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DIAGONAL SUBMATRICES OF MATRIX MAPS

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The first question answered in this paper is: if $A: \lambda \rightarrow \mu$ is a linear operator between sequence spaces, with a matrix representation (a_{ij}), does it follow that the associated diagonal matrix $(a_{ij}\delta_{jj})$ maps λ into μ? An affirmative answer is given if λ is a normal (or monotone) sequence space and μ is a perfect sequence space.

Morever, if λ, μ are normed sequence spaces, under what conditions will the following inequality hold for all matrix maps (a_{ij}) from λ to μ: $\|(a_{ij})\| \geqq \|(a_{ij}\delta_{ij})\|$ (where $\|\cdot\|$ denotes the operator sup norm)?

We apply our answer to the first problem to give another proof for a theorem of S. Mazur.

Section 1 gives the definitions, notations, and some computations. Section 2 considers the first question. Section 3 gives the application to Mazur's theorem. Definitions of terms relating to sequence spaces are from Köthe (1).

1. Vector spaces of sequences, over the real or complex number system, which contain all finitely supported sequences are called sequence spaces and are denoted by λ and μ. Let x be a sequence $(x(1), \cdots, x(i), \cdots)$ in λ. By the support of x we mean the set of all indices i for which $x(i) \neq 0$. We use m to denote the space of all bounded sequences and ω to denote the space of all sequences. We say that a sequence of vectors $x_1, \cdots, x_k, \cdots$, in λ is disjointly supported if the family of supports $S(k)$ of x_k is disjoint and we use $x_1 \vee x_2 \vee \cdots \vee x_k \vee$ to denote the sequence $x \in \omega$ defined by $x(i) = x_k(i)$ if there is a k for which $i \in S(k)$ and $x(i) = 0$, otherwise.

Let c denote a sequence $(c(1), \cdots, c(i), \cdots)$ in ω. The sequence $(c(1)x(1), \cdots, c(i)x(i), \cdots)$ is denoted by $c \cdot x$. A sequence space λ which satisfies: $c \cdot x \in \lambda$ for all $c \in m$ and all $x \in \lambda$ is called a normal sequence space.

A monotone sequence space λ is a sequence space satisfying $c \cdot x \in \lambda$ for all $x \in \lambda$ and $c \in m$ where $c(i) = 1, -1$, or 0 (all i). The sequence $(1, 1, \cdots, 1, \cdots)$ is denoted by $\bar{1}$.

δ_{ij} denotes the Kronecker delta. If S is a set of indices and λ is a normal sequence space, the linear projection $\pi_s: \lambda \rightarrow \lambda$ is defined by setting $\pi_s(x)$ to be the vector in λ whose i-th coordinate is x_i when $i \in S$ and is 0 when $i \notin S$. If S is the set of all indices $m \leqq i \leqq n$, we shall write $\pi_{[m,n]}$.

DEFINITION 1.1. Let λ, μ be sequence spaces. A linear map

$A: \lambda \to \mu$ is said to have matrix representation (a_{ij}) if $\sum_j a_{ij}x(j)$ converges for all $x \in \lambda$ and if

$$A(x) = \left(\sum_j a_{1j}x(j), \cdots, \sum_j a_{ij}x(j), \cdots\right).$$

By the associated diagonal of A, we mean the linear map $D: \lambda \to \omega$ with matrix representation $(\delta_{ij}a_{ij})$. We write r_i to denote the sequence consisting of the i-th row of (a_{ij}) and c_j to denote the sequence consisting of the j-th column of (a_{ij}).

Linear operators with matrix representation are referred to as matrix maps.

A sign distribution σ on n places is any sequence $\sigma = (x_1, \cdots, x_i, \cdots, x_n)$ where $x_i = \pm 1$ for all i. Two sign distributions σ, τ are said to be distinct if $\sigma \neq \tau$ and $\sigma \neq -\tau$.

LEMMA 1.2. *Let* $\mathscr{S} = \{\sigma_1, \cdots, \sigma_i, \cdots, \sigma_{2^{n-1}}\}$ *be a family of* 2^{n-1} *distinct sign distributions on* n *places. Then*:

$$\sum_{1 \leq i \leq 2^{n-1}} \left| \sum_{1 \leq k \leq n} \sigma_i(k)x(k) \right| \geqq 2^{n-1} \max\{|x(1)|, \cdots, |x(n)|\},$$

where $x(1), \cdots, x(n)$ *are complex numbers.*

For the proof, see (1.3) of [3] where one uses the triangle inequality instead of the Lemma (1.2) in [3].

