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NORM DECREASING HOMOMORPHISMS BETWEEN GROUP ALGEBRAS

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The norm decreasing homomorphisms φ of $L^1(F)$ into $M(G)$ for locally compact groups F and G have been characterized by F. P. Greenleaf using an integral representation. In this note the authors improve and unify some of the results and proofs of structure theorems in the previous literature. Necessary and sufficient conditions that φ have a canonical factorization of a general type are expressed in terms of the extensibility of a φ-associated character on a φ-related closed normal subgroup. In particular, an explicit factorization of φ can be obtained when either F or G is Abelian. Also investigated is the structure of norm decreasing homomorphisms φ with range in $L^1(G)$.

With F and G denoting (throughout this note) locally compact Hausdorff groups, the norm decreasing homomorphisms of the group algebra $L^1(F)$ into the measure algebra $M(G)$ have been characterized by Glicksberg [2] and Cohen [1] for Abelian groups and in the general setting by Greenleaf [3]. The characterization obtained for nonAbelian groups is less tractable than that obtained in the Abelian case. (Compare Theorem 2.1 of [2] with Theorem 4.2.2 of [3].) In this note the authors give necessary and sufficient conditions for a nonzero norm decreasing homomorphism $\varphi: L^1(F) \to M(G)$ to have certain factorizations analogous to those obtained by Glicksberg [2, Theorems 2.1, 2.9] and Greenleaf [3, Theorem 5.1.5]. Furthermore, those φ with range in $L^1(G)$ are investigated, and simpler proofs of Greenleaf's characterization of the epimorphisms and monomorphisms between L^1-group algebras (cf., [3, Theorem 5.2.1, Cor. 5.1.6], [4, Theorem 2.1]) are provided.

In the interest of brevity we adopt the notations and definitions of [3]. In addition, if X and Y are locally compact Hausdorff spaces and $\theta: X \to Y$ is continuous, then $\theta_*: M(X) \to M(Y)$ denotes the canonical norm decreasing linear map defined by $\langle \theta_*(\mu), f \rangle = \langle \mu, f \circ \theta \rangle$ for all f in $C_0(Y)$, or equivalently, by $\theta_*(\mu)(B) = \mu(\theta^{-1}(B))$ for all Borel subsets, B, of Y; if $\theta: F \to G$ is a continuous homomorphism then θ_* is also multiplicative. If K is a compact subgroup or closed normal subgroup of G, then $m_{G/K}$ denotes the positive left invariant measure on G/K so that m_G, m_K, and $m_{G/K}$ are canonically related [6, Chaps. 3, 8]. $L^1(G/K)$ denotes the usual Lebesgue space with respect to $m_{G/K}$. If K is compact, then $\pi_K^*: M(G/K) \to M(G)$ is the norm decreasing linear map defined by

$\langle \pi_K^*(\mu), f\rangle = \langle \mu, (\pi_K)_*(f)\rangle$ for all f in $\mathscr{K}(G)$, the space of continuous, C-valued, compactly supported, functions on G, where under the identification $L^1(G) \subset M(G)$, $(\pi_K)_* f(gK) = \int_K f(gk)dm_K(k)$ for $g \in G$. Let $\hat{G}$ denote the group of continuous homomorphisms of G into the circle group S^1. If α is in $\hat{G}$, then A_α denotes the isometric automorphism of $M(G)$ defined by $\langle A_\alpha(\mu), f\rangle = \langle \alpha\mu, f\rangle = \langle \mu, \alpha f\rangle$ for all f in $C_0(G)$.

Let $\bar{\varphi}: (M(F), (so)) \to (M(G), (\sigma))$ denote the unique norm decreasing extension of φ; let $H_0 = \bigcup \{\text{support}\,\bar{\varphi}(\delta_x): x \in F\}$, a subgroup of G; then $\bar{\varphi}(\delta_e) = \rho m_K$ where K is a compact subgroup of G normal in H_0 and ρ is in $\hat{K}$[3, § 4.2]. Let $\zeta: F \to H_0/K$ be the mapping defined by $\zeta(x) = \pi_K(\text{supp}\,\bar{\varphi}(\delta_x))$. The continuity properties of $\bar{\varphi}$ show that ζ is an epimorphism of F onto H_0/K and is continuous as a map of F into G/K. According to [3, Lemma 5.1.2] there is a unique topology τ on H_0 making $H = (H_0, \tau)$ into a locally compact group, K a compact subgroup of H, the monomorphic inclusion $j: H \to G$ continuous, and the algebraic epimorphism ζ a continuous open epimorphism $\theta: F \to H/K$. (Algebraically, ζ and θ are the same map, but their topological properties differ.) Finally, recall ([3, § 4.2]) that $\bar{\varphi}(\delta_x) = \lambda_{x,g}\delta_g * \rho m_K$, where $\pi_K(g) = \zeta(x)$ for g in H_0 and $|\lambda_{x,g}| = 1$.

