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Review of

ROBERT GOLDBLATT, *MATHEMATICS OF MODALITY*

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WIM RUITENBURG

The author collects a total of 11 of his papers on modal logic into the 11 chapters of this publication. Of these, 8 essentially are mildly edited conversions into LaTeX of previously published papers. Of the other three papers, chapter 8 is a significant extension of a previous paper on the "Henkin method". Chapter 9 on infinitary rules of inference is new; and so is chapter 11, which covers certain relationships between modal logic and first-order logic. First let us have a quick overview of the chapters.

Chapter 1, entitled *Metamathematics of Modal Logic*, originally appeared in two parts as [5, 6]. For their Mathematical Reviews, see 58 #27331a and b. It is a slightly expanded version of the author's PhD thesis. This well-written paper still is a significantly up-to-date introduction to propositional modal logic and Kripke model theory.

Chapters 2 and 3, entitled *Semantic Analysis of Orthologic* and *Orthomodularity is Not Elementary* respectively, originally appeared as [4, 11]. Their Mathematical Reviews numbers are 55 #5398 and 85e:03154.

Chapter 4, entitled *Arithmetical Necessity, Provability and Intuitionistic Logic*, originally appeared as [7]. Its Mathematical Reviews number is 80h:03026.

Chapter 5, entitled *Diodorean Modality in Minkowski Spacetime*, originally appeared as [8], and its Mathematical Reviews number is 82a:03018.

Chapter 6, entitled *Grothendieck Topology as Geometric Modality*, originally appeared as [9]. Its Mathematical Reviews number is 83d:03069.

Chapter 7, entitled *The Semantics of Hoare's Iteration Rule*, originally appeared as [10]. Its Mathematical Reviews number is 85i:03086.

Chapter 8, entitled *An Abstract Setting for Henkin Proofs*, is a significant extension of a paper of the same name that originally appeared as [12]. The Mathematical Reviews number of this original paper is 86f:03021. The significant extension involves additional applications of the Abstract Henkin Principle, namely, completeness for the Barcan formula in quantificational modal logic, and completeness for propositional modal logics with infinitary rules.

Chapter 9, entitled *A Framework for Infinitary Modal Logic*, is new. Consider a modal logic that is complete in the sense that consistent sentences are satisfiable, but not in the sense that all sets of consistent sentences are satisfiable. This chapter considers ways of extending such logics by infinitary rules of inference. Applications are given that are proof-theoretic accounts of some of the results of chapter 8 on modal logics with infinitary rules.

Chapter 10, entitled *The McKinsey Axiom is Not Canonical*, originally appeared as [13]. Its Mathematical Reviews number is 93c:03017.

Chapter 11, entitled *Elementary Logics are Canonical and Pseudo-Equational*, is new. A normal propositional modal logic Λ is *elementary* if it is determined by a first-order definable class of Kripke models. Here it is shown that if Λ is elementary, then it is determined by the class of Kripke frames axiomatized by the first-order theory of the canonical frame of Λ. Additionally, necessary and sufficient conditions are given for a canonical logic to be elementary.

All articles are well-written and accessible; a strong 'trademark' of this author. But who is this book for? It is neither a historical book, nor a true introduction to the field of modal logic and its metamathematics. So let us look for an audience.

A specialist in the field can find most of the material in the literature, and buying a whole book for just the few new parts makes no economic sense. At least for the buyer. He may prefer to order the book for the library, since the new parts are interesting contributions to the field and should be easily available. As specialists have alternative options, we now turn to beginners in the field, including beginning graduate students. For them this book has several attractions. Even today chapter 1, which covers almost a third of the contents of this volume, is an acceptably up-to-date introduction to modal propositional logic and model theory. After reading the essential parts of this chapter the reader can, with little need for additional help, continue with several of the later chapters, and so get a good idea of some of the questions and answers in this field of research. Naturally, sections like chapter 4 need significant additional reading outside this book. Nevertheless, beginners could benefit relatively more from this publication than specialists would.

There are a few things that could have been done better, in particular on behalf of beginners in the field. The 'old' papers form research work that all dates back to the 1970s or early 1980s. Since then progress has been made, although of course not equally much in all directions. But unfortunately the author left the old papers almost unchanged, except for the expansion involving chapter 8. The more limited option of adding references to more recent work hasn't been exploited either. Below is a small sample of references, only to illustrate the point. For example, the author could have added a reference to papers like [16] to chapter 3, to illustrate the connections with physics. References to more recent books, like to [14], would stimulate further reading. Very recent papers like [15] may have appeared too late for inclusion. The author could have added references to [1, 2, 3] in chapter 4, if only for historical context. The author could have added a reference to [18] to chapter 8, or a reference to [17] to chapter 10, but these publications may have appeared too late for inclusion.

Conclusion: This is a well-written collection of older research work, with a number of interesting new results added. Specialists may not want to purchase this whole volume just for the new results. Beginners should be prepared to do additional reading outside this book, but this volume doesn't offer help as to where to look for this extra reading.

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Marquette University, Department of Mathematics, P.O. Box 1881, Milwaukee, WI 53201