LEMMA 1.3. *Let* λ, μ *be sequence spaces of dimension* n. *Let* $A: \lambda \to \mu$ *be a matrix map represented by* (a_{ij}). *For each* $u \in \lambda$, $v' \in \mu^x$, *we can find a sign distribution* σ *on* n *places and* $y' \in \mu^x$ *so that*:

(a) $|y'(l)| = |v'(l)|$ *for all* $l = 1, 2, \cdots, n$.

(b) *If* x *is defined by* $x(k) = \sigma(k)u(k)$, *then*

$$(A(x), y') \geqq \sum_{1 \leq k \leq n} |a_{kk}u(k)v'(k)|.$$

Proof. Let $\mathscr{S} = \{\sigma_1, \cdots, \sigma_{2^{n-1}}\}$ be a family of 2^{n-1} distinct sign distributions on n places. Lemma (1.2) gives:

$$|v'(l)| \sum_{1 \leq i \leq 2^{n-1}} \left| \sum_{1 \leq k \leq n} \sigma_i(k)a_{lk}u(k) \right| \geqq 2^{n-1} |a_{ll}u(l)| \cdot |v'(l)|$$

for all $l = 1, \cdots, n$. Hence, we can find a sign distribution σ_1 so that:

$$\text{(1.3.1)} \qquad \sum_{1 \leq l \leq n} |v'(l)| \left| \sum_{1 \leq k \leq n} \sigma_i(k)a_{lk}u(k) \right| \geqq \sum_{1 \leq l \leq n} |a_{ll}u(l)| \cdot |v'(l)|.$$

Choose y' to satisfy:

$$y'(l) \sum_{1 \leqq k \leqq n} \sigma_i(k) a_{lk} u(k) = |v'(l)| \left| \sum_{1 \leqq k \leqq n} \sigma_i(k) a_{lk} u(k) \right| .$$

Thus, $|y'(l)| = |v'(l)|$. By (1.3.1), we get:

$$(A(x), y') \geqq \sum_{1 \leqq l \leqq n} |a_{ll} u(l) v'(l)| .$$

DEFINITION 1.4. A normed (Banach) sequence space λ is said to be a normed (Banach) ideal if it is a normal sequence space satisfying:

$$\|c \cdot x\| \leqq \|x\| \sup\{|c(i)|: i = 1, 2, \cdots\} \tag{1.4.1}$$

for all $c \in m$, $x \in \lambda$. A normed monotone sequence space is said to be monotonely normed if (1.4.1) holds for all $c \in m$ satisfying $c(i) = 1, -1$, or 0 (all i).

REMARK 1.5. If λ is a monotonely normed sequence space of dimension n and μ is a normed ideal of dimension n, then Lemma (1.3) shows that: $\|A\| \geqq \|D\|$ where D is the associated diagonal of A.

DEFINITION 1.6. Let $A = (a_{ij})$ be a matrix. Let

$$1 = m(1) \leqq n(1) < \cdots < m(k) \leqq n(k) < \cdots$$

be a sequence of indices. The submatrix of A whose (i, j)-th entry is a_{ij} when there is a k for which $m(k) \leqq i, j \leqq n(k)$ and whose (i, j)-th entry is 0, otherwise, is called the diagonal block submatrix of A determined by $m(k), n(k)$, and is denoted by B. Denote

$$\pi_{[m(k), n(k)]} A \pi_{[m(k), n(k)]}$$

by $A^{[k]}$. Note that $A^{[k]} = B^{[k]}$.

2. Throughout this section, λ and μ are spaces of sequences over the real numbers.

THEOREM 2.1. *Let λ be a monotone sequence space. Let μ be any sequence space. If $A: \lambda \to \mu$ is a matrix map and D is the associated diagonal submatrix, then $D(\lambda) \subset \mu^{xx}$.*

Proof. Suppose $D(\lambda) \not\subset \mu^{xx}$. Then we can find $u \in \lambda$, $v' \in \mu^x$ so that

$$\sum_i |a_{ii} u(i) v'(i)| = \infty .$$

Define a sequence of indices

$$1 = m(1) \leqq n(1) < \cdots < m(k) \leqq n(k) \cdots$$

inductively. Suppose (a_{ij}) is the matrix representation of A. Set $m(1) = 1$. Assume: $m(1) \leqq \cdots < m(k)$ have been chosen.

