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λ-LARGE SUBGROUPS OF C_λ-GROUPS

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If L is a fully invariant subgroup of the p-primary group G, and if $G = B + L$ for all basic subgroups B of G, then L is called a large subgroup of G; this definition is due to R. Pierce. In light of K. Wallace's generalization of the concept of basic subgroup to that of a λ-basic subgroup, we extend Pierce's definition by defining the fully invariant subgroup L to be a λ-large subgroup of G if $G = B + L$ for all λ-basic subgroups B of G. Our main theorems are: (1) L is a λ-large subgroup of the C_λ-group G if and only if $L = G(v)$ where v denotes an increasing sequence of ordinals less than λ satisfying the gap condition. (2) If L is a λ-large subgroup of the C_λ-group G, then G/L is a totally projective group, and L is a C_μ-group where μ denotes the length of $L/p^\lambda G$. (3) If L is a λ-large subgroup of the C_λ-group G, then L is a totally projective group only if G is a totally projective group.

1. Preliminaries. All our groups are additively written, abelian, p-primary groups for some prime p. Most of the terminology and notation we use can be found in [2].

DEFINITION 1. [10] Let λ denote a limit ordinal and B a subgroup of the p-primary group G. Then B is called a λ*-basic subgroup* of G if B is a totally projective group of length at most λ, B is a p^λ-pure [8] subgroup of G, and G/B is divisible. A reduced p-primary group G is a C_λ*-group* if $G/p^\alpha G$ is a totally projective group for all α less than λ.

Wallace has shown in [10] that the p-primary group G contains a proper λ-basic subgroup if and only if λ is cofinal with ω (the first infinite ordinal) and G is a C_λ-group. Thus λ will henceforth denote a limit ordinal cofinal with ω. If G is a C_λ-group of length less than λ, then G is necessarily a totally projective group. Since properties of these groups are well-known, we shall restrict our attention to C_λ-groups of length at least λ. By applying results in [8], we can prove the following.

PROPOSITION 1. *A subgroup B of G is a λ-basic subgroup if and only if* (1) *B is a totally projective group of length λ,* (2) *$G[p] \subseteq p^\alpha G + B[p]$ for all α less than λ, and* (3) *there is no subgroup H of G properly containing B such that $H[p] = B[p]$.*

DEFINITION 2. If L is a fully invariant subgroup of the C_λ-group G, then L is called a *λ-large subgroup* of G if $G = B + L$ for all λ-basic subgroups B of G.

Note that the ω-large subgroups are just the large subgroups that Pierce studies in [9]. It follows from Proposition 1 that $p^\alpha G$ is a λ-large subgroup of G whenever α is less than λ. In addition, a straightforward argument shows that $p^n L$ is a λ-large subgroup of G whenever L itself possesses this property and n is a positive integer; further on, we shall show that $p^\alpha L$ is also a λ-large subgroup if α is less than the length of $L/p^\lambda G$.

DEFINITION 3. [2] Let $v = (\sigma(0), \sigma(1), \cdots, \sigma(n), \cdots)$ denote a sequence of ordinals and perhaps symbols ∞ such that for any k and t, $\sigma(k) < \sigma(k+1)$ if $\sigma(k)$ is an ordinal and $\sigma(t+1) = \infty$ if $\sigma(t) = \infty$. We say that v *satisfies the gap condition* (for the p-primary group K) if $\sigma(n) + 1 < \sigma(n+1)$ for some n implies that the Ulm invariant of K corresponding to $\sigma(n)$ is nonzero. If v satisfies the gap condition for the reduced p-primary group K and if each ordinal is less than the length of K, then v is called a *U-sequence for K.*

DEFINITION 4. [3] If $v = (\sigma(0), \sigma(1), \cdots, \sigma(n), \cdots)$ is a sequence of ordinals and perhaps symbols ∞, and if K is a p-primary group, then $K(v)$ denotes $\{x \in K: h_K^*(p^n x) \geqq \sigma(n)$ for each $n\}$. Note that $\sigma(n) = \infty$ for all n larger than some fixed integer k if and only if $p^{k+1}(K(v)) = 0$.