Lemma 1. *Let $F_\varphi = \{x \in F: \bar{\varphi}(\delta_x) = \lambda_x \rho m_K$ for some $|\lambda_x| = 1\}$, and let $\gamma_\varphi: F_\varphi \to S^1$ be defined by $\gamma_\varphi(x) = \lambda_x$. Then F_φ is a closed normal subgroup with* Ker $\zeta = F_\varphi$ *and γ_φ is in $\hat{F}_\varphi$.*

Proof. $x \in F_\varphi$ iff $\bar{\varphi}(\delta_x) = \lambda_x \delta_e * \rho m_K = \lambda_{x,e}\delta_e * \rho m_K$ iff $\lambda_x = \lambda_{x,e}$ and $\zeta(x) = \pi_K(e) = e/K$. Thus, $F_\varphi = \text{Ker}\ \zeta$ and, therefore, it is a closed normal subgroup. As φ is a homomorphism and is ((so), (σ))-continuous on norm bounded sets, it is easy to see that γ_φ is in $\hat{F}_\varphi$.

We now state the main results reserving their proofs till later.

Theorem 1. (i) *γ_φ has an extension to a character γ in $\hat{F}$ iff $\varphi = j_* A_\beta \pi_K^* \theta_* A_\gamma$ where K is a compact normal subgroup of a locally compact group H, $\theta: F \to H/K$ is a continuous open epimorphism, $j: H \to G$ is a continuous monomorphism, and $\gamma \in \hat{F}$, $\beta \in \hat{H}$.*

(ii) *ρ has an extension to a character α in $\hat{G}$ iff $\varphi = A_\alpha \pi_K^* \zeta_* A_\gamma$ where K is a compact subgroup normal in a subgroup H_0 of G, $\zeta: F \to H_0/K \subset G/K$ is a continuous epimorphism, and $\gamma \in \hat{F}$, $\alpha \in \hat{G}$.*

Theorem 2. *Let H and G be locally compact groups, let $K \subset H$ and $J \subset G$ be compact subgroups, let L be a subgroup of G with J*

normal in L, *let* $\psi: H \to L/J \subset G/J$ *be a continuous epimorphism with the relative topology on* L/J, *and let* $\beta \in \hat{H}$ *with* $\operatorname{Ker} \beta \supset \operatorname{Ker} \psi$. *Then the following are equivalent*:

(i) L *is an open/closed subgroup of* G *and* ψ *is an open map*;

(ii) $\psi_*(L^1(H)) \subset L^1(G/J)$;

(iii) $\psi_* A_\beta \pi_K^*(L^1(H/K)) \subset L^1(G/J)$;

(iv) ψ^{-1} *satisfies property* N_{loc}, i.e., *the* ψ*-inverse image of an* $m_{G/J}$*-locally null set in* G/J *is* m_H*-locally null in* H; *and,*

(v) *if* H *and* G *are Abelian, then the dual homomorphism* $\hat{\psi}: (G/J)^\wedge \to \hat{H}$ *is a proper map*, i.e., *the* $\hat{\psi}$*-inverse image of a compact subset is compact.*

COROLLARY 1. (i) $\gamma_\varphi \equiv 1$ *iff* $\varphi = j_* A_\beta \pi_K^* \theta_*$.

(ii) $\bar{\varphi}(\delta_e) \geqq 0$ *iff* $\varphi = \pi_K^* \zeta_* A_\gamma$.

(iii) φ *is order preserving iff* $\varphi = \pi_K^* \zeta_*$.

(iv) $\langle \varphi(f), \bar{\alpha} \rangle \neq 0$ *for some* f *in* $L^1(F)$ *and some* α *in* $\hat{G}$ *iff* $\varphi = A_\alpha \pi_K^* \zeta_* A_\gamma$.

