creation and evaluation of certain programming languages.

A more detailed look at the logical systems explored by Russell between 1905 and 1907 is undertaken by Gregory Landini in his contribution, "Russell's Substitutional Theory". Russell had argued in *The Principles of Mathematics* that everything that can be named, mentioned or counted must be an "individual", capable of occurring as logical subject in a proposition, and hence, concluded that a proper logic must employ only one style of variable. The substitutional theories of 1905-07 represent an attempt to reconcile this attitude with the need for finding a solution to the logical paradoxes. The theories center around a four-place relation, written $p/a\,;b\,!\,q$, which means that the proposition q results from the substitution of the entity b for the entity a wherever it occurs as logical subject in the proposition p. On this theory, both classes and propositional functions are excluded as entities, but one can in effect do the work of higher-order quantification by quantifying over two entities: a proposition and an entity in it to be replaced by other entities. For example, rather than considering a function x *is human*, one can consider the "matrix" consisting of the proposition *Socrates is human* and Socrates. The theory yields results very similar to a simple type-theory, and Russell's paradox is excluded because there is no way to represent a propositional function taking "itself" as argument, as something such as $p/a\,;p/a\,!\,q$ is ungrammatical. Philosophically, it provides an explanation for what goes wrong with the paradoxes without positing different ontological types of entities about which the same things cannot meaningfully be asserted.

Landini then goes on to discuss the problems that lead Russell to abandon the substitutional theory, principally certain paradoxes of propositions, including one analogous to the Cantorian paradox of propositions discussed in Appendix B of *The Principles of Mathematics*, which lead Russell to abandon quantified propositions as entities, and another which Landini calls "the p_o/a_o paradox". These paradoxes, although involving the notion of a proposition understood as an intensional entity, are not straightforwardly semantic or intentional. Landini contends that they shed light on Russell's motivation for adopting ramified type-theory, and show that Ramsey's widely heralded distinction between logical paradoxes and semantic/epistemic paradoxes is not as straightforward as it may seem. Landini also argues that the substitutional theory represents the "conceptual linchpin" between Russell's early work in *The Principles of Mathematics* and the mature logic of *PM*, and that while *PM* abandons the substitutional approach of employing only one style of variable, the underlying contention that there exists only one type of entity is retained in the nominalist semantics Russell intended for *PM*.

Understanding the development of Russell's views helps in addressing certain longstanding criticisms such as those mentioned in the piece contributed by Andrew Irvine and Martin Godwyn entitled "Bertrand Russell's Logicism". They begin with a discussion of previous logicist thinkers (those also holding the view that some or all or mathematics was reducible to logic), especially Leibniz, Dedekind and Frege. Unfortunately, again, they do not make it clear in what precise ways these thinkers influenced Russell. They then briefly discuss Russell's type-theory and its use in solving the paradoxes facing more naïve forms of logicism. Next, they turn to the ontological themes of Russell's logicism, and in particular, his claim that numbers, and classes generally, are to be understood as "logical constructions" or "logical fictions" that disappear on analysis, *i.e.*, that propositions apparently about such entities as classes or numbers, when properly understood, are analyzed as quantificational statements about individuals and propositional functions. They lastly tackle the epistemology of Russell's logicism, noting that although Russell believed that the propositions of pure mathematics could be deduced from more general principles of logic, Russell did not believe that the logical principles were always epistemologically more fundamental. Russell thought that simple arithmetical claims such as $2 + 3 = 5$ were more epistemologically certain than many logical axioms, and that one reason accepting certain logical doctrines is that they, and no obvious rivals, lead to already-accepted mathematical truths as consequences.

Godwyn and Irvine express sympathy with Quine's complaint that Russell succeeded not so much in reducing logic to mathematics, but at most logic to set theory, because Russell's construction of classes out of propositional functions introduced entities every bit as logically contentious as sets themselves, since Russell vacillated (or so Quine claims) between understanding propositional functions as mere open sentences and understanding them as attributes (see [9], pp. 122-23). However, sympathetic readers of Russell have in recent years attempted to reply to Quine's complaints (see, *e.g.*, [4], [7], [8], [13]). Indeed, there is significant evidence that, although Russell was a realist about simple universals like *redness*, he understood universals to be individuals and was not committed to any entities corresponding to complex open sentences, and that a propositional function was simply a linguistic item (see, *e.g.*, [5]). Statements involving quantification over propositional functions was to be given a substitutional or nominalist semantics so that their truth depends on the truth of their substitution instances. Hence, Russell's construction of classes does not postulate any additional entities beyond those involved in a proper understanding of elementary propositions.

