Modern Logic
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H. JEROME KEISLER ET AL., *MATHEMATICAL LOGIC AND COMPUTABILITY*

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vii + 484 pp. with computer disk

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In 1976 Prindle, Weber & Schmidt published a book that stood out in the traditionally homogenized realm of calculus texts. That book was H. Jerome Keisler's *Elementary Calculus*. Adapting Abraham Robinson's nonstandard analysis for first-year students, Keisler presented an infinitesimal-based development of the material. Whatever the other pros and cons of the book, its distinctive viewpoint set it apart as a genuine alternative to the usual clones.

Keisler's *Mathematical Logic and Computatibility*, however, lacks that flair. Perhaps the authorship by committee of this undergraduate text helps account for its hint of blandness. The cover credits Keisler as the author, but the list of titles in the flyleaf also names Joel Robbin. And the title page adds the names of Arnold Miller, Kenneth Kunen, Terrence Millar, and Paul Corazza as contributors. Given so many authors, one is not surprised that this book does not possess the individuality of *Elementary Calculus*.

Not that a text needs to be path-breaking to be a valuable addition to the literature. Books can certainly distinguish themselves by doing things well rather than differently. But Keisler and company have produced a work of uneven quality, with some nice features but also with some very definite drawbacks. This applies both to the text proper and to the accompanying computer software package.

Before describing these in more detail, let me first say what *Mathematical Logic and Computability* is and is not. The authors have clearly targeted an audience of upper-division mathematics majors, rather than aiming for a broader market including students in, say, computer science or philosophy. The treatment presupposes an appropriate level of mathematical maturity, while content prerequisites are minimal. An appendix covers the necessary rudiments of naive set theory, functions, cardinality, and so on. (As the authors rightly point out, this material gets short shrift in the curriculum at many institutions, with unfortunate results.) From the title one might expect a broad-based coverage of the various subfields of mathematical logic, as in Mendelson's standard text, for example. But such is not the case. The book centers around the basic soundness, completeness, compactness, and incompleteness theorems; several other areas of logic receive little attention. The authors give no mention of basic model-theoretic topics like Löwenheim-Skolem, nonstandard analysis(!), or elementary equivalence. Likewise, such set-staples as cardinal and ordinal arithmetic and the Axiom of Choice, for all practical purposes, do not appear.

The first of *Mathematical Logic and Computability*'s five chapters deals with propositional logic, the next two with predicate logic (Chapter 2 without function symbols and equality and Chapter 3 with them). Emphasizing the similarities and differences between their respective logics, these three chapters unfold in parallel fashion, proceeding from syntax to semantics, soundness, completeness, and compactness. Proofs are formalized via tableaus, which suit the authors' purposes well, providing a simple, convenient formalism within both the book and the software. These chapters include some nice material intended to help students relate formal proof methods to everyday mathematical practice. For example, the text analyzes informal proofs of the infinitude of primes and Rolle's Theorem in terms of proof strategies (contradiction, case-splitting) whose formal tableau counterparts are presented.

Chapter 4 introduces computability in terms of register machines. Again, the choice of formalism works well, especially with a simple modification that smooths the handling of Gödel numbers. Unfortunately, the authors' enthusiasm for register machines gets the better of them. This chapter provides a narrow and somewhat skewed view of computability theory by dwelling on register machines to the exclusion of other matters. After all, the text's title features computablity, and the preface claims logic and computability as the book's two main topics. Yet computability theory *per se* appears very spottily. One of the central concepts in the field, that of a recursively enumerable set, shows up only in the exercises to Chapter 5, and with an incorrect definition at that! Nor do the authors even mention fundamentals like relative computability and reducibilities, the Recursion Theorem, or computational complexity. A student could easily get the impression that computability theory exists only to serve Gödel incompleteness — the book gives no hint of such applications as Hilbert's Tenth Problem or the word problem for groups — and that it consists mainly of writing programs for register machines.

The final chapter focuses on incompleteness and related results. The first sections of the chapter present Tarski's Theorem, Gödel's First Incompleteness Theorem (with two different proofs), and Church's Theorem. In the last sections a modal logic for provability is introduced. From this standpoint the authors examine the First Incompleteness Theorem yet again, as well as the Second Incompleteness Theorem. An exercise also walks the reader through a proof of Löb's Theorem. (In general, the authors have included several interesting exercises throughout the book.)

As a whole, *Mathematical Logic and Computability* contains a huge numbers of errors, several of moderate seriousness. False statements, inaccuracies, and gaps abound, creating a potential nuisance for an instructor teaching from the book and an outright hazard for a novice using it for self-study. Particularly glitch-ridden are the text's register-machine programs and pseudocode, which on more than one occasion hang in infinite loops. At times the authors also shy away from standard nomenclature, another drawback for students using the book on their own.

In addition, readers of *Modern Logic* will particularly regret that the authors provide no historical context for their material. Indeed, even attributions are by and large lacking. The question of who devised tableaus or register machines, much less when and why, gets no answer here. To take an especially glaring example, Turing's name does not appear a single time, not even in the discussion of the halting problem. (In case you're wondering: no, Turing machines never receive mention either.)

So much for the text proper. What about the software package? As a rule, and in accord with the authors' expressed intentions, the software fills a supplementary rôle, and so the book can be read and used independently of the programs. One jarring exception to this rule occurs when the authors state a result about tableaus in terms of the conventions used by the software to display tableaus on a color monitor. Also, occasionally the details of proofs show up in the form of register-machine programs on the disk.

The programs come in both DOS and Windows versions to run on IBM PCs or compatibles with at least 320K memory. There is not a Mac version. Of the four programs in the package, three deal with tableaus, the other with register machines. The workhorse of the tableau program, TABLEAU, lets the user construct and extend tableaus. Each chapter of the book except the fourth contains several exercises geared to the use of TABLEAU in constructing proofs.

PREDCALC displays graphs of sets of ordered triples (from a universe of up to eight elements) satisfying first-order formulas. Again, the authors designed several exercises in Chapter 2 around PREDCALC. The demonstration program COMPLETE automatically completes tableaus of propositional logic. Finally, GNUMBER simulates register machines; it features in the exercises of Chapter 4. The disk also includes several data, documentation, and installation files.

Like the text proper, the software calls for mixed reviews. On the plus side, the programs—at least the Windows version that I tested—are quite easy to learn and use. The documentation, both on-line and in the book, contributes to this user-friendliness. However, the DOS programs lack the tutorials present in the Windows versions. On the minus side lie some operational flaws. For instance, documentation to the contrary, TABWIN (the Windows TABLEAU) would not let me add a hypothesis to a tableau that had been extended. Since one cannot delete or modify hypotheses of extended tableaus either, this puts quite a crimp in the kind of experimentation that the authors intended the package to encourage. Nor does the code always properly safeguard against user errors. When I inadvertently clicked on the DOS COMPLETE rather than its Windows counterpart COMPWIN, my machine froze and I had to reboot.

In summary, *Mathematical Logic and Computability* exhibits some good features, but not enough to compensate for its shortcomings. Its positive aspects could provide a basis for a nice textbook; however, that would take a substantial amount of revision and error-correcting.

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