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Review of

CHARLES L. SILVER, *FROM SYMBOLIC LOGIC . . . TO MATHEMATICAL LOGIC*

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xxi + 378 pp. ISBN 0-6971-4175-6

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For Silver, symbolic logic consists of sentential calculus and first-order predicate calculus without identity, but mathematical logic covers various advanced topics, like first-order number theory and the limitative theorems of Gödel and Church. In more familiar terminology, 'symbolic logic' refers to elementary logic without identity, and 'mathematical logic' to metamathematics. Thus, the title of this textbook informs that its material proceeds from elementary logical calculi to metamathematics. The author warns that this organization is also parallel to the increasing technical complexity of the text.

The book is divided into eleven chapters. Chapter 0 is devoted to mathematical preliminaries (sets, mathematical induction). Chapter 1 presents the formal syntax of sentential logic (SL, for brevity) together with a review of inference rules of this system. In the next section, we find short-cut rules for SL; also, an axiomatic version is briefly sketched. Chapter 3 deals with the semantics of SL through Boolean interpretation. Various facts are proved about Boolean models of SL, including compactness. Connections of syntax and semantics of SL are investigated in Chapter 6. In particular, we have here proofs of consistency (by purely syntactic resources), soundness and strong completeness; compactness is also revisited. The next two chapters are devoted to syntax and semantics of predicate logic (PL, for brevity). Among questions discussed, we have inference rules, derivations from assumptions, interpretations, truth for an interpretation, satisfiability, validity, invalidity, and logical consequence. Chapter 6 terminates the review of symbolic logic in the book.

Mathematical logic begins with Chapter 7 in which connections between the syntax and semantics of PL are shown. In particular, proofs of strong soundness, consistency, strong completness and compactness are given. Chapter 8 is mixed in its character. It contains twofold information. First, we find in it an outline of various first-order theories (the theory of equality, the theory of orderings, group theory, theory of addition and three theories of arithmetic). Secondly, the author introduces several general concepts, like mathematical completeness, mechanical procedure, decidability, axiomatizability and extension. This chapter is seen as preparatory for the following Chapters 9–11 in which first-order number theory is systematically studied. Chapter 9 gives the syntax of first-order arithmetic of natural numbers (the system $\mathcal{N}$). The last section shows how to represent numerical relations on $\mathcal{N}$ in $\mathcal{N}$ itself. Models of first-order theories are investigated in Chapter 10. Isomorphism, elementary equivalence, cardinality of first-order languages, Löwenheim-Skolem-Tarski-Vaught theorems, first-order capture (i.e. expressive power), categoricity and completeness. Łoś's theorem and ultraproduct construction are among topics of this chapter. The last chapter gives an informal account as well as a formal proof of Gödel's undecidability theorems and related topics (the Church thesis, undecidability of PL, the Tarski undefinability of truth theorem, recursive functions); Chaitin's theorem (an information-theoretic version of the first Gödel undecidability theorem) is briefly sketched in the Postscript.

This textbook has several virtues. It is self-contained, well organized and comprehensive. The author actually fulfills his main promise and very consistently proceeds from less to more advanced topics. Everything is explained very carefully. More advanced and complicated topics are at first informally introduced and then rigorously developed. If a given problem exceeds the assumed and realized level of technical sophistication, but is worthy of being included, the author presents just a very sketchy treatment of it (e.g. ultraproduct construction). Finally, the book offers many interesting exercises.

The reviewer has two critical remarks, one general and one more specific. The general one concerns something ideological in a sense. The author very correctly ascribes great significance to relations between syntax and semantics. Unfortunately, he does not complete his remarks by giving a clear conclusion which seems to stem from limitative results: semantics is mathematically much more powerful than syntax. Reading Silver's textbook, I had an impression that semantics was for him only a preparatory step for syntax. My concrete remark concerns the concept of logical name which replaces the concept of individual constant. I see no clear motivation behind this novelty. We have a theorem which states that logic does not distinguish any extralogical concepts: whatever can be proven on an object α, can also be demonstrated on any other object. If the author thinks about logical names as invariant modulo logical theorems, this intuition is exactly captured by the standard concept of individual constant. If not, his idea is not quite transparent. However, it is rather a minor point. The book decidedly makes a very good impression.

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