

**Pacific
Journal of
Mathematics**

VOLUME XIV

1964

PACIFIC JOURNAL OF MATHEMATICS

EDITORS

ROBERT OSSERMAN

Stanford University
Stanford, California

M. G. ARSOVE

University of Washington
Seattle 5, Washington

J. DUGUNDJI

University of Southern California
Los Angeles 7, California

LOWELL J. PAIGE

University of California
Los Angeles 24, California

ASSOCIATE EDITORS

E. F. BECKENBACH

B. H. NEUMANN

F. WOLF

K. YOSIDA

SUPPORTING INSTITUTIONS

UNIVERSITY OF BRITISH COLUMBIA

CALIFORNIA INSTITUTE OF TECHNOLOGY

UNIVERSITY OF CALIFORNIA

MONTANA STATE UNIVERSITY

UNIVERSITY OF NEVADA

NEW MEXICO STATE UNIVERSITY

OREGON STATE UNIVERSITY

UNIVERSITY OF OREGON

OSAKA UNIVERSITY

UNIVERSITY OF SOUTHERN CALIFORNIA

STANFORD UNIVERSITY

UNIVERSITY OF TOKYO

UNIVERSITY OF UTAH

WASHINGTON STATE UNIVERSITY

UNIVERSITY OF WASHINGTON

* * *

AMERICAN MATHEMATICAL SOCIETY

CALIFORNIA RESEARCH CORPORATION

SPACE TECHNOLOGY LABORATORIES

NAVAL ORDNANCE TEST STATION

CONTENTS

Tom M. Apostol and Herbert S. Zuckerman, <i>On the functional equation $F(mn)F(m, n) = F(m)F(n)f((m, n))$</i>	377
Richard Arens, <i>Normal form for a Pfaffian</i>	1
Reinhold Baer, <i>Irreducible groups of automorphisms of abelian groups</i>	385
E. Balslev and T. W. Gamelin, <i>The essential spectrum of a class of ordinary differential operators</i>	755
H. S. Bear, <i>An abstract potential theory with continuous kernel</i> .	407
Homer F. Bechtell, <i>Pseudo-Frattini subgroups</i>	1129
E. F. Beckenbach, <i>Superadditivity inequalities</i>	421
R. H. Bing, <i>The simple connectivity of the sum of two disks</i> . .	439
T. K. Boehme and A. M. Bruckner, <i>Functions with convex means</i>	1137
J. H. Bramble and L. E. Payne, <i>Bounds for derivatives in elliptic boundary value problems</i>	777
A. M. Bruckner, See T. K. Boehme and A. M. Bruckner.	
H. D. Brunk, <i>Integral inequalities for functions with nondecreasing increments</i>	783
L. Bungart, <i>Boundary kernel functions for domains on complex manifolds</i>	1151
Herbert Busemann, <i>Length-preserving maps</i>	457
G. T. Cargo, See O. Shisha and G. T. Cargo.	
L. Carlitz, <i>Rings of arithmetic functions</i>	1165
D. S. Carter, <i>Uniqueness of a class of steady plane gravity flows</i> .	1173
William Edward Christilles, <i>A result concerning integral binary quadratic forms</i>	795
Charles V. Coffman, <i>Non-linear differential equations on cones in Banach spaces</i>	9
H. S. Collins, <i>Characterizations of convolution semigroups of measures</i>	479
Paul Conrad, <i>The relationship between the radical of a lattice-ordered group and complete distributivity</i>	493
P. Crawley and B. Jónsson, <i>Refinements infinite direct decompositions of algebraic systems</i>	797
R. A. Dean and Robert H. Oehmke, <i>Idempotent semigroups with distributive right congruence lattices</i>	1187
Don Deckard and Carl Percy, <i>On continuous-valued functions on a Stonian space</i>	857
Ralph E. DeMarr, <i>Order convergence in linear topological spaces</i> .	17
R. F. Dickman, L. R. Rubin and P. M. Swingle, <i>Another characterization of the n-sphere and related result</i>	871
P. H. Doyle, <i>A sufficient condition that an arc in S^n be cellular</i> .	501