(2.1.1) Choose $n(k)$ so that
(1) $m(k) \leqq n(k)$
(2) $\sum_{m(k) \leqq i \leqq n(k)} |a_{ii}u(i)v'(i)| > k$.

(2.1.2) Choose $m(k+1)$ to satisfy
(1) $n(k) < m(k+1)$
(2) $\sum_{m(k+1) \leqq j < \infty} |a_{ij}u(j)v'(i)| < 1/2^k n(k)$ whenever $m(k) \leqq i \leqq n(k)$
(3) $\sum_{m(k+1) \leqq i < \infty} |a_{ij}u(j)v'(i)| < 1/2^k n(k)$ whenever $m(k) \leqq j \leqq n(k)$.
Here, we used the fact that the i-th row r_i of (a_{ij}) is a sequence in λ^x to obtain (2) and the assumption that $A(\lambda) \subset \mu$ and that $v' \in \mu^x$ to obtain (3).

Let $\sigma_1, \cdots, \sigma_k, \cdots$, be elements in m so that σ_k is supported on $[m(k), n(k)]$, $|\sigma_k(j)| = 1$ if j is in the support, and so that

$$\sum_{m(k) \leqq i \leqq n(k)} |v'(i)| \left| \sum_{m(k) \leqq j \leqq n(k)} a_{ij}\sigma_k(j)u(j) \right| \geqq \sum_{m(k) \leqq i \leqq n(k)} |a_{ii}u(i)v'(i)| \,.$$

Here, we used (1.3.1). Hence, by (2.1.1),

$$\sum_k \left(\sum_{m(k) \leqq i \leqq n(k)} |v'(i)| \left| \sum_{m(k) \leqq j \leqq n(k)} a_{ij}\sigma_k(j)u(j) \right| \right) \geqq \sum_k \sum_{m(k) \leqq i \leqq n(k)} |a_{ii}u(i)v'(i)| = \infty \,.$$

If we can show that the term on the left, which we denote by Δ, is finite, then a contradiction results and the hypothesis $D(\lambda) \not\subset \mu^{xx}$ is false.

Define $\sigma = \sigma_1 \vee \cdots \vee \sigma_k \vee \cdots$. Let $\tau \in m$ be defined by

$$\tau(i) = \operatorname{sign}\left(v(i) \sum_{m(k) \leqq j \leqq n(k)} a_{ij}\sigma_k(j)u(j) \right)$$

whenever $m(k) \leqq i \leqq n(k)$ and by $\tau(i) = 0$, otherwise. Let

$$S_k = \{i \colon m(k') \leqq i \leqq n(k') \text{ for some } k' > k\} \,.$$

Observe that (2.1.2) (2) and (3) give:

$$\sum_k \sum_{j \in S_k} \sum_{m(k) \leqq i \leqq n(k)} |a_{ij}u(j)v'(i)| < \infty$$
$$\sum_k \sum_{i \in S_k} \sum_{m(k) \leqq j \leqq n(k)} |a_{ij}u(j)v'(i)| < \infty$$

By inspection, $\Delta = (A(\sigma \cdot u), \tau \cdot v')$

$$- \sum_k \sum_{j \in S_k} \sum_{m(k) \leqq i \leqq n(k)} a_{ij}u(j)v'(i)$$
$$- \sum_k \sum_{i \in S_k} \sum_{m(k) \leqq j \leqq n(k)} a_{ij}u(j)v'(i) \,.$$

Since λ and μ^x are monotone sequence spaces and since $A(\lambda) \subset \mu$, we must have: $\sigma \cdot u \in \lambda$ so that

$$(A(\sigma \cdot u), \tau \cdot v') < \infty .$$

Thus $\Delta < \infty$.

