should be replaced by $\overset{\wp}{z}$, and $\tilde{Z}$ should be replaced by $\overset{\wp}{Z}$. The symbols $\tilde{\mathfrak{A}}_m$ and $\tilde{\mathfrak{A}}^0_m$ should be replaced throughout by $\overset{\wp}{\mathfrak{A}}_m$ and $\overset{\wp}{\mathfrak{A}}^0_m$, respectively; however, $\tilde{\mathfrak{A}}_n$ and $\tilde{\mathfrak{A}}^0_n$ remain unchanged. The first equation of line 14 page 235 should be $'\overset{\wp}{\mathfrak{A}}_n = \tilde{\mathfrak{A}}'_n$."

Correction to

DUALITY AND TYPES OF COMPLETENESS IN LOCALLY CONVEX SPACES

WILLIAM B. JONES

Volume 18 (1966), 525-544

Proposition 2.14 is an obvious consequence of Lemma 2.8.

p. 538, line 5: The second equality is false in general for all α (see [4]).

Some misprints:

p. 526	§ 2 should start "$(\alpha, \beta) - \cdots$"
	line 3 of § 2, "α" instead of "a"
p. 528	last line, remove final "}"
p. 532	line 14, second "ε" should be "$\in$"
p. 535	line 2, should read $\cdots \leqq \frac{\varepsilon}{r}(r - \cdots$
p. 537	line 8, second "$=$" should be "$-$"
p. 541	line 9, "λ_0" instead of "1_0"

Correction to

UNIQUENESS AND EXISTENCE PROPERTIES OF BOUNDED OBSERVABLES

S. P. GUDDER

Volume 19 (1966), 81-93

The author recently discovered that the proof of the corollary to Theorem 4.5 is incorrect, thus invalidating Theorem 4.6. We show now that Theorem 4.6 is still true for a class of observables with infinite spectra and prove a generalization of Theorem 4.5.

An observable x is *semi-bounded above* (*below*) if there is a number

$-\infty < c < \infty$ such that $\sigma(x) \subset \{\lambda : \lambda \leqq c\}$ $(\sigma(x) \subset \{\lambda : \lambda \geqq c\})$. The following not only generalizes Theorem 4.5 but gives a much simpler proof.

THEOREM 1.1. *Let x and y be observables on a quite full logic which are semi-bounded above and suppose that $m(x)$ exists if and only if $m(y)$ exists and in that case $m(x) = m(y)$. Then* $\lambda_0 = \max\{\lambda : \lambda \in \sigma(x)\} = \max\{\lambda : \lambda \in \sigma(y)\}$ *and* $x(\lambda_0) = y(\lambda_0)$.

Proof. The first part of the conclusion follows just as in Theorem 4.5. Now suppose $m[x(\lambda_0)] = 1$, and $m[y(\lambda_0)] \neq 1$. Then there is a number $\mu < \lambda_0$ such that $m[y(-\infty, \mu)] > 0$. Now since $m(x)$ exists, so does $m(y)$ and we have

$$\lambda_0 = m(x) = m(y) = \int_{(-\infty,\lambda_0]} \lambda m[y(d\lambda)] = \left(\int_{(-\infty,\mu)} + \int_{[\mu,\lambda_0]}\right)\lambda m[y(d\lambda)]$$
$$\leqq \mu m[y(-\infty, \mu)] + \lambda_0 m[y[\mu, \lambda_0)] < \lambda_0 .$$

which is a contradiction. Thus $m[y(\lambda_0)] = 1$ whenever $m[x(\lambda_0)] = 1$ and hence $x(\lambda_0) \leqq y(\lambda_0)$. By symmetry $x(\lambda_0) = y(\lambda_0)$.

Of course the same result holds for observables which are semi-bounded from below.

