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ON NORMALOID OPERATORS

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The purpose of the present paper is to extend an earlier theorem of the author's on hyponormal operators to the following, on normaloid operators.

THEOREM. *Let N be an operator such that $N - zI$ is normaloid for all complex values of z. If $AN = N^*A$, for an arbitrary operator A, for which $0 \notin \mathrm{Cl}(W(A))$, then $N = N^*$.*

2. Notations. We consider bounded linear operators defined on a Hilbert space H. As usual, the symbols $s(T)$, $\Sigma(T)$, $W(T)$ and $\mathrm{Cl}(W(T))$ stand for the spectrum of an operator T, the closed convex hull of $s(T)$, the numerical range of T and the closure of $W(T)$ respectively.

An operator T is said to be normaloid if $\|T\| = \sup\{|z|; z \in s(T)\}$ and hyponormal, if $T^*T - TT^* \geqq 0$. It is known that if T is hyponormal, then T is normaloid and $T - zI$ is also hyponormal for all complex numbers z.

When the original version of this paper was submitted, the referee told me of [3] then existing as a preprint and this makes possible the following shorter proof.

Proof of Theorem. Since $AN = N^*A$ and $0 \notin \mathrm{Cl}(W(A))$, $s(N)$ is real [3]. Also $\Sigma(N) = \mathrm{Cl}(W(N))$ for such a normaloid operator N [1]. Hence $\mathrm{Cl}(W(N))$ is real. This completes the proof of theorem.

The corresponding result for hyponormal operators now follows as corollary from this theorem and the remark made above.

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