LEMMA 2.2. *Let X be a normed space. Let (a_{ij}) be an $n \times n$ matrix of vectors in X. Then there is a sign distribution σ on n places and a vector $y' \in l_\infty^n$ where $|y'(i)| = 1$ for $1 \leqq i \leqq n$ such that:*

$$\left\| \sum_{1<i\leqq n} y(i) \sum_{1\leqq j\leqq n} \sigma(j) a_{ij} \right\| \geqq \left\| \sum_{1\leqq k\leqq n} a_{kk} \right\| .$$

Proof. For each $x' \in X'$, define a matrix A of real numbers by setting the i, j-th term to be (a_{ij}, x'). Choose $u = \bar{1}$, $v' = \bar{1}$ in (1.3) to get the existence of a sign distribution σ on n places and a sequence y' so that $|y'(i)| = 1$ for all $i = 1, 2, \cdots, n$ and

$$\left(\sum_i y'(i) \sum_j \sigma(j) a_{ij}, x' \right) = (A(\sigma), y') \geqq \sum_{1\leqq k\leqq n} |(a_{kk}, x')| .$$

By the Hahn-Banach theorem, this implies:

$$\left\| \sum_i y'(i) \sum_j \sigma(j) a_{ij} \right\| \geqq \left\| \sum_{1\leqq k\leqq n} a_{kk} \right\| .$$

THEOREM 2.3. *Let $1 = m(1) \leqq n(1) < \cdots m(k) \leqq n(k) \cdots$ be a sequence of indices. Let λ be a monotonely normed sequence space and μ be a sequence space. Let $A: \lambda \to \mu$ be a matrix map. Then, the associated diagonal block submatrix B, determined by $\{m(k), n(k)\}$, of A satisfies: $B(\lambda) \subset \mu^{xx}$ and $\|A\| \geqq \|B\|$ where $\|\cdot\|$ denotes the operator sup norm.*

Proof. Let π_k^1 denote $\pi_{[m(k),n(k)]}$ and π_k^2 denote $\pi_{[n(k)+1,m(k+1)-1]}$. Define a_{ij} to be $\pi_l^1 A \pi_k^1$, $\pi_l^2 A \pi_k^1$, $\pi_l^1 A \pi_k^2$, or $\pi_l^2 A \pi_k^2$ depending on whether

(1) $i = 2k - 1$ and $j = 2l - 1$,

(2) $i = 2k - 1$ and $j = 2l$,

(3) $i = 2k$ and $j = 2l - 1$ or

(4) $i = 2k$ and $j = 2l$

respectively. The vectors a_{ij} are to be regarded as linear operators from λ to μ. Let $\mathscr{D}$ denote the diagonal submatrix of (a_{ij}). Let $\mathscr{A}^{[k]}$ denote the $2k \times 2k$ submatrix of (a_{ij}) whose i, j term is a_{ij} if $1 \leqq i, j \leqq 2k$. Let $\mathscr{D}^{[k]}$ be the diagonal submatrix of $\mathscr{A}^{[k]}$. We may regard $\mathscr{A}^{[k]}$ (and also $\mathscr{D}^{[k]}$) as an operator:

$$\mathscr{A}^{[k]}(x) = \sum_{1 \leq j \leq 2k} \sum_{1 \leq i \leq 2k} a_{ij}(x) .$$

If we set $X =$ all bounded linear operators from λ to μ^{xx}, then we get from Lemma (2.2) that:

$$\| \mathscr{A}^{[k]} \| \geqq \| \mathscr{D}^{[k]} \| .$$

Here, we used the fact that μ^{xx} is a normed ideal and that λ is monotonely normed. Assuming, momentarily, that $\mathscr{D}(\lambda) \subset \mu^{xx}$, we get that since $B = \pi_s \mathscr{D}$ where $S = \{i: m_k \leqq i \leqq m_k \text{ for some } k\}$ we must also have that $B(\lambda) \subset \mu^{xx}$: it is easy to see that:

$$\| A \| \geqq \| \mathscr{A}^{[k]} \| \geqq \| \mathscr{D}^{[k]} \|$$

and so,

$$\| A \| \geqq \lim_k \| \mathscr{D}^{[k]} \| = \| \mathscr{D} \| .$$

Thus,

$$\| A \| \geqq \| \mathscr{D} \| \geqq \| \pi_s \mathscr{D} \| = \| B \|$$

because μ^{xx} is a normed ideal.

