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A CONNECTED HAUSDORFF SPACE WHICH IS NOT CONTAINED IN A MAXIMAL CONNECTED SPACE

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An example of a countable connected Hausdorff space (X, σ) is given which has the property that for every topology γ strictly larger than σ, where (X, γ) is connected, there exists a topology γ', strictly larger than γ, such that (X, γ') is connected. There also exists uncountable connected Hausdorff spaces which have this property.

The purpose of this note is to present an example of a connected Hausdorff space which cannot be embedded into a maximal connected space.

1. **Preliminaries.** R will be used to denote the real line with the usual topology, τ. Let X be a set and let A and B be families of subsets of X, then $A \vee B$ denotes the topology generated by the subbase $P = \{C \mid C \in A \text{ or } C \in B\}$.

DEFINITION 1 (see [4]). A connected topological space (X, τ) is maximal connected if for every topology τ_1, where τ_1 is strictly larger than τ, (X, τ_1) is not connected.

REMARK 1. Let X be a set and τ_n, $n = 1, 2, 3, \cdots$, be a sequence of topologies on X such that (X, τ_n) is connected for $n = 1, 2, \cdots$. If τ is the topology which is the least upper bound of the sequence τ_n, $n = 1, 2, \cdots$, then (X, τ) is not necessarily connected (see problem 5, page 155, [1]).

DEFINITION 2. A topological space (X, τ) is maximal perfect if (X, τ) has no isolated points and, for every topology τ_1 strictly larger than τ, (X, τ_1) has an isolated point.

LEMMA 1. *If (X, τ) is any topological space without isolated points, then there exists a topology $\tau_1 \supset \tau$ such that (X, τ_1) is maximal perfect.*

Proof. Let $\{\tau_\alpha\}_{\alpha \in A}$ be a linearly ordered family of topologies on X such that, for each $\alpha \in A$, $\tau_\alpha \supset \tau$ and (X, τ_α) has no isolated points. Let σ be the topology generated by $\{\tau_\alpha\}_{\alpha \in A}$. If there exists some $x \in X$ such that $\{x\} \in \sigma$, then, since $\{\tau_\alpha\}_{\alpha \in A}$ is linearly ordered, $\{x\} \in \tau_\alpha$, for some α. Therefore, (X, σ) has no isolated points. It therefore

follows by Zorn's lemma that there exists a topology $\tau_1 \supset \tau$ such that (X, τ_1) is maximal perfect.

2. **Particular maximal perfect topologies on sets of rationals.** In constructing the topological space (X, σ) referred to in the title, we will make use of the fact that the restriction of σ to certain subsets of X is maximal perfect. However, in order to make full use of this fact, we need to know something about the structure of the open subsets of these maximal perfect subspaces. Consequently, we now put a particular maximal perfect topology on a dense subspace of the rationals. We will make use of this model in constructing the required topological space.

Let P denote a dense subset of the rationals with the topology τ inherited from R.

DEFINITION 3. A set $G \subset P$ is an N-set if $G = \phi$ or if for each $x \in G$ and for every $b > x$, $\{y \in G \mid x < y < b\}$ has nonempty τ-interior.

DEFINITION 4. A collection $\mathscr{G}$ of N-sets which is closed under finite intersections is an N-family.

It follows from Zorn's lemma that every N-family is contained in a maximal N-family. For each $x \in P$, let $I_x = \{y \in P \mid x \leqq y < x + 1\}$. Put $B = \{I_x \mid x \in P\} \cup \{0 \subset P \mid 0 \in \tau\}$. B is contained in an N-family, B_1. Let $\mathscr{M}$ be a maximal N-family containing B_1.

If D is a subset of P, then D^c denotes the complement of D in P. Let $Q = \{D \subset P \mid D^c$ is τ-nowhere dense in $P\}$. Then Q is contained in a filter $\mathscr{F}$, where $\mathscr{F}$ is maximal with respect to the properties that, (i) $Q \subset \mathscr{F}$ and (ii) if $F \in \mathscr{F}$, then F is τ-dense in P. Throughout, $\mathscr{M}$ and $\mathscr{F}$ will be used, respectively, to denote a maximal family of N-sets and a maximal filter of dense sets constructed as outlined above.