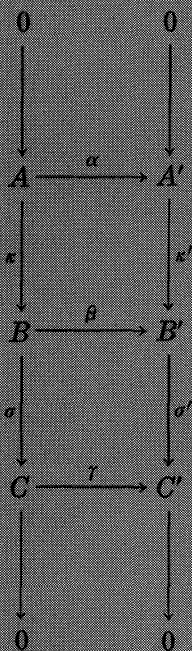
Lester Dubins and David Freedman, <i>Measurable sets of measures</i>	1211
P. L. Duren, <i>On the spectrum of a Toeplitz operator</i>	21
R. E. Edwards, <i>Endomorphisms of function-spaces which leave stable all translation-invariant manifolds</i>	31
Erik Ellentuck, <i>Infinite products of isols</i>	49
E. E. Enochs, <i>A note on reflexive modules</i>	879
C. L. Vanden Eynden, See Lowell A. Hinricks, Ivan and C. L. Vanden Eynden.	
Carl Faith and Yuzo Utumi, <i>Intrinsic extensions of rings</i>	505
Vladimir Filippenko, <i>On the reflection of harmonic functions and of solutions of the wave equation</i>	883
William J. Firey, <i>Some applications of means of convex bodies</i>	53
David Freedman, See Lester Dubins and David Freedman.	
W. Fulks, <i>An approximate Gauss mean value theorem</i>	513
D. J. H. Fuller, <i>Mappings of bounded characteristic into arbitrary Riemann surfaces</i>	895
Curtis M. Fulton, <i>Clifford vectors</i>	917
Haim Gaifman, <i>Concerning measures on Boolean algebras</i>	61
T. W. Gamelin, See E. Balslev and T. W. Gamelin.	
Richard C. Gilbert, <i>Extremal spectral functions of a symmetric operator</i>	75
R. P. Gilbert, <i>On a class of elliptic partial differential equations in four variables</i>	1223
I. Glicksberg, <i>Maximal algebras and a theorem of Rudó</i>	919
Harry Gonshor, <i>On abstract affine near-rings</i>	1237
Edward E. Grace, <i>Cut points in totally non-semi-locally-connected continua</i>	1241
Edward E. Grace, <i>On local properties and $G\delta$ sets</i>	1245
R. L. Graham, <i>On finite sums of reciprocals of distinct nth powers</i>	85
Hwa S. Hahn, <i>On the relative growth of differences of partition functions</i>	93
Kyong T. Hahn, <i>Minimum problems of Plateau type in the Bergman metric space</i>	943
Arshag Hajian, <i>Strongly recurrent transformations</i>	517
K. A. Hardie, <i>A proof of the Nakaoka-Toda formula</i>	1249
Morisuke Hasumi and T. P. Srinivasan, <i>Doubly invariant subspaces, II</i>	525
Allan Hayes, <i>A representation theory for a class of partially ordered rings</i>	957
Lowell A. Hinricks, Ivan Niven and C. L. Vanden Eynden, <i>Fields defined by polynomials</i>	537
Lowell A. Hinricks, <i>Open ideals in $C(X)$</i>	1255
I. I. Hirschman, Jr., <i>Extreme eigen values of Toeplitz forms associated with Jacobi polynomials</i>	107