To see that $\mathscr{D}(\lambda) \subset \mu^{xx}$, write: $x_{2k-1} = \pi_k^1(x)$, $x_{2k} = \pi_k^2(x)$ for each $x \in \lambda$. Similarly, define $y'_{2k-1} = \pi_k^1(y')$ and $y'_{2k} = \pi_k^2(y')$ for each $y' \in \mu^x$. For each $x \in \lambda$ and each $y' \in \mu^x$, let κ denote

$$\{u \in m: u(1)x_1 \vee \cdots \vee u(i)x_1 \vee \cdots \in \lambda\} .$$

Let ν^x denote

$$\{v' \in m: v'(1)y'_1 \vee \cdots \vee v'(i)y'_1 \vee \cdots \in \mu^x\} .$$

Then κ is monotonely normed and $(\nu^x)^x$ is a Banach ideal if κ and ν^x are given the respective induced norms from λ and μ^x.

The matrix $M = (a_{ij}(x_j), y'_1)$ defines a matrix map from κ to ν^{xx} if we set:

$$M(u) = \left(\sum_{1 \leqq j < \infty} u(j)(a_{ij}(x_j), y'_1), \cdots, \sum_{1 \leqq j < \infty} u(j((a_{ij}(x_j), y'_1), \cdots \right)$$

for each $u \in \kappa$. If D is the associated diagonal of M, then (2.1) gives $D(\kappa) \subset \nu^{xx}$, since $\bar{1} \in \kappa$ and $\bar{1} \in \nu^r$, we get:

$$\sum_i (a_{ii}(x_i), y'_1) < \infty .$$

Thus:

$$(\mathscr{D}(x), y') = \sum_i (\mathscr{D}(x_i), y'_1) = \sum_i (a_{ii}(x_i), y'_1) < \infty$$

holds for all $x \in \lambda$, $y' \in \mu^x$. $\therefore$ $\mathscr{D}(\lambda) \subset \mu^{xx}$.

REMARKS 2.4. Although c_0 is not a perfect sequence space, it is nevertheless true that if λ is any normal sequence space (over the reals) and $A: \lambda \to c_0$ is a matrix map, then the associated diagonal D of A maps λ into c_0. For, if $A(\lambda) \subset c_0$, then Theorem (2.1) shows $D(\lambda) \subset m$. Let (a_{ij}) be the matrix representation of A. If r_i denotes the i-th row of (a_{ij}), it is not hard to check that $\{r_i\}$ is a 0-convergent in the weak topology on l_1 from l_∞. Thus, $\{r_i\}$ is 0-convergent in the l_1 norm. In particular,

$$\lim |a_{ii}| \leqq \lim_i l_1(r_i) = 0 . \qquad D(\lambda) \subset c_0 .$$

(Essentially the same argument holds even when $\lambda \not\subset m$.)

The techniques used in (2.1) and (2.3) are similar to that of [3]. The results there were given for $\lambda = l_p$, $\mu = l_r$ and sharper conclusions were derived insofar as we were able to establish a criterion for $\|A\| > \|D\|$ whenever $1 \leqq r < p \leqq \infty$.

We note also that (2.3) above may be reinterpreted as showing that the projection from the space of bounded matrix maps onto the subspace of the associated diagonals is of norm one, provided that λ is monotonely normed, while μ is a normed ideal and a perfect sequence space. Actually, we have required μ to be a normed ideal because we are proving these results in such a way as to cover sequence spaces over the complex scalars. If real scalars are being used, then it suffices to assume that μ is monotonely normed. The crucial use of these hypotheses is in facilitating the use of Lemma (1.3) where the definition of y' following (1.3.1) is the deciding issue.

3. This section gives an independent proof of Theorem (2.1) for the case where λ is normal. With this hypothesis on λ, we are able to reduce the problem to an argument involving matrix maps between Banach sequence spaces and thereby avoid the involved computations of (2.1). We allow λ and μ to be sequence spaces over the reals or the complex numbers. The rest of the section gives the application to the problem of Mazur.

THEOREM 3.1. *Let λ be a normal sequence space. Let μ be any sequence spaces. Let $A: \lambda \to \mu$ be a matrix map. Then the associated diagonal matrix D of A maps λ into μ^{xx}.*

Proof. If $u \in \lambda$ and $v' \in \mu^x$, define κ, ν to be Banach sequence spaces by setting:

$$\kappa = \{x \in \omega : x \cdot u \in m\}$$

$$\|x\|_\kappa = \sup\left\{\frac{|x(k)|}{|u(k)|} : k = 1, 2, \cdots\right\}$$

$$\nu = \left\{ y \in \omega \colon \sum_k |\, y(k)v'(k) \,| < \infty \right\}$$
$$\|\, y \,\|_\nu = \sum_k |\, y(k)v'(k) \,| \; .$$

Here, we have assumed without loss of generality, that $u(k) \neq 0$, $v'(k) \neq 0$ for all k. It is easy to check that κ and ν are Banach ideals. Clearly, $A(\kappa) \subset \nu$ and since any matrix map between Banach sequence spaces is bounded (see Corollary 5, p. 204 of [4]), A is also a bounded operator.