In [3] Kaplansky shows that each fully invariant subgroup of a fully-transitive, p-primary group K has the form $K(v)$ where v is a U-sequence for K. In [9] Pierce proves that a fully invariant subgroup is a large subgroup of K if and only if it has the form $K(v)$ where v is a U-sequence for K consisting of nonnegative integers. In the next two sections, we shall show that λ-large subgroups are similarly determined by U-sequences of ordinals less than λ.

2. C_λ-groups of length λ. Our immediate objective is to show that C_λ-groups of length λ are fully transitive.

DEFINITION 5. [7] Call a reduced, p-primary group G of length β *σ-summable* if $G[p] = \cup\{S(n): n < \omega\}$ where $S(n) \subseteq S(n+1)$ and $S(n) \cap p^{\alpha(n)} G = 0$ for some increasing sequence of ordinals $\{\alpha(n): n < \omega\}$ having supremum β.

The following generalized Kulikov Criterion plays a crucial role in the development of results in this section and in the study of the structure of λ-large subgroups which we begin in Section 4.

PROPOSITION 2. [7] *A p-primary group G of length λ (cofinal with ω) is a totally projective group if and only if G is a σ-summable C_λ-group.*

PROPOSITION 3. *If H is a subgroup of a C_λ-group G and $H \cap p^\alpha G = 0$ for some α less than λ, then H is contained in some λ-basic subgroup of G.*

Proof. If α is the first ordinal satisfying $H \cap p^\alpha G = 0$, then we let $\{\alpha(n): n < \omega\}$ denote an increasing sequence of ordinals greater than α having supremum λ. We construct an increasing sequence of subgroups $\{S(n): n < \omega\}$ with the property that $S(n)$ is maximal in $G[p]$ with respect to the property $S(n) \cap p^{\alpha(n)}G = 0$. If B is maximal in $G[p]$ with respect to the properties $B \supseteq H$ and $B[p] = \cup\{S(n): n < \omega\}$, then B is σ-summable and satisfies the second and third conditions of Proposition 1. For each α less than λ, $B/p^\alpha B$ is isomorphic to $G/p^\alpha G$ and thus B is a C_λ-group. Proposition 2 implies that B is a totally projective group.

PROPOSITION 4. *Let H denote a finite subgroup of the C_λ-group G. If $H \cap p^\alpha G = 0$ for some α less than λ, then $G = A \oplus K$ where $A \supseteq H$ and A is a direct summand of some λ-basic subgroup of G.*

Proof. According to the preceding proposition, H is contained in a λ-basic subgroup B of G. Since B is of length λ and λ is a limit ordinal, we can write $B = \oplus\{B(i): i \in I\}$ where, for each $i \in I$, $B(i)$ is a totally projective group of length less than λ. There is a finite subset J of I such that $H \subseteq \oplus\{B(j): j \in J\}$; let A denote this sum. If $K = (\oplus\{B(i): i \in I - J\}) + p^\beta G$, where β is the maximum of the lengths of the groups $B(j)$ for $j \in J$, then $G = A \oplus K$.

PROPOSITION 5. *Every C_λ-group of length λ is fully transitive.*

Proof. Suppose that x and y are elements in the C_λ-group G of length λ, where $h^*_G(p^n x) \leqq h^*_G(p^n y)$ for all n. We need only show the existence of an endomorphism of G sending x to y. By Proposition 4, $G = A \oplus K$ where $\langle x, y\rangle \subseteq A$ and A is a direct summand of a λ-basic subgroup of G; thus A is a totally projective group. Since totally projective groups are fully transitive and since $h^*_A(a) = h^*_G(a)$ for all $a \in A$, $f(x) = y$ for some endomorphism f of A. There is an obvious extension of f to an endomorphism of G.

The next proposition can be proved by applying Proposition 3 and generalizing the proof of Lemma (1.2) in [9].

