

Pacific Journal of Mathematics

SOLVABILITY OF GROUPS OF ODD ORDER

by

WALTER FEIT (Cornell University)

and

JOHN G. THOMPSON (University of Chicago)

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