Chen-Jung Hsu, <i>Remarks on certain almost product spaces</i> . . .	163
G. S. Innis, Jr., <i>Some reproducing kernels for the unit disk</i> . . .	177
J. R. Isbell, <i>Natural sums and Abelianizing</i>	1265
Ronald Jacobowitz, <i>Multiplicativity of the local Hilbert symbol</i> .	187
B. Jónsson, See P. Crawley and B. Jónsson.	
J. M. C. Joshi, <i>On a generalized Stieltjes transform</i>	969
J. M. C. Joshi, <i>Inversion and representation theorems for a generalized Laplace transform</i>	977
Paul Kelly, <i>On some mappings related to graphs</i>	191
G. W. Kimble, <i>A characterization of extremals for general multiple integral problems</i>	1283
W. A. Kirk, <i>On curvature of a metric space at a point</i>	195
N. Kishore, <i>A representation of the Bernoulli number B_n</i> . . .	1297
M. R. Krom, <i>A decision procedure for a class of formulas of first order predicate calculus</i>	1305
G. J. Kurowski, <i>On the convergence of semi-discrete analytic functions</i>	199
Richard G. Laatsch, <i>Extensions of subadditive functions</i>	209
W. B. Laffer and H. B. Mann, <i>Decomposition of sets of group elements</i>	547
Peter Lappan, <i>Identity and uniqueness theorem for automorphic functions</i>	1321
Lorraine D. Lavalley, <i>Mosaics of metric continua and of quasi-peans spaces</i>	1327
John A. Lindberg, Jr., <i>Algebraic extensions of commutative Banach algebras</i>	559
W. Ljunggren, <i>On the Diophantine equation $Cx^2 + D = y^n$</i>	585
M. Donald MacLaren, <i>Atomic orthocomplemented lattices</i>	597
Mark Mahowald, <i>On the normal bundle of a manifold</i>	1335
Moshe Marcus, <i>Transformations of domains in the plane and applications in the theory of functions</i>	613
H. B. Mann, See W. B. Laffer and H. B. Mann.	
V. Marić, <i>On some properties of solutions of $\Delta\psi + A(r^2)X\Delta\psi + C(r^2)\psi = 0$</i>	217
J. D. McKnight, Jr., <i>Kleene quotient theorems</i>	1343
E. K. McLachlan, <i>Extremal elements of the convex cone B_n of functions</i>	987
C. K. Megibben, <i>On high subgroups</i>	1353
Robert A. Melter, <i>Contributions to Boolean geometry of p-rings</i> .	995
Philip Miles, <i>B^* algebra unit ball extremal points</i>	627
Philip Miles, <i>Derivations on B^* algebras</i>	1359
W. H. Mills, <i>Polynomials with minimal value sets</i>	225
George J. Minty, <i>On the monotonicity of the gradient of a convex function</i>	243

George J. Minty, <i>On the solvability of nonlinear functional equations of 'Monotonic' type</i>	249
J. B. Muskat, <i>On the solvability of $x^e \equiv e \pmod{p}$</i>	257
Zeev Nehari, <i>On an inequality of P. R. Beesack</i>	261
W. F. Newns, <i>On the difference and sum of a basic set of polynomials</i>	639
Ivan Niven, See Lowell A. Hinricks, Ivan Niven and C. L. Vanden Eynden.	
Robert H. Oehmke, See R. A. Dean and Robert H. Oehmke.	
J. Marshall Osborn, <i>A generalization of power-associativity</i>	1367
B. L. Osofsky, <i>Rings all of whose finitely generated modules are injective</i>	645
T. G. Ostrom, <i>Nets with critical deficiency</i>	1381
L. E. Payne, see J. H. Bramble and L. E. Payne.	
Carl Pearcy, See Don Deckard and Carl Pearcy.	
C. R. Putnam, <i>Toeplitz matrices and invertibility of Hankel matrices</i>	651
Elvira Strasser Rapaport, <i>On the defining relations of a free product</i>	1389
R. M. Redheffer and E. G. Straus, <i>Degenerate elliptic equations</i>	265
James R. Retherford, <i>Basic sequences and the Paley-Wiener criterion</i>	1019
Abraham Robinson, <i>On generalized limits and linear functionals</i>	269
Kenneth Rogers, <i>A note on orthogonal latin squares</i>	1395
B. W. Roos, <i>On a class of singular second order differential equations with a non linear parameter</i>	285
L. R. Rubin, See R. F. Dickman, L. R. Rubin and P. M. Swingle.	
Tôru Saito, <i>Ordered completely regular semigroups</i>	295
Tôru Saito, "Regular elements in an ordered semigroup"	1505
Shôichiro Sakai, <i>Weakly compact operators on operator algebras</i>	659
D. W. Sasser, <i>Quasi-positive operators</i>	1029
Parfeny P. Saworotnow, <i>On continuity of multiplication in a complemented algebra</i>	1399
J. Schönheim, <i>On coverings</i>	1405
J. Segal, "Convergence of inverse system"	1505
Victor L. Shapiro, <i>Bounded generalized analytic functions on the torus</i>	1413
A. Sharma, See J. L. Walsh and A. Sharma.	
O. Shisha, <i>On the structure of infrapolynomials with prescribed coefficients</i>	1039
O. Shisha and G. T. Cargo, <i>On comparable means</i>	1053
O. Shisha, See J. L. Walsh and O. Shisha.	
Edward Silverman, <i>A problem of least area</i>	309
James E. Simpson, <i>Nilpotency and spectral operators</i>	665