Proof. (i), (ii) and (iii) are all obvious from Theorem 1. If $\langle \varphi(f), \alpha \rangle \neq 0$ for some f and α, then the map $f \to \langle \varphi(f), \bar{\alpha} \rangle$ is a nonzero multiplicative linear functional on $L^1(F)$ and hence ([5, (23.7)]) there is a γ in $\hat{F}$ such that $\langle \varphi(f), \bar{\alpha} \rangle = \langle f, \gamma \rangle$ for all f in $L^1(F)$. Let $f \geqq 0$ be in $L^1(F)$ and $||f||_1 = 1$. Then $\langle A_\alpha^{-1} \varphi A_\gamma^{-1}(f), 1 \rangle = \langle \varphi A_\gamma^{-1}(f)\rangle, \bar{\alpha} \rangle = \langle \bar{\gamma} f, \gamma \rangle = ||f||_1 = 1$. As $||A_\alpha^{-1} \varphi A_\gamma^{-1}(f)||_1 \leqq 1$, it must be that $A_\alpha^{-1} \varphi A_\gamma^{-1}(f) \geqq 0$ and hence $A_\alpha^{-1} \varphi A_\gamma^{-1}$ is order preserving. Thus $\varphi = A_\alpha \pi_K^* \zeta_* A_\gamma$. Conversely, if $\varphi = A_\alpha \pi_K^* \zeta_* A_\gamma$, then for all $g \geqq 0$ in $L^1(F)$ and with $f = \bar{\gamma} g$, we have $\langle \varphi(f), \bar{\alpha} \rangle = \langle \pi_K^* \zeta_*(g), 1 \rangle$ and this is clearly nonzero for some $g \geqq 0$.

COROLLARY 2. (i) *If* F *is Abelian, then every nonzero* $\varphi: L^1(F) \to M(G)$ *is of the form* $\varphi = j_* A_\beta \pi_K^* \theta_* A_\gamma$.

(ii) *If* G *is Abelian, then every nonzero* $\varphi: L^1(F) \to M(G)$ *is of the form* $\varphi = A_\alpha \pi_K^* \zeta_* A_\gamma$.

Proof. This follows immediately from Theorem 1 since it is well known that characters on closed subgroups of Abelian groups extend to the whole group.

COROLLARY 3. (i) $\varphi = j_* A_\beta \pi_K^* \theta_* A_\gamma$ *maps* $L^1(F)$ *into* $L^1(G)$ *iff* $j(H)$ *is open/closed in* G *and* $j: H \to G$ *is an open map.*

(ii) $\varphi = A_\alpha \pi_K^* \zeta_* A_\gamma$ *maps* $L^1(F)$ *into* $L^1(G)$ *iff* H_0 *is an open/closed subgroup of* G *and* $\zeta: F \to G/K$ *is an open mapping.*

Proof. To prove (i), note that as $\theta_* A_\gamma(L^1(F)) = L^1(H/K)$, we have $j_* A_\beta \pi_K^*(L^1(H/K)) \subset L^1(G)$ if $\varphi(L^1(F)) \subset L^1(G)$ and so Theorem 2 applies with $\psi = j$, $L = H_0$, and $J = (e)$; the converse of (i) is well known. If $\varphi = A_\alpha \pi_K^* \zeta_* A_\gamma$ maps $L^1(F)$ into $L^1(G)$, then $\pi_K^* \zeta_*(L^1(F)) \subset L^1(G)$. Since $(\pi_K)_* \circ \pi_K^*$ is the identity map, we have $\zeta_*(L^1(F)) \subset (\pi_K)_*(L^1(G)) \subset L^1(G/K)$. Now, Theorem 2 applies with $H = F$, $J = K$, $L = H_0$, and $\psi = \zeta$; again, the converse of (ii) is well known.

An immediate corollary to Corollary 3 (i) is

COROLLARY 4 (Greenleaf [4, Theorem 2.1]). *φ is a monomorphism of $L^1(F)$ into $L^1(G)$ iff $\varphi = j_* A_\beta \pi_K^* \theta_*$ where $\theta: F \cong H/K$ and j is a topological isomorphism of H onto an open/closed subgroup of G.*

COROLLARY 5 (Greenleaf [3, Theorem 5.2.1]). *φ is an epimorphism of $L^1(F)$ onto $L^1(G)$ iff $\varphi = \Lambda T_{F_0} A_\gamma$ where F_0 is a closed normal subgroup of F, $T_{F_0} = (\pi_{F_0})_*|_{L^1(F)}$, $\gamma \in \hat{F}$, and $\Lambda: L^1(F/F_0) \cong L^1(G)$ is an isometric isomorphism.*

Proof. If φ is an epimorphism, then $\langle \varphi(f), 1 \rangle \neq 0$ for some f in $L^1(F)$ and so $\varphi = \pi_K^* \zeta_* A_\gamma$ by Corollary 1 (iv). By Corollary 3 (ii), ζ is an open map and H_0 is an open/closed subgroup of G. Since elements in $\pi_K^*(L^1(G/K))$ are constant on the cosets of K in G and φ is an epimorphism, we must have $K = \{e\}$ and $\varphi = \zeta_* A_\gamma$. Since ζ maps onto the open/closed subgroup H_0, it follows immediately from the definition of ζ_* that $\zeta_*(L^1(F))$ is supported on H_0 in G. However, $\zeta_*(L^1(F)) = \zeta_*(A_\gamma(L^1(F))) = \varphi(L^1(F)) = L^1(G)$ and hence $H_0 = G$ and $\zeta: F \to G$ is a continuous open epimorphism. Let $\zeta = \lambda \circ \pi_{F_0}$ where $F_0 = \operatorname{Ker} \zeta$ and $\lambda: F/F_0 \cong G$. Then $\varphi = \zeta_* A_\gamma = \lambda_* T_{F_0} A_\gamma$ where $\lambda_*: L^1(F/F_0) \cong L^1(G)$.