PROPOSITION 6. *If B is a λ-basic subgroup of the C_λ-group G, where* *the length of G is not less than λ, and if A and C are fully invariant subgroups of G, then $(A+B)\cap C=(A\cap C)+(B\cap C)$.*

COROLLARY 1. *Suppose that B is a λ-basic subgroup of G, $x\in G$, and α is less than λ. If $o(x)$ denotes the exponential order of x, then $x=b+g$ for some $g\in p^{\alpha}G$ and $b\in B$ satisfying $o(b)\leqq o(x)$.*

Proof. If $o(x)=m$, set $A=p^{\alpha}G$ and $C=G[p^m]$ and then apply the preceding proposition.

DEFINITION 6. Let $v=(\sigma(0),\sigma(1),\cdots,\sigma(n),\cdots)$ denote a U-sequence for the p-primary group K. Then v is called a *U_{β}-sequence* for K if each $\sigma(n)$ is an ordinal less than β.

THEOREM 1. *Suppose that G is a C_{λ}-group of length λ. Then L is a λ-large subgroup of G if and only if $L=G(v)$ where v is a U_{λ}-sequence for G.*

Proof. Suppose first that L is λ-large in G. Since G is fully transitive and L is fully invariant, then $L=G(v)$ where $v=(\sigma(0),\sigma(1),\cdots,\sigma(n),\cdots)$ is a U-sequence for G. Thus, if $\sigma(n)$ is an ordinal for some n, then $\sigma(n)$ is less than λ; however, all of the symbols $\sigma(n)$ are ordinals since λ-large subgroups must be unbounded.

Conversely, suppose that $L=G(v)$ where v is a U_{λ}-sequence for G. It suffices to show that $G\subseteq B+G(v)$. If $x\in G$ and $o(x)=m$, then by Corollary 1, we can write $x=b+g$ where $b\in B$, $g\in p^{\sigma(m)}G$, and $o(b)\leqq o(x)$. It follows that $g\in G(v)$.

Note that in the preceding proof we have shown that $G(v)$ is λ-large whenever v is a U_{λ}-sequence, even when the length of G exceeds λ. The following corollary is useful in the study of λ-large subgroups of C_{λ}-groups having length greater than λ which we begin in §3.

COROLLARY 2. *If G is a C_{λ}-group of length λ, then L is a λ-large subgroup of G if and only if L is an unbounded, fully invariant subgroup of G.*

3. C_{λ}-groups of length greater than λ. Whenever L is a λ-large subgroup of a C_{λ}-group G, L contains $p^{\lambda}G$; this follows from Proposition 6 by setting $A=L$ and $C=p^{\lambda}G$. Now, if $p^n(L/p^{\lambda}G)=0$, we can show that $p^{\lambda}G=p^nL$; however this gives us a decomposition $G=B\oplus p^{\lambda}G$ for any λ-basic subgroup B of G since p^nL is also a λ-large subgroup. Since C_{λ}-groups are reduced, we must conclude that $L/p^{\lambda}G$ is an unbounded subgroup of $G/p^{\lambda}G$. It is important to our development to show that $L/p^\lambda G$ is a λ-large subgroup of $G/p^\lambda G$ whenever L is a λ-large subgroup of G; according to Corollary 2, we need only show that $L/p^\lambda G$ is a fully invariant subgroup of $G/p^\lambda G$. Most of this section is devoted to accomplishing that goal.

Many of our subsequent results can best be formulated in topological language. Therefore we introduce the following definition.

DEFINITION 7. [6] The *λ-topology* is defined on the p-primary group K by taking the family of subgroups $\{p^\alpha K: \alpha < \lambda\}$ as neighborhoods of the identity. If H is a subset of K, then H'_K will denote the closure of H in K with respect to the λ-topology on K; whenever the containing group is obvious, we will simply write H'.

PROPOSITION 7. *If F is a fully invariant subgroup of G, and if B is a λ-basic subgroup of G, then $F \subseteq (F \cap B)'$. Moreover, if G has length λ and F is unbounded, then $F = (F \cap B)'$.*

Proof. If α is less than λ, then we set $A = p^\alpha G$ and $C = F$ and apply Proposition 6 to get $F \subseteq (F \cap B) + p^\alpha G$, and thus $F \subseteq (F \cap B)'$. If G has length λ, then we write $F = G(v)$ where $v = (\sigma(0), \sigma(1), \cdots, \sigma(n), \cdots)$ is a U_λ-sequence for G. Now if $x \in (F \cap B)'$ where $o(x) = m$, say, then by applying Corollary 1, we can write $x = b + g$ where $g \in p^{\sigma(m)}G$, $b \in B(v)$. It follows that $x \in G(v)$.