R. C. Sine, <i>Spectral decomposition of a class of operators</i>	333
Maurice Sion, <i>A characterization of weak convergence</i>	1059
Morton L. Slater and Robert J. Thompson, <i>A permanent inequality for positive functions on the unit square</i>	1069
Walter L. Smith, <i>On the elementary renewal theorem for non- identically distributed variables</i>	673
David A. Smith, <i>On fixed points of automorphisms of classical Lie algebras</i>	1079
T. P. Srinivasan, See Morisuke Hasumi and T. P. Srinivasan.	
T. P. Srinivasan, <i>Doubly invariant subspaces</i>	701
James D. Stafney, <i>Arens multiplication and convolution</i>	1423
Sherman K. Stein, <i>Homogeneous quasigroups</i>	1091
Daniel J. Sterling, <i>Coverings of algebraic groups and lie algebras of classical type</i>	1449
E. G. Straus, See R. M. Redheffer and E. G. Straus.	
J. D. Swift, <i>Chains and graphs of Ostrom planes</i>	353
P. M. Swingle, See R. F. Dickman, L. R. Rubin and P. M. Swingle.	
J. Roger Teller, <i>On the extensions of lattice-ordered groups</i>	709
John G. Thompson, <i>2-signalizers of finite groups</i>	363
R. C. Thompson, <i>Unimodular group matrices with rational integers as elements</i>	719
R. J. Thompson, See Morton L. Slater and Robert J. Thompson.	
Yuzo Utumi, See Carl Faith and Yuzo Utumi.	
J. L. Walsh and A. Sharma, <i>Least squares and interpolation in roots of unity</i>	727
J. L. Walsh and O. Shisha, <i>On the location of the zeros of some infrapolynomials with prescribed coefficients</i>	1103
R. J. Warne, <i>Homomorphisms of d-simple inverse semigroups with identity</i>	1111
Charles E. Watts, <i>A Jordan-Hölder theorem</i>	731
R. Westwick, <i>Linear transformations on Grassmann spaces</i>	1123
Harold Widom, <i>On the spectrum of a Toeplitz operator</i>	365
Alfred B. Willcox, <i>Silov type C algebras over a connected locally compact abelian group II</i>	1463
Kung-Wei Yang, <i>On seme finite groups and their cohomology</i>	735
Adil Yaqub, <i>On the ring-logic character of certain rings</i>	741
B. Yood, <i>Faithful-representations of normed algebras II</i>	1475
Paul R. Young, <i>A note on pseudo-creative sets and cylinders</i>	749
A. Zabrodsky, <i>Covering spaces of paracompact spaces</i>	1489
Herbert S. Zuckerman, See Tom M. Apostol and Herbert S. Zuckerman.	

Pacific Journal of Mathematics



Richard Arens, <i>Normal form for a Pfaffian</i>	1
Charles V. Coffman, <i>Non-linear differential equations on cones in Banach spaces</i>	9
Ralph E. DeMarr, <i>Order convergence in linear topological spaces</i>	17
P. L. Duren, <i>On the spectrum of a Toeplitz operator</i>	21
R. E. Edwards, <i>Endomorphisms of function-spaces which leave stable all translation-invariant manifolds</i>	31
Erik Ellentuck, <i>Infinite products of isols</i>	49
William J. Firey, <i>Some applications of means of convex bodies</i>	53
Haim Gaifman, <i>Concerning measures on Boolean algebras</i>	61
Richard C. Gilbert, <i>Extremal spectral functions of a symmetric operator</i>	75
R. L. Graham, <i>On finite sums of reciprocals of distinct nth powers</i>	85
Hwa S. Hahn, <i>On the relative growth of differences of partition functions</i>	93
I. I. Hirschman, Jr., <i>Extreme eigen values of Toeplitz forms associated with Jacobi polynomials</i>	107
Chen-Jung Hsu, <i>Remarks on certain almost product spaces</i>	163
G. S. Innis, Jr., <i>Some reproducing kernels for the unit disk</i>	177
Ronald Jacobowitz, <i>Multiplicativity of the local Hilbert symbol</i>	187
Paul Kelly, <i>On some mappings related to graphs</i>	191
W. A. Kirk, <i>On curvature of a metric space at a point</i>	195
G. J. Kurowski, <i>On the convergence of semi-discrete analytic functions</i>	199
Richard G. Laatsch, <i>Extensions of subadditive functions</i>	209
V. Marić, <i>On some properties of solutions of $\Delta\psi + A(r^2)X\nabla\psi + C(r^2)\psi = 0$</i>	217
W. H. Mills, <i>Polynomials with minimal value sets</i>	225
George J. Minty, <i>On the monotonicity of the gradient of a convex function</i>	243
George J. Minty, <i>On the solvability of nonlinear functional equations of 'Monotonic' type</i>	249
J. B. Muskat, <i>On the solvability of $x^e \equiv e \pmod{p}$</i>	257
Zeev Nehari, <i>On an inequality of P. R. Beesack</i>	261
R. M. Redheffer and E. G. Straus, <i>Degenerate elliptic equations</i>	265
Abraham Robinson, <i>On generalized limits and linear functionals</i>	269
B. W. Roos, <i>On a class of singular second order differential equations with a non linear parameter</i>	285
Tôru Saitô, <i>Ordered completely regular semigroups</i>	295
Edward Silverman, <i>A problem of least area</i>	309
R. C. Sine, <i>Spectral decomposition of a class of operators</i>	333
J. D. Swift, <i>Chains and graphs of Ostrom planes</i>	353
John G. Thompson, <i>2-signalizers of finite groups</i>	363
Harold Widom, <i>On the spectrum of a Toeplitz operator</i>	365