Although the various chapters of the *Companion* differ in quality and their faithfulness to Russell's writings, overall the *Companion* is sure to be a wonderful resource for both scholars and students. If nothing else, the *Companion* may serve to revive interest and discussion of Russell's work in logic, which has largely been ignored and forgotten by contemporary working logicians. It deserves renewed consideration. Recent scholarship, the *Companion* included, drawing upon dozens of hitherto unpublished manuscripts, shows that many of the approaches to logic considered by Russell while attempting to solve Russell's paradox bear striking similarities to approaches still popular today. In early 1903, he made various attempts to produce a consistent axiomatization of arithmetic by modifying Frege's Basic Law V, beginning with Frege's own ill-fated modification from the appendix added to volume II to the *Grundgesetze der Arithmetik*. The systems he invented in mid-1903 were very similar to later untyped Lambda Calculi and Combinatory logics. According to Russell's "zig-zag theory" of 1904, certain open formulae were thought to comprehend classes but not others, which readily invites comparison with Quine's systems NF and ML. Near the same time, Russell also considered an approach he called "limitation of size", in which classes are only postulated to exist if their members were limited to an easily specified and small range: here, the natural comparison is to "iterative" set theories such as ZF and NBG. In creating and evaluating the various approaches he tried, it is clear that Russell was not looking simply for a system devoid of contradictions, but also for a philosophically plausible logical system that avoided *ad hoc* assumptions or restrictions. The reasons Russell had for rejecting systems similar to those now more prevalent in mathematics, and settling instead on the ramified type-theory of *PM*, are still worthy of consideration and may yet be instructive in the philosophy of logic and the philosophy of mathematics.

Unfortunately, Russell's ultimate reasons for preferring his final approach over others are still not very well known or well understood. There are a number of reasons for this. The most obvious reason is simply that most of these manuscripts were not published in Russell's lifetime, and at the time he was writing, there simply wasn't an audience for his thoughts. Another reason is that, by contemporary standards, Russell's own presentation of his type-theory and its philosophical rationale, is rather imprecise and clumsy. There is (as yet) no universally agreed upon reconstruction of the syntax and axiomatization of *PM*, and most later commentators, including Urquhart in the *Companion*, end up assimilating it to the formulations of later logicians, such as Church and Myhill. However, there is increasing evidence that what Russell intended was quite a bit different from these later type-theories. In his own interpretation of *PM*, Church holds that Russell's logic is committed to both propositional functions and propositions understood as intensional entities. Realizing this is out of sorts with *PM*'s Introduction, where Russell outlines his "multiple relations theory of judgment", and denies the existence of proposition as single entities, Church (see [1], p. 748n) concludes that the Introduction is out of sorts with the body of *PM*, and cannot be taken seriously. However, more recent evidence suggests that Russell's abandonment of propositions as single entities was a result of his failure to solve the paradoxes of propositions plaguing the substitutional theory, and that his reason for favoring the type-theory of *PM* was in part that he saw it as compatible with his newfound eschewal of propositions.

I know of no contemporary logician whose work is infused with the same sort of philosophical spirit as Russell's. One can only assume he would be highly disappointed with the near dominance of iterative set-theories in contemporary mathematics, and with the fact that these theories seem to be preferred for no reason other than their practical utility. Russell's own exploration into class/set theory began within a tradition continuous with the Boolean algebraic approach as exposited in the work of Whitehead and Peano. In that tradition, "classes" were largely conceived as the extensions of concepts, and the logic of classes was thought to be the proper interpretation or reconstruction of all categorical and syllogistic reasoning. When it became evident to Russell (on the basis of the logical paradoxes) that a class could not be naïvely assumed always to exist as the extension of a concept, and no philosophically plausible way of limiting the assumptions about their existence was found, he concluded that classes were not genuine entities, but instead "logical fictions" or "logical constructions". In contemporary "set theory", the notion of a set is mostly divorced from the notion of the extension of a concept, and the postulates adopted in a certain set theory regarding their existence are usually motivated only by practicality rather than philosophical or logical argument. In general, Russell objected to theories that simply set limits on the postulation of the existence of classes without reconciling the limitation with a general philosophical theory about the nature of logic generally. In 1905 he wrote to Jourdain claiming that "I think ... other mathematicians do not realize how any limitation such as you propose [on the postulation of classes or sets] involves a modification of the fundamentals of logic, since these, as commonly accepted, exclude any such limitation" ([3], p. 46). Witnessing the birth of mainstream set theory in the work of logicians such as von Neumann, Zermelo and König, Russell wrote to Jourdain complaining that "I have given up expecting much of solutions [of the set-theoretical paradoxes]" (*ibid.*, p. 54). After all his work looking for something better, Russell had every right to complain. This is in keeping with his later remark that postulating mathematical entities rather than finding philosophical or logical arguments for their existence should be regarded as having "the advantages of theft over honest toil" ([10], p. 71).

It is high time, I think, to re-open the question as to what the philosophical standards should be for evaluating logical or set-theoretic systems aimed at providing the foundations for mathematics. Hopefully, the Cambridge *Companion to Bertrand Russell* will help to promote Russell's own philosophical approach to the creation and evaluation of logical systems. At the very least, it should serve to renew interest in a unique and influential figure in the history of logic and the history of philosophy. There is no work currently available that would provide a better overall guide to Russell's philosophy and views in logic. It is certainly worth a read for students and scholars in philosophy and logic alike.

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