In general, fully invariant subgroups are not closed in the λ-topology. For example, $p^{\lambda+1}G$ is a fully invariant subgroup of G and has $p^\lambda G$ as its closure. On the other hand, the following proposition shows that λ-large subgroups of G are closed even when G has length exceeding λ.

PROPOSITION 8. *If B is a λ-basic subgroup of G, and if L is a λ-large subgroup of G, then $L = (L \cap B)'$.*

Proof. Since $G = L + B$ and $L \subseteq (L \cap B)'$, by the modular law $(L \cap B)' = (L + B) \cap (L \cap B)' = L + B \cap (L \cap B)'$. Thus it suffices to prove that $B \cap (L \cap B)' \subseteq L \cap B$. But by purity, $B \cap (L \cap B)' = (L \cap B)'_B$. Thus the result follows from Proposition 7, once we see that $L \cap B$ is fully invariant subgroup of B. Now if $z \in L \cap B$ and f is an endomorphism of B, then by applying the technique of Proposition 4, we can obtain a subgroup A of B where $\langle z_2, f(z_2)\rangle \subseteq A$ and $G = A \oplus K$. Thus there is an endomorphism of A mapping z_2 to $f(z_2)$ which extends to an endomorphism of G. Since L is a fully invariant subgroup of G, $f(z_2) = b$ is in L. Thus $x = b + z_1$ is in L, and $(L \cap B)'$ is contained in L.

COROLLARY 3. *If F is an unbounded, fully invariant subgroup of B, where B is a λ-basic subgroup of G, then F'_G is a fully invariant subgroup of G.*

Proof. We can write $F = B(v)$ where v is a U-sequence for B. But v is also a U-sequence for G. By Proposition 8, $G(v) = (G(v) \cap B)' = B(v)'$.

Note that the proof of Proposition 8 shows that $F \cap B$ is a fully invariant subgroup of B whenever F is a fully invariant subgroup of G and B is a λ-basic subgroup of G. This observation is important to our study which now turns to the quotients $L/p^\lambda G$.

PROPOSITION 9. *L is a λ-large subgroup of G if and only if $L/p^\lambda G$ is a λ-large subgroup of $G/p^\lambda G$.*

Proof. Suppose first that L is a λ-large subgroup of G. We have seen that $L/p^\lambda G$ is unbounded; since $G/p^\lambda G$ is a C_λ-group of length λ, we need only show that $L/p^\lambda G$ is a fully invariant subgroup. If B is a λ-basic subgroup of G, then $(L/p^\lambda G) \cap ((B + p^\lambda G)/p^\lambda G)$ is equal to $((L \cap B) + p^\lambda G)/p^\lambda G$. The latter quotient is an isomorphic copy of $L \cap B$, which is unbounded by Proposition 8, while $(B + p^\lambda G)/p^\lambda G$ is isomorphic to B. Thus $(L/p^\lambda G) \cap ((B + p^\lambda G)/p^\lambda G)$ is an unbounded, fully invariant subgroup of $(B + p^\lambda G)/p^\lambda G$, a λ-basic subgroup of $G/p^\lambda G$. By Proposition 8, $(L \cap B)' = L$ and thus the closure of $((L \cap B) + p^\lambda G)/p^\lambda G$ in $G/p^\lambda G$ is just $L/p^\lambda G$. Since $(B + p^\lambda G)/p^\lambda G$ is a λ-basic subgroup of $G/p^\lambda G$, Corollary 3 guarantees that $L/p^\lambda G$ is a fully invariant subgroup of $G/p^\lambda G$.

On the other hand, if $L/p^\lambda G$ is a λ-large subgroup of $G/p^\lambda G$, then we can easily show that L is a fully invariant subgroup of G. If B is a λ-basic subgroup of G, then $G = B + L$ since $G/p^\lambda G = ((B + p^\lambda G)/p^\lambda G) + (L/p^\lambda G)$.