PACIFIC JOURNAL OF MATHEMATICS

EDITORS

ROBERT OSSERMAN
Stanford University
Stanford, California

J. DUGUNDJI
University of Southern California
Los Angeles 7, California

M. G. ARSOVE
University of Washington
Seattle 5, Washington

LOWELL J. PAIGE
University of California
Los Angeles 24, California

ASSOCIATE EDITORS

E. F. BECKENBACH

B. H. NEUMANN

F. WOLF

K. YOSHIDA

SUPPORTING INSTITUTIONS

UNIVERSITY OF BRITISH COLUMBIA
CALIFORNIA INSTITUTE OF TECHNOLOGY
UNIVERSITY OF CALIFORNIA
MONTANA STATE UNIVERSITY
UNIVERSITY OF NEVADA
NEW MEXICO STATE UNIVERSITY
OREGON STATE UNIVERSITY
UNIVERSITY OF OREGON
OSAKA UNIVERSITY
UNIVERSITY OF SOUTHERN CALIFORNIA

STANFORD UNIVERSITY
UNIVERSITY OF TOKYO
UNIVERSITY OF UTAH
WASHINGTON STATE UNIVERSITY
UNIVERSITY OF WASHINGTON
* * *
AMERICAN MATHEMATICAL SOCIETY
CALIFORNIA RESEARCH CORPORATION
SPACE TECHNOLOGY LABORATORIES
NAVAL ORDNANCE TEST STATION

Mathematical papers intended for publication in the *Pacific Journal of Mathematics* should be typewritten (double spaced), and on submission, must be accompanied by a separate author's résumé. Manuscripts may be sent to any one of the four editors. All other communications to the editors should be addressed to the managing editor, L. J. Paige at the University of California, Los Angeles 24, California.

50 reprints per author of each article are furnished free of charge; additional copies may be obtained at cost in multiples of 50.

The *Pacific Journal of Mathematics* is published quarterly, in March, June, September, and December. Effective with Volume 13 the price per volume (4 numbers) is \$18.00; single issues, \$5.00. Special price for current issues to individual faculty members of supporting institutions and to individual members of the American Mathematical Society: \$8.00 per volume; single issues \$2.50. Back numbers are available.

Subscriptions, orders for back numbers, and changes of address should be sent to Pacific Journal of Mathematics, 103 Highland Boulevard, Berkeley 8, California.

Printed at Kokusai Bunken Insatsusha (International Academic Printing Co., Ltd.), No. 6, 2-chome, Fujimi-cho, Chiyoda-ku, Tokyo, Japan.

PUBLISHED BY PACIFIC JOURNAL OF MATHEMATICS, A NON-PROFIT CORPORATION

The Supporting Institutions listed above contribute to the cost of publication of this Journal, but they are not owners or publishers and have no responsibility for its content or policies.