LEMMA 2. *Let $\sigma = \mathscr{M} \vee \mathscr{F}$. Then $\tau \subset \sigma$ and (P, σ) is a maximal perfect space.*

Proof. Since $\tau \subset \mathscr{M}$, it follows that $\tau \subset \sigma$. Let $x \in P$ and let $x \in F \cap M$, where $F \in \mathscr{F}$ and $M \in \mathscr{M}$. Since M contains a τ-open set U and $F \cap U$ is τ-dense in U, it follows that $F \cap M \neq \{x\}$. Therefore, (P, σ) is perfect.

Suppose (P, σ) is not maximal perfect. Then there exists a topology γ on P such that γ is strictly larger than σ and (P, γ) is perfect. Let $V \in \gamma$ such that $V \notin \sigma$. Let x_0 be an arbitrary fixed element of V. For every $b > x_0$, $\{x \in P \mid x_0 < x < b\} \cap V \neq \phi$. [Otherwise, for some $b > x_0$, $V \cap \{x \in P \mid x_0 - 1 < x < b\} \cap I_{x_0} = \{x_0\}$, where $I_{x_0} =$

$\{x \in P \mid x_0 \leqq x < x_0 + 1\}$, and $\{x_0\}$ would be an element of γ which is impossible.]

Suppose there exists some $b > x_0$ such that $D = \{x \in P \mid x_0 < x < b\} \cap V$ is τ-nowhere dense in $\{x \in P \mid x_0 < x < b\}$. Then $D \in \gamma$ and it follows from the construction of $\mathscr{F}$ that $D^c \in \mathscr{F}$. If y is a fixed element of D, then $(D^c \cup \{y\}) \in \mathscr{F}$ and $D \cap (D^c \cup \{y\}) = \{y\} \in \gamma$, which is impossible. Therefore, for every $b > x_0$, $\{x \in P \mid x_0 < x < b\} \cap V$ is τ-dense in $I \cap P$, for some open interval $I \subset R$. Let $\{J_n\}_{n=1}^{\infty}$ be a family of disjoint τ-open subsets of P whose union is the τ-interior of the τ-closure of $V \cap \{x \in P \mid x > x_0\}$.

For each positive integer n, there exists an $F \in \mathscr{F}$ such that $V \cap J_n = F \cap J_n$. [Suppose there exists some n such that for each $F \in \mathscr{F}$, $V \cap J_n \neq F \cap J_n$. Since $\mathscr{F}$ is a maximal filter of τ-dense subsets of P and since $V \cap J_n$ is τ-dense in J_n, this implies that there exists some $F_1 \in \mathscr{F}$ such that $(V \cap J_n) \cap F_1$ is not τ-dense in J_n. Hence, there exists some τ-open set $L \subset J_n$ such that $L \cap F_1 \cap (V \cap J_n) = \phi$. If x is a fixed element of $L \cap (V \cap J_n)$, then $(F_1 \cup \{x\}) \in \mathscr{F}$ and $(F_1 \cup \{x\}) \cap L \cap (V \cap J_n) = \{x\} \in \gamma$, which is impossible.]

Put $G = \{x_0\} \cup \{\bigcup_n^{\infty} J_n\}$. G is an N-set. Also, $G \in \mathscr{M}$. [For, if $G \notin \mathscr{M}$, $G \cap G_1$ is not an N-set for some $G_1 \in \mathscr{M}$. This implies there exists some $x_1 \in G \cap G_1$ and some $b > x_1$ such that $\{x \in P \mid x_1 < x < b\} \cap G \cap G_1$ does not contain a τ-open set. It is clear that $x_1 = x_0$. It follows as a consequence of how G was constructed from V, that $C = V \cap \{x \in P \mid x_0 < x < b\} \cap G_1$ is a τ-nowhere dense subset of P. Also, $C \in \gamma$. $C^c \in \mathscr{F}$ and if $y \in C$, then $(C^c \cup \{y\}) \cap C = \{y\} \in \gamma$. If C is empty, then, since $\{x \in P \mid x_0 \leqq x < b\} \in \mathscr{M}$, it follows that $V \cap \{x \in P \mid x_0 \leqq x < b\} \cap G_1 = \{x_0\} \in \gamma$. A contradiction, therefore, $G \in \mathscr{M}$.]