The theorem is proved if we show that $D(\kappa) \subset \nu$. Let $\|\cdot\|$ denote the operator sup norm. If D does not map κ into ν, then:

$$\sum_k |\, a_{kk}u(k)v'(k) \,| = \infty \; .$$

By (1.5), we have:

$$\begin{aligned} \infty > \|\, A \,\| &\geqq \sup_n \|\, \pi_{[1,n]} A \pi_{[1,n]} \,\| \\ &\geqq \sup_n \|\, \pi_{[1,n]} D \pi_{[1,n]} \,\| \geqq \|\, D \,\| \\ &\geqq \sum_k |\, a_{kk}u(k)v'(k) \,| = \infty \; . \end{aligned} \tag{3.1.1}$$

Hence, we must have had: $D(\kappa) \subset \nu$.

If μ is a normed sequence space we give μ^{xx} the norm generated by (unit ball of $\mu)^{xx}$.

THEOREM 3.2. *Let λ be a monotonely normed sequence space. Let μ be any sequence space. Let $A\colon \lambda \to \mu$ be a bounded matrix map. If D is the associated diagonal of A, then $D(\lambda) \subset \mu^{xx}$ and $\|\, A \,\| \geqq \|\, D \,\|$, where the* sup norm *is computed with respect to the norms of λ and μ^{xx}.*

Proof. By (2.1), $D(\lambda) \subset u^{xx}$. Observe that

$$\|\, D \,\| = \lim_n \|\, \pi_{[1,n]} D \pi_{[1,n]} \,\|$$

and argue as in (3.1.1), observing that $\pi_{[1,n]}(\lambda)$ is a monotonely normed sequence (of dimension n) and that $\pi_{[1,n]}(\mu)$ is a normed ideal (of dimension n) so that the appeal to (1.5) is legitimate.

S. Mazur posed and affirmatively answered the following problem: If (a_{ij}) is a matrix of complex numbers so that $\sum_i |\sum_j a_{ij}x(j)\,| < \infty$ for all $x \in m$, does it follow that $\sum_i |\, a_{ii} \,| < \infty$?

His answer, found in Pelczyński-Szlenk [2], gives a stronger conclusion. In this section, we apply our results in § 2 to give a more general version of the problem.

THEOREM 3.3. *Let* $1 \leqq p, r \leqq \infty$. *Let* g *denote* ∞, $pr/(p-r)$, *or* r *depending on whether* $1 \leqq p \leqq r \leqq \infty$, $1 \leqq r < p < \infty$, *or* $1 \leqq r < \infty$ *and* $p = \infty$ *respectively. Let* $A: l_p \to l_r$ *be a matrix map. Then* $l_g(a_{11}, \cdots a_{ii}, \cdots) < \infty$. *In fact, if* D *is the associated diagonal of* A, *then* $D(l_p) \subset l_r$ *and* $\|A\| \geqq \|D\|$.

Proof. Note that for the case $p = \infty$, $r = 1$, we have $g = 1$ so that this theorem contains the affirmative answer of Mazur. Observing that $l_r = l_r^{xx}$, we conclude from (2.1) that $D(l_p) \subset l_r$. In (2.2) of [3], it was computed that $\|D\| = l_g(a_{11}, \cdots, a_{ii}, \cdots)$ and an inspection of the proof shows that $D(l_p) \subset l_r$ if and only if $\|D\| < \infty$. Finally,

$$\begin{aligned} \|A\| &\geqq \lim_n \|\pi_{[1,n]} A \pi_{[1,n]}\| \\ &\geqq \lim_n \|\pi_{[1,n]} D \pi_{[1,n]}\| \\ &= \lim_n l_g(a_{11}, \cdots, a_{nn}) = \|D\| \end{aligned}$$

where (3.2) was used to obtain the second inequality. We note in passing that a Baire category argument proves every matrix map (between Banach sequence spaces) to be bounded (see Corollary 5, p. 204 of [4]) so that in the theorem above, $\|D\| \leqq \|A\| < \infty$.

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