THEOREM 2. *L is a λ-large subgroup G if and only if $L = G(v)$, where v is a U_λ-sequence for G.*

Proof. L is λ-large in G if and only if $L/p^\lambda G$ is λ-large in $G/p^\lambda G$. Thus L is λ-large in G if and only if $L/p^\lambda G = (G/p^\lambda G)(v)$ for some U_λ-sequence for $G/p^\lambda G$; however v is also a U_λ-sequence for G, and $(G/p^\lambda G)(v) = G(v)/p^\lambda G$. Hence L is λ-large in G if and only if $L/p^\lambda G = G(v)/p^\lambda G$.

4. The structure of λ-large subgroups. It is shown in [1] that some of the solutions to the open statement "A large subgroup L

of G has property P if and only if G has property P" are these properties: direct sum of cyclic groups, direct sum of countable groups, and totally projective group. In this section we study the relation between the structure of λ-large subgroups and the structure of the containing groups. Note that if G is a totally projective group of length $\Omega + \omega_2$ where Ω denotes the first uncountable ordinal, then $p^{\Omega+\omega}G$ is a direct sum of cyclic groups and $p^{\Omega}G$ is a direct sum of countable groups. Since each of these subgroups is a λ-large subgroup of G, we see that inheritance of structure in our general C_λ-theory is not as widespread as that in the classical theory.

PROPOSITION 10. *If L is a λ-large subgroup of G and if α is less than the length of $L/p^\lambda G$, then $p^\alpha L$ is also a λ-large subgroup of G.*

Proof. Let μ denote the length of $L/p^\lambda G$. We can assume that α is not less than ω and write $\alpha = \omega + \sigma$ and $\mu = \omega + \beta$, where $\sigma < \beta$. If $L = G(v)$, where $v = (\sigma(0), \sigma(1), \cdots, \sigma(n), \cdots)$, and if $\delta = \sup\{\sigma(n): n < \omega\}$, then $p^\omega L = p^\delta G$; hence $p^\alpha L = p^\sigma(p^\omega L) = p^{\delta+\sigma}G$ where $\delta + \sigma$ is less than λ.

PROPOSITION 11. ([4], [5], [2]). *Let F denote a fully invariant subgroup of the totally projective group K. Then F and K/F are totally projective and the length of K/F does not exceed the length of K.*

COROLLARY 4. *G/L is a totally projective group whenever L is a λ-large subgroup of G.*

Proof. If B is a λ-basic subgroup of G, then G/L is isomorphic to $B/(L \cap B)$ where $L \cap B$ is a fully invariant subgroup of the totally projective group B.

THEOREM 3. *If L is a λ-large subgroup of G, then L is a totally projective group only if G is a totally projective group.*

Proof. We first consider the case where G has length λ. Our proof is inductive on λ; if $\lambda = \omega$, then the result follows from Theorem 4.3 in [1]. Thus we assume the conclusion for all limit ordinals β less than λ where β is a limit ordinal cofinal with ω. If $L = G(v)$ where $v = (\sigma(0), \sigma(1), \cdots, \sigma(n), \cdots)$ is a U_λ-sequence for G, then we set $\delta = \sup\{\sigma(n): n < \omega\}$ and consider two cases.

Case 1. $\delta < \lambda$. In this case we note that $(G/p^\delta G)(v) = L/p^\delta G = L/p^\omega L$ is a totally projective group and is a δ-large subgroup of the

C_δ-group $G/p^\delta G$. By the induction hypothesis, $G/p^\delta G$ is a totally projective group as is $p^\delta G = p^\omega L$. Hence G is a totally projective group.