Now, put $T = [\bigcup_{n=1}^{\infty} \{V \cap J_n\}] \cup \{\bigcup_{n=1}^{\infty} J_n\}^c$. For each τ-open set $U \subset P$, T is τ-dense in U. $\mathscr{F}$ is a maximal filter of τ-dense subsets of P. If $T \notin \mathscr{F}$, then there exists some $F' \in \mathscr{F}$ such that $T \cup F'$ is not τ-dense in P. By the definition of T, this implies that $F' \cap (V \cap J_n)$ is not dense in J_n for some n. However, this is impossible since $V \cap J_n = F \cap J_n$, for some $F \in \mathscr{F}$, and $\mathscr{F}$ is a filter of τ-dense subsets of P. Therefore $T \in \mathscr{F}$. Clearly $x_0 \in G \cap T$ and $G \cap T \subset V$. Also, $G \cap T \in \sigma$, since $G \in \mathscr{M}$ and $T \in \mathscr{F}$. Therefore, $G \cap T \subset V$ is a σ-open neighborhood of x_0, which, since x_0 is an arbitrary element of V, contradicts the assumption that $V \notin \sigma$. Hence, (P, σ) is maximal perfect and the lemma is established.

3. Example. We now give an example of a countable connected Hausdorff space (X, σ) which is not contained in any maximal connected space. The space (X, σ) is a modification of a countable connected Urysohn space which was constructed by P. Roy (see [3]).

Let $\{E_i\}_{i=+\infty}^{-\infty}$ be a countable, disjoint collection of dense subsets

of rational numbers indexed by the set of all integers. For each integer n, let $P_n = \{(x, n) \mid x \in E_n\}$. Let w denote an ideal point. Let $X = \{w\} \cup \{\bigcup_{n=-\infty}^{+\infty} P_n\}$. We may consider X as a collection of points in the plane lying on horizontal lines with integer ordinates together with the point w. We now construct a neighborhood system for the points of X.

(a) If n is even, then put a maximal perfect topology σ_n on P_n exactly as described in § 2.

(b) Let $n \neq 1$ be an odd integer and let $p = (x, n) \in P_n$. Then, for each positive integer m, let

$$U_m(p) = \left\{(y, n+1) \in P_{n+1} \,\middle|\, |x - y| < \frac{1}{m}\right\} \cup \left\{(y, n-1) \in P_{n-1} \,\middle|\, |x - y| < \frac{1}{m}\right\} \cup \{p\} .$$

(c) Let $n = 1$ and $p = (x, n) \in P_n$. If $x \notin (\pi/2, \pi)$, then, for each positive integer m, define $U_m(p)$ as in (b). If $\pi/2 < x < \pi$, then, for each positive integer m, let $U_m(p) = \{(y, n-1) \in P_{n-1} \mid |x - y| < 1/m\} \cup \{p\}$.

(d) If $p = w$, then, for each positive integer m, let $U_m(p) = \{p\} \cup \{(x, n) \mid |n| \geqq 2m\}$ be a neighborhood of p.

It is clear that the neighborhood system described above generates a topology σ on X. X is countable and it is easily seen that (X, σ) is a Hausdorff space. We state the following lemmas (a slight modification of Lemmas 1, 2, and 3 of [3]) without proof. However, it is not difficult to verify that these lemmas also hold for the space (X, σ).

LEMMA 3. *Suppose $p = (x, n)$, where n is an even integer. Let U be a basic open set containing p and let m be an integer.*

(i) *If $p \notin \{(x, n) \mid n = 2$ and $\pi/2 < x < \pi\}$, then $(\mathrm{cl}_\sigma U) \cap P_m \neq \phi$, if and only if $|m - n| \leqq 1$.*

(ii) *If $p = (x, 2)$, where $x \in (\pi/2, \pi)$, then $(\mathrm{cl}_\sigma U) \cap P_m \neq \phi$, if and only if $m = 2$ or 3.*

(iii) *$w \notin \mathrm{cl}_\sigma U$.*

LEMMA 4. *Suppose $p = (x, n)$, where n is an odd integer. Let U be a basic open set containing p and let m be an integer.*

(i) *If $p \notin \{(x, n) \mid n = 1$ or 3 and $\pi/2 < x < \pi\}$, then $(\mathrm{cl}_\sigma U) \cap P_m \neq \phi$, if and only if, $|m - n| \leqq 2$.*

(ii) *If $p = (x, 1)$, where $x \in (\pi/2, \pi)$, then $(\mathrm{cl}_\sigma U) \cap P_m \neq \phi$, if and only if, $m = 1$, 0 or -1.*

(iii) *If $p = (x, 3)$, where $x \in (\pi/2, \pi)$, then $(\mathrm{cl}_\sigma U) \cap P_m \neq \phi$ if and only if, $m = 2$, 3, 4 or 5.*

(iv) $w \notin \mathrm{cl}_\sigma U$.

