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ERRATA: TYPICAL CONTINUOUS FUNCTIONS ARE NOT CHAOTIC IN THE SENSE OF DEVANEY

Abstract

The author gives corrected statements of results in [M] and a corrected proof of Theorem 1 of [M].

In this note, necessary changes, clarifications and corrections are given to the results of [M]. Here we prove the result in Theorem 1 [M], which is renamed below as Lemma 1, for compact n-cubes in Euclidean n-space. In the end, we obtain a stronger result in that the set of functions in C which are not topologically transitive, is shown to be a *dense open* subset of C. We also obtain a new result in Theorem 5 below. Hereafter, instead of C, we let $C(K,K)$ denote the set of all continuous functions of the form $f : K \to K$, where K is a compact n-cube in E^n, where E^n denotes Euclidean n-space with the usual metric. That is, K is the Cartesian product of n compact intervals in the real line. We put the uniform topology on the function space $C(K,K)$. Then since the range space K is metrizable, we have the supremum metric; so that for elements f and h in $C(K,K)$, $D(f,h) = \sup\{d((f(x),h(x)) : x \in K\}$. For any set E, $\mathrm{Cl}(E)$ will denote the closure of E, and ∂E will denote the boundary of E.

Lemma 1. *Let K be a compact n-cube in Euclidean n-space E^n, with the usual metric d. Let $C(K,K)$ denote the set of all continuous functions of the form $f : K \to K$ with the supremum metric. Then there is a dense open subset W in $C(K,K)$ such that every function f in W has the property that for some nonempty open subset $U \subset K$, $f(\mathrm{Cl}(U)) \subset U$.*

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PROOF. To show that W is open in $C(K,K)$, let h be any function in W. Then for some open set $U \subset K$, $h(\mathrm{Cl}(U)) \subset U$, where $U \neq K$ and U is not dense in K. Since $h(\mathrm{Cl}(U))$ and ∂U are both compact subsets of K, there is a positive distance $\mu > 0$ between them. Let $N_{\mu/2}(h)$ be an $\mu/2-$neighborhood of h in $C(K,K)$. Then for any f in $N_{\mu/2}(h)$, $f(\mathrm{Cl}(U)) \subset U$. Hence, W is open in $C(K,K)$.

To show that W is dense in $C(K,K)$, fix $\epsilon > 0$, and let h be any element in $C(K,K)$, where h is not necessarily an element of W. Since K has the fixed-point property, h has a fixed point, say x_0, in K. Since h is uniformly continuous on K, there exists $\delta^* > 0$ such that for all x_1, x_2 in K, $d(x_1,x_2) < \delta^*$ implies that $d(h(x_1),h(x_2)) < \epsilon$. Choose $\delta > 0$ such that $\delta \leq \delta^*$ and $\delta < \mathrm{diam}(K)$. Let $B(x_0,\delta/2) = B_0$ be the open ball of radius $\delta/2$ about x_0. Let $B(x_0,\delta) = B_1$ be the open ball of radius δ about x_0. We now define a continuous function $f : K \to K$ by

$$f(x) = \begin{cases} x_0 & \text{if } x \in \mathrm{Cl}(B_0) \\ (2-2t)x_0 + (2t-1)h(x) & \text{if } x \in \mathrm{Cl}(B_1)\backslash B_0 \\ h(x) & \text{if } x \in K\backslash B_1 \end{cases}$$

where t is defined so that for any $x \in \mathrm{Cl}(B_1)\backslash B_0$, $t = \dfrac{d(x,x_0)}{\delta}$. Then $f \in C(K,K)$, and $D(h,f) < \epsilon$, and by $B_0 = U$, we have $f(\mathrm{Cl}(U)) \subset U$. Hence, W is a dense open subset of $C(K,K)$. □

Theorem 2. *There exists a dense open subset W in $C(K,K)$ such that every function in W is not topologically transitive, and hence not chaotic in the sense of Devaney.*

We now make the obvious changes and corrections in the statement of other results in [M]. Theorem 6 of [M] is restated as follows.

Corollary 3. *There exists a dense open subset W in $C(K,K)$ such that every function in W has an asymptotically stable set.*

By application of the above results and Lemma 1 of [BC], we can combine Corollary 7, Lemma 8, and Theorem 10 into the following.

Corollary 4. *There exists an dense open subset W in $C(K,K)$ such that every function f in W has the property that $CR(f) \neq K$, where $CR(f)$ denotes the chain recurrent set of f.*

We now let $C(I,I)$ denote the set of all continuous functions of the form $f : I \to I$, where I is a compact interval in the real line. As noted in [M], the set of all functions in $C(I,I)$ which are Block-Coppel chaotic, comprise an open subset of $C(I,I)$. In [K], although different terminology is used, Kloeden essentially shows that the set of functions of the form $f : I \to I$ which are Block-Coppel chaotic, make up a dense subset of $C(I,I)$. It follows that the set of Block-Coppel chaotic functions in $C(I,I)$ is dense and open. In the next result, we let $P(I,I)$ denote the set of all polynomials on a compact interval I in the real line. We now show that there is a dense open subset of all polynomials in $P(I,I)$ which are Block-Coppel chaotic, but not chaotic in the sense of Devaney.

Theorem 5. *Let $P(I,I)$ denote the set of all polynomials on the compact interval I. There exists a dense open subset $S \subset P(I,I)$ such that each polynomial in S is Block-Coppel chaotic but not chaotic in the sense of Devaney.*

Proof. $P(I,I)$ is dense in $C(I,I)$. Since there is a dense open subset $W \subset C(I,I)$ such that every function in W is not chaotic in the sense of Devaney, in the relative topology on the function space $P(I,I)$, the set of polynomials which are not chaotic in the sense of Devaney, is dense and open in $P(I,I)$. Similarly, there is a dense open subset of Block-Coppel-chaotic polynomials in $P(I,I)$. Since the intersection of a finite number of dense open sets is again dense and open, the theorem is proved. □

References

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