Case 2. $\delta = \lambda$. In this case, $p^\omega L = p^\delta G = 0$ and, according to the generalized Kulikov Criterion, L is σ-summable; thus $L[p] = \cup\{S(n)\colon n < \omega\}$ where $S(n) \subseteq S(n+1)$ and $S(n) \cap p^n L = 0$ for each $n < \omega$. Since $p^{\sigma(0)}G[p] = L[p]$, we can show that $p^{\sigma(0)}G$ is a σ-summable C_μ-group of length μ, where μ is a limit ordinal cofinal with ω. For each positive integer n, there is an ordinal $\mu(n)$ less than μ, the length of $p^{\sigma(0)}G$, such that $\sigma(n) = \sigma(0) + \mu(n) < \sigma(0) + \mu = \lambda$. From familiar properties of ordinals, it follows that $\mu = \sup\{\mu(n)\colon n < \omega\}$ and hence μ is cofinal with ω. Since $S(n) \cap p^{\mu(n)}(p^{\sigma(0)}G)[p] \subseteq S(n) \cap p^{\sigma(n)}G[p] \subseteq S(n) \cap p^n L = 0$, we see that $p^{\sigma(0)}G$ is σ-summable. Let β denote an ordinal less than μ. Since G is a C_λ-group, $G/p^\beta(p^{\sigma(0)}G)$ and $p^{\sigma(0)}(G/p^\beta(p^{\sigma(0)}G))$ are totally projective groups. Hence $p^{\sigma(0)}G/p^\beta(p^{\sigma(0)}G)$ is a totally projective group and $p^{\sigma(0)}G$ is a C_λ-group. Thus, by the generalized Kulikov Criterion, $p^{\sigma(0)}G$ is a totally projective group as is $G/p^{\sigma(0)}G$. So G possess this property.

In general, if G has length greater than λ, then $0 \neq p^\delta G = p^\omega L$ where $\delta = \sup\{\sigma(n)\colon n < \omega\}$. Thus $L/p^\omega L = L/p^\delta G$ is a totally projective group and δ-large in $G/p^\delta G$. By the argument given above, $G/p^\delta G$ is a totally projective group as is $p^\delta G = p^\omega L$.

Proposition 12. *If L is a λ-large subgroup of G and B is a λ-basic subgroup of G, then $L \cap B$ is a μ-basic subgroup of B, where μ denotes the length of $L/p^\lambda G$.*

Proof. Let $L = G(v)$ where v is a U_λ-sequence for G. Then $L \cap B = B(v)$ is a fully invariant subgroup of the totally projective group B. By Proposition 11, $L \cap B$ is a totally projective group. If $\delta = \sup\{\sigma(n)\colon n < \omega\}$ and β has the property that $\lambda = \delta + \beta$ and $\mu = \omega + \beta$, then $p^\mu(B(v)) = p^\beta(p^\omega B(v)) = p^\beta(p^\delta B) = p^\lambda B = 0$. Thus the length of $B(v) = L \cap B$ does not exceed μ.

In order to show that $L[p] \subseteq (L \cap B)[p] + p^\beta L$ for all β less than μ, we first note that $p^\beta L$ is λ-large in G. Thus, by Proposition 6, $p^\beta G[p] = p^\beta G[p] \cap (B + p^\beta L) = p^\beta B[p] + p^\beta L[p]$. Since B is a λ-basic subgroup of G, $G[p] = p^\beta G[p] + B[p] = B[p] + p^\beta L[p]$. Suppose now that $x \in L[p]$. Then $L[p] = L \cap G[p] = L \cap (B[p] + p^\beta L[p]) = (L \cap B)[p] + p^\beta L[p]$ (by the modular law).

All that remains is to show that there is no subgroup H of L properly containing $L \cap B$ such that $H[p] = (L \cap B)[p]$. It suffices to show that $pL \cap (L \cap B) \subseteq p(L \cap B)$ or $pL \cap B \subseteq p(B(v))$. So suppose that $pz \in pL \cap B$ for some $z \in L$. Then $pz \in p^{\sigma(1)}G \cap B \subseteq p^{\sigma(0)+1}B$, and

$pz = pb$ for some $b \in p^{\sigma(0)}B$. It follows that $b \in B(v)$ and that $pL \cap B \subseteq p(B(v))$.

COROLLARY 5. *If L is a λ-large subgroup of the C_λ-group G and if μ denotes the length of $L/p^\lambda G$, then L is a C_μ-group.*

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