LEMMA 5. *Suppose $p = w$, m is a positive integer greater than 1, and n is an integer. Then*

(i) $\mathrm{cl}_\sigma U_m(p) \supset P_n$, *if* $|n| \geqq 2m - 1$; *and* (ii) $\{\mathrm{cl}_\sigma U_m(p)\} \cap P_n = \phi$, *if* $|n| < 2m - 1$.

LEMMA 6. *(X, σ) is connected.*

The proof of Lemma 6 is omitted since it is essentially the same as that given in [3].

Let γ be any topology on X such that γ is larger than or equal to σ and (X, γ) is connected. We will now show that there exists a topology γ' on X such that γ' is strictly larger than γ and (X, γ') is connected.

Let I be any open interval in R and, for each integer n, let $I_n = \{(x, n) \in P_n \mid x \in I\}$. For each odd integer n, let $U(I_n) = \{x \in I \mid (x, n) \in I_n, (x, n) \in \mathrm{cl}_\gamma P_{n+1}$ and $(x, n) \in \mathrm{cl}_\gamma P_{n-1}\}$. Throughout, I will denote an open interval of the real line with the usual topology. For each integer n, I_n is a subset of X and $U(I_n)$ is a subset of I for each odd integer n.

LEMMA 7. *If I is any open interval contained in $(\pi/2, \pi)$, then $U(I_n)$ is dense in I for all odd integers n, except for $n = 1$.*

Proof. It follows from condition (c) in the definition of the neighborhood base for the topology σ on X, that for every open interval $I \subset (\pi/2, \pi)$, $U(I_1) = \phi$. Suppose there exists some interval $I \subset (\pi/2, \pi)$ and an odd integer $n \neq 1$ such that $U(I_n)$ is not dense in I. We may assume without loss of generality that $I = (a, b)$, where a and b are irrationals, $U(I_n) = \phi$ and $n > 1$. Put $H_n = \{p \in I_n \mid p \notin \mathrm{cl}_\gamma P_{n+1}\}$. Let $H = H_n \cup \{\bigcup_{k=2}^{n-1} I_k\}$. We will show that H is both γ-open and γ-closed. Suppose $p \in H_n$, then $p \notin \mathrm{cl}_\gamma P_{n+1}$, so there exists a γ-open neighborhood, V, of p which consists of p and a subset of I_{n-1}. Hence $V \subset H$. Clearly, if $p \in I_k$, $k = 2, \cdots, n - 1$, there exists a σ-open set containing p and contained in $\bigcup_{k=2}^{n-1} I_k$. Therefore, H is γ-open.

If $p \in I_{n+1}$, then p is not a σ-limit point of H and, since $\sigma \subset \tau$, it follows that $p \notin \mathrm{cl}_\gamma H$. If $p \in I_n$ and $p \notin H$, then p is a γ-limit point of I_{n+1} and, since $U(I_n) = \phi$, p must have a γ-open neighborhood which consists of p and a subset of I_{n+1}. Hence $p \notin \mathrm{cl}_\gamma H$. It follows from the construction of σ, that if $p \in P_1$ or $p \in P_0$, then $p \notin \mathrm{cl}_\gamma H$. Also, if $p \in P_k$, for $k = 2, \cdots, n - 1$, and $p \notin H$, there exists a σ-open set containing p which does not meet H. Therefore, H is γ-closed.

This contradicts the assumption that (X, γ) is connected and the lemma is established.

We now make use of the preceding lemma to put a topology γ' on X such that γ' is strictly larger than γ. Let J be a fixed open interval contained in $(\pi/2, \pi)$. It follows from Lemma 7 that $U(J_n)$ is dense in J for all odd integers, except for $n = 1$. Let y_0 be a fixed element of $U(J_3)$ and put $y = (y_0, 3)$. Let V be an open neighborhood of y in γ. It follows that $V \cap P_4$ and $V \cap P_2$ are both nonempty. Put $G = \{y\} \cup \{V \cap P_2\}$. Clearly $G \notin \gamma$. Put $\gamma' = \gamma \vee G$. We will show (Theorem 1) that (X, γ') is connected. Throughout the remainder of this note, G, γ', J, and y will be reserved for the roles assigned to them in this paragraph.