The next corollary is an interesting parallel of Corollary 1.2 and Theorem 2.3 of [4].

COROLLARY 6. (i) (Greenleaf) *$L^1(F)$ has a nonzero norm decreasing homomorphic image in $M(G)$ iff $F/F_0 \cong H/K$ where F_0 is closed normal subgroup of F, K is a compact normal subgroup of a locally compact group H, and H is continuously isomorphic to a* (not necessarily closed) *subgroup of G.*

(ii) *$L^1(F)$ has a nonzero norm decreasing homomorphic image in $L^1(G)$ of either of the types described in Theorem 1 iff $F/F_0 \cong H/K$ where F_0 is a closed normal subgroup of F and K is a compact normal subgroup of a locally compact group H which is topologically isomorphic to an open/closed subgroup of G.*

Proof. Part (i) is immediate from the construction of $H = (H_0, \tau)$

and $\theta: F \to H/K$. Part (ii) follows from Corollary 3.

It is clear that if $\varphi = A_\alpha \pi_K^* \zeta_* A_\gamma$, then $\varphi = j_* A_\beta \pi_K^* \theta_* A_\gamma$ where $\beta = \alpha \circ j$, and so $j_* A_\beta \pi_K^* \theta_* A_\gamma$ is the more general factorization of the two types. There naturally arises the question of existence of a φ not of this general form. In light of Theorem 1 and its corollaries, an example has been difficult to find and has eluded the authors. The supporting evidence is favorable in view of the well known fact that in general characters on closed subgroups need not extend to the whole group.

Proof of Theorem 1. Part (i). If φ has the indicated factorization, then $\bar{\varphi} = j_* A_\beta \pi_K^* \theta_* A_\gamma$ and

$$\bar{\varphi}(\delta_x) = \beta(g)\gamma(x)\delta_{j(g)} * (\beta \circ j^{-1})m_{j(K)} ,$$

where $\pi_K(g) = \theta(x)$ for x in F and g in H. It follows that $i = (\beta \circ j^{-1})m_{j(K)}$ is the unit of $\Gamma = \bar{\varphi}(\{\delta_x : x \in F\})$, and $\bar{\varphi}(\delta_x) = \lambda_x i$ for $|\lambda_x| = 1$ iff $\bar{\varphi}(\delta_x) = \gamma(x)i$. Thus, $\gamma(x) = \gamma_\varphi(x)$ on F_φ. Conversely, suppose γ_φ has an extension to a character γ in $\hat{F}$. Let $\psi = \varphi \circ A_\gamma^{-1}$, a norm decreasing homomorphism of $L^1(F)$ into $M(G)$ with $\bar{\psi} = \bar{\varphi} A_\gamma^{-1}$. Since $\bar{\psi}(\delta_x) = \overline{\gamma(x)}\bar{\varphi}(\delta_x)$ for all x in F, we have that $\bigcup \{\operatorname{supp} \bar{\psi}(\delta_x) : x \in F\} = H_0$, $\bar{\psi}(\delta_e) = \bar{\varphi}(\delta_e) = \rho m_K = i$, and $\bar{\varphi}$ and $\bar{\psi}$ determine the same $\zeta: F \to H_0/K \subset G/K$ and $\theta: F \to H/K$. Furthermore, if $\bar{\psi}(\delta_x) = \lambda i$ for some $|\lambda| = 1$, then $\gamma(x)\lambda i = \gamma(x)\bar{\psi}(\delta_x) = \bar{\varphi}(\delta_x)$. Therefore, $x \in F_\varphi$ and $\gamma_\varphi(x) = \gamma(x)\lambda$. Since γ_φ is the restriction of γ to F_φ, $\lambda = 1$. Thus, $\bar{\psi}(\{\delta_x : x \in F\}) \cap S^1 i = \{i\}$ and by Theorem 5.1.5 of [3], $\psi = j_* A_\beta \pi_K^* \theta_*$; therefore, $\varphi = \psi A_\gamma = j_* A_\beta \pi_K^* \theta_* A_\gamma$.

Part (ii). If ρ has an extension to a character α in $\hat{G}$, consider $\psi = A_\alpha^{-1}\varphi$, a norm decreasing homomorphism