THEOREM 1.2. *Let x and y be bounded observables on a quite full logic and suppose the spectrum of x has at most one limit point. If $m(x) = m(y)$ for all $m \in M$ then $x = y$.*

Proof. The most general such x has a point $\lambda_0 \in \sigma(x)$ which is a limit point from both above and below of elements of $\sigma(x)$. The other cases will follow in a similar manner. We can assume without loss of generality that $\lambda_0 = 0$. Let the points of $\sigma(x)$ be ordered as follows: $\mu_1 < \mu_2 < \cdots < \lambda_0 < \cdots < \lambda_2 < \lambda_1$. Now by Theorem 1.1 $\max\{\lambda : \lambda \in \sigma(y)\} = \lambda_1$ and $y(\lambda_1) = x(\lambda_1)$. Now let $x_1 = x - \lambda_1\chi_{\lambda_1}(x)$ and let $y_1 = y - \lambda_1\chi_{\lambda_1}(y)$. Letting f be the identity function $f(\lambda) = \lambda$ we have for $E \in B(R)$

$$x_1(E) = (f - \lambda_1\chi_{\lambda_1})(x)(E) = x[(f - \lambda_1\chi_{\lambda_1})^{-1}(E)]$$
$$= \begin{cases} x(E) \wedge x(\lambda_1)' & \text{if } 0 \in E \\ x(E) \vee x(\lambda_1) & \text{if } 0 \in E \end{cases} \qquad \cdots\cdots (1).$$

It is now easy to see that

$$\sigma(x_1) = \sigma(x) \cap \{\lambda_1\}'; \; x_1(\lambda_i) = x\lambda_i), \; i = 2, 3, \cdots ;$$

and

$$x_1(\mu_i) = x(\mu_i), \; i = 1, 2, \cdots .$$

Now

$$m(x_1) = m(x) - \lambda_1 m[x(\lambda_1)] = m(y) - \lambda_1 m[y(\lambda_1)] = m(y_1) .$$

Applying Theorem 1.1, $\lambda_2 = \max\{\lambda : \lambda \in \sigma(y_1)\}$ and $y_1(\lambda_2) = x_1(\lambda_2) = x(\lambda_2)$. It now follows by applying (1) to y_1 and y that λ_2 is the second largest number in $\sigma(y)$ and $y(\lambda_2) = y_1(\lambda_2) = x(\lambda_2)$. Continuing this process with the λ_i's and also the μ_i's we have $\{\lambda_i, \mu_i: i = 1, 2, \cdots\} \subset \sigma(y)$ and $y(\lambda_i) = x(\lambda_i)$, $y(\mu_i) = x(\mu_i)$, $i = 1, 2, \cdots$. Since λ_0 is a limit point of the λ_i's it follows that $\lambda_0 \in \sigma(y)$, $\{\lambda_i, \mu_i: i = 1, 2, \cdots\} = \sigma(y)$ and

$$\begin{aligned} y(\lambda_0) &= y(\{\lambda_i, \mu_i: i = 1, 2, \cdots\}') = [\Sigma y(\lambda_i) + \Sigma y(\mu_i)]' \\ &= [\Sigma x(\lambda_i) + \Sigma x(\mu_i)]' = x(\lambda_0) . \end{aligned}$$

Hence $y = x$.

A similar technique may be used to prove:

COROLLARY 1.3. *Let x and y be observables on a quite full logic which are semi-bounded from above (below) and suppose the spectrum of x has no finite limit point (this includes the possibility of a limit point at $-\infty(+\infty)$). Suppose $m(y)$ exists if and only if $m(x)$ exists and in that case $m(y) = m(x)$. Then $x = y$.*

We close with a slightly strengthened form of Lemma 6.2 [1].

LEMMA 1.4. *If L is quite full and has Property E, then L is a lattice and $m(a) = m(b) = 1$ implies $m(a \wedge b) = 1$.*

Proof. That L is a lattice follows from Lemma 6.2 [1]. If $m(a) = m(b) = 1$, then $m(x_a + x_b) = m(a) + m(b) = 2$ and hence $1 = m[(x_a + x_b)\{2\}] = m(a \wedge b)$.

This last lemma is of interest since it rules out the counter-example of Section 5 [1] and is thus a possible sufficient condition for Property E.