LEMMA 8. *If n is an integer and $V \subset P_{2n}$, where $V \in \gamma'$, then there exists some interval $I \subset R$ such that $\{x \mid (x, 2n) \in V\}$ is dense in I.*

Proof. Suppose not. Then there exists some integer n and a γ'-open set $V \subset P_{2n}$ such that $\{x \mid (x, 2n) \in V\}$ is not dense in any subinterval of R. Since $P_{2n} \in \sigma$ and $(P_{2n}, \sigma \mid P_{2n})$ is maximal perfect, it follows that $V \in \sigma$ [otherwise, some point of V would be an isolated point of γ']. Put $F = \{(x, 2n) \in P_{2n} \mid (x, 2n) \notin V\}$. Since $\{x \mid (x, 2n) \in V\}$ is not dense in any subinterval of R, it follows from the definition of the maximal filter $\mathscr{F}$ on P_{2n} that $F \in \mathscr{F}$. If $p \in V$, then $F \cup \{p\} \in \mathscr{F}$ and $(F \cup \{p\}) \in \sigma$. Therefore, $(F \cup \{p\}) \cap V = \{p\}$ and $\{p\} \in \sigma$. This contradiction establishes the lemma.

LEMMA 9. *Suppose n is an integer and V and U are open subsets of P_{2n}. If $\{x \mid (x, 2n) \in V\}$ and $\{x \mid (x, 2n) \in U\}$ are both dense in some open interval I, then $V \cap U \neq \phi$.*

Proof. Suppose $V \cap U = \phi$. Since $\{x \mid (x, 2n) \in V\}$ and $\{x \mid (x, 2n) \in U\}$ are both dense in I, we may assume that $V = I_{2n} \cap F_1$ and $U = I_{2n} \cap F_2$, where $F_1, F_2 \in \mathscr{F}$ and $\mathscr{F}$ is the maximal filter of dense sets used to construct the neighborhood system of P_{2n}. If $p \in V$, then $F_2 \cup \{p\} \in \mathscr{F}$. Hence, $U \cup \{p\} = (F_2 \cap \{p\}) \cap I_{2n}$ is a σ-open subset of P_{2n}. However, $(U \cup \{p\}) \cap V = \{p\}$ and $\{p\} \in \sigma$. This contradiction establishes the lemma.

LEMMA 10. *Let U be a γ'-open neighborhood of w. If I is any open interval contained in J, then there exists an integer $n \geq 2$ such that $I_n \cap U \neq \phi$.*

Proof. Suppose for some γ'-open set U containing w, there exists

an interval $I \subset J$, $I = (a, b)$, where a and b are irrational numbers, such that $I_n \cap U = \phi$, for each integer $n \geqq 2$. Since every γ'-open set containing w contains a γ-open neighborhood of w, we may assume that $U \in \gamma$. Put $H = \bigcup_{n=2}^{+\infty} I_n$. It follows that H is γ-open. H is also γ-closed. For, if $p \in I_1$, p has a σ-open neighborhood which does not meet H, and by assumption, w has a γ-open neighborhood which does not meet H. This contradicts the assumption that (X, γ) is connected and the lemma follows.

COROLLARY. *Let $I \subset J$. If U is any γ'-open set containing w, then there exists a infinite set of positive integers, N, such that $I_n \cap U \neq \phi$, for each $n \in N$.*

LEMMA 11. *Let n be an integer such that $n \geqq 1$. Let V be an open subset of P_{2n} such that $\{x \mid (x, 2n) \in V\}$ is dense in an open interval $I \subset J$. If $V \subset K$, where K is an open and closed set in (X, γ'), then $I_k \subset K$, for all $k > 2n$.*

Proof. First we will show that I_{2n+1} and I_{2n+2} are both contained in K. $U(I_{2n+1}) = \{x \in I \mid p = (x, 2n + 1) \in I_{2n+1}$ and p is a γ-limit point of both I_{2n} and $I_{2n+2}\}$. It follows from Lemma 7 and the choice of J, that, since $I \subset J$, $U(I_{2n+1})$ is dense in I. Let $p = (x, 2n + 1)$, where $x \in U(I_{2n+1})$. Let U be a γ'-open set containing p (we may, without loss of generality, assume that $U \in \gamma$). Since $U \cap I_{2n} \neq \phi$, it follows from Lemma 8 that there exists some open interval $I' \subset I$ such that $\{x \mid (x, 2n) \in U\}$ is dense in I'. Also, $\{x \mid (x, 2n) \in V\}$ is dense in I'. Therefore, by Lemma 9, $V \cap U \neq \phi$. Since $V \subset K$, $U \cap K \neq \phi$ and p is therefore a limit point of K. Since K is γ'-closed, $p \in K$. Therefore, $\{p \mid p = (x, 2n + 1)$ and $x \in U(I_{2n+1})\} \subset K$.

