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Review of
***SOLVABILITY, PROVABILITY, DEFINABILITY: THE COLLECTED WORKS OF EMIL L. POST*,**
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After Tarski's *Collected Papers* (1986), the same publisher produced a volume with all the works of Emil Post, another giant of modern logic. The volume contains, besides an introduction by Martin Davis, Post's mathematical as well as logical papers. His mathematical papers clearly show that he was not only a logician (more precisely: a mathematical logician) but also a creative mathematician. In fact, his first research did not concern logic. Post's mathematical papers are these: The generalized gamma functions (1918); Discussion of problem 433 [on the fractional differential equations] (1919); General differentiation (1924); Polyadic groups (1940). Moreover, he published several abstracts of which two are philosophically relevant: Visual intuition in Lobachevsky space (1923); Visual intuition on spherical and elliptic space: Einstein's finite universe (1923).

However, Post's fame as a scholar is mainly connected with his seminal logical achievements. The volume contains: Introduction to a general theory of elementary propositions (1921); Finite combinatory processes - formulation I (1936); The two-valued iterative systems of mathematical logic (1941); Absolutely unsolvable problems and relatively undecidable propositions - account of an anticipation (written in 1941, published in 1965); Formal reductions of the general combinatorial decision problem (1943); Recursively enumerable sets of positive integers and their decision problems (1944); A variant on a recursively unsolvable problem (1946); Recursive unsolvability of a problem of Thue (1947).

Post also published several abstracts on logical matters. One of them has significance for general discussions on conceptual foundations for metamathematics. In "Solvability, definability, provability; history of an error" (1953), Post proposes to add definability to other absolute

metamathematical properties, like solvability and provability. Moreover, he insists that a natural sequence of these properties should be the one given in the title of the abstract.

Post's most important results are contained in his doctoral dissertation (1921) and in papers belonging to recursion theory. He proved the completeness theorem for propositional calculus, introduced many-valued logics (related to Post algebras), and combinatorial processes, a device which is provably equivalent to Turing machines and other formulations of the theory of recursive functions. Also, his works on unsolvability are remarkable. The lesser-known monograph on iterative systems of mathematical logic continues his doctoral dissertation and studies generators of sets of two-valued functions closed under compositions. A careful reading of Post's dissertation is insightful for two important historical remarks. In the introduction Post explicitly states the difference between logic and metalogic. He writes: "We here wish to emphasize that the theorems of this paper are *about* the logic of propositions but are *not included* therein. [...] whereas the propositions of 'Principia' are particular assertions introduced for their interest and usefulness in later portions of the work, those of the present paper are about the set of *all* such possible assertions." (p. 22) The second remark is even more important. It concerns an intuitive basis of Post's many-valued logic. In the last sections of the dissertation (12–16), he gives some intuitions which lie behind his formal constructions, namely analogy with m dimensional space and an observation that logical values of the basic (i.e., two-valued) system correspond to complete truth and falsity. Most comments of Post's research in many-valued logic ignore these remarks. As far as I know, the Kneales' ([3], p. 569) is the only review of Post's ideas which mentions his intuitions concerning the relation of many-valued logic to two-valued logic. Other historical accounts either are silent on the points (for example, [4]) or contain statements that Post's many-valued constructions were purely formal, without any intuitive motivations behind them (for example, [6] and unfortunately, my own [5]). Although Post's intuitions were not as extensive as those of Łukasiewicz, they should be noted.

The long paper "Absolutely unsolvable problems and relative undecidable propositions – account of an anticipation" has a special historical value. It shows that Post conjectured the incompleteness of formal systems as early as in 1924. He wrote this paper in 1941 in order to make his idea public, but the work was rejected by the editor of the *American Journal of Mathematics* (H. Weyl). Post's paper is

important not only as a historical document, but also because it contains stimulating philosophical remarks which can refresh discussions on absolute and relative concepts in metamathematics.

The editors wrote a nice introduction which presents Post's life (very dramatic, indeed) as well as his works in logic and mathematics. One item of information should be corrected. Davis writes that Post was born in 1897 in Augustow, Poland. However, there was no Poland as an independent state in 1897; this area belonged to Russia at the time; it was part of Poland before 1795 and after 1918. There is a nice story connected with the "Polish" roots of Post. (This story was reported by Mostowski, see [1], p. 20). When Tarski met Post for the first time (probably in 1939 or 1940) he congratulated him for being the only person who had proved very important results on sentential logic without having anything to do with Poland. Post replied "Oh, no, I was born in Bialystok, and it is a town in the east of Poland." In my earlier writing (in particular, in [5], p. 314), I repeated the version given by Mostowski. Now, it should be corrected by replacing "Bialystok" by "Augustov", because I believe that the editor used documents concerning Post's curriculum vitae.

Everybody interested in the history of logic should be greatly indebted to the publisher and the editor for this very important collection.

References

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[2] J. Crossley, *Algebra and Logic*, Berlin, Springer-Verlag, 1975a.
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[5] J. Woleński, *Logic and Philosophy in the Lvov-Warsaw School*, Dordrecht, Kluwer, 1989.
[6] A. A. Zinov'ev, *Philosophical Problems of Many-Valued Logic*, Dordrecht, Kluwer, 1963.

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