Put $Q = \{x \in I \mid q = (x, 2n + 2) \in I_{2n+2}$ and $q \in K\}$. Since $U(I_{2n+1})$ is dense in I, Q is also dense in I. Let $z \in I_{2n+2}$. It will be shown that z is a γ'-limit point of K. Let H be a γ'-open set containing z. We may assume that $H \subset P_{2n+2}$. Again, by Lemma 8, there exists an interval $I' \subset I$ such that $\{x \mid (x, 2n + 2) \in H\}$ is dense in I'. Since Q is dense in I, $K \cap I'_{2n+2} \neq \phi$. This implies that $K \cap I'_{2n+2} \in \gamma'$ and, by Lemma 8, we may assume without loss of generality, that $K \cap I'_{2n+2}$ is dense in I'_{2n+2}. By Lemma 9, $(K \cap I'_{2n+2}) \cap H \neq \phi$. Therefore, z is a γ'-limit point of K and since K is γ'-closed, $z \in K$. Hence, $I_{2n+2} \subset K$.

Now, suppose there exists some $p \in I_{2n+1}$ such that $p \notin K$. Then there exists a γ'-open set L containing p such that $L \cap I_{2n+2} = \phi$ and $L \cap V = \phi$. Using Lemmas 8 and 9, this can be shown to be impossible. Hence $I_{2n+1} \subset K$. It can now be shown by induction that, for all $k > 2n$, $I_k \subset K$.

THEOREM 1. *(X, γ') is connected.*

Proof. Let y be that element of J_3, where $J \subset (\pi/2, \pi)$, such that $\gamma' = \gamma \vee G$, $y \in G$ and $G \notin \gamma$. Suppose there exists a set $K \subset X$ such that K is both open and closed in (X, γ'). Assume $y \notin K$. This implies that $K \in \gamma$. Since (X, γ) is connected, K is not closed in (X, γ), Therefore, if U is a γ-open neighborhood of y, $U \cap K \neq \phi$. Hence, $U \cap (P_2 \cap K) \neq \phi$ or $U \cap (P_4 \cap K) \neq \phi$. Since $G = \{y\} \cup (V \cap P_2)$, where V is a fixed γ-open neighborhood of y, and y is not a γ'-limit point of K, it follows that $U \cap (P_4 \cap K) \neq \phi$. By Lemma 8, there exists a γ'-open set $U_4 \subset U \cap (P_4 \cap K)$ such that $\{x \mid (x, 4) \in U_4\}$ is dense in an open interval $I \subset J$. Lemma 11 implies, that, for all $k > 4$, $I_k \subset K$. Then, by the corollary following Lemma 10, $w \in K$.

Since $y \notin K$, $y \in K^c$ and K^c is also open and closed in (X, γ'). Let H be a γ'-open neighborhood of y such that $H \subset K^c$. Then $H \cap P_2 \neq \phi$. This implies, by Lemma 8, that there exists a γ'-open set $H_2 \subset H \cap P_2 \cap K^c$ such that $\{x \mid (x, 2) \in H_2\}$ is dense in some open interval $I' \subset J$. Again, it follows from Lemmas 10 and 11 that $w \in K^c$. This contradicts the fact that $K \cap K^c = \phi$. Therefore, (X, γ') is connected.

It is now and immediate consequence of the following theorem (X, σ) cannot be embedded into any maximal connected space.

THEOREM 2. (See [2]). *Every connected subspace of a maximal connected space is maximal connected.*

REMARK 2. If for each n, E_n is an uncountable dense subset of the irrationals, it follows that one can then construct an uncountable connected Hausdorff space which cannot be embedded into any maximal connected space.

Question. What connected Hausdorff spaces have the property that they cannot be embedded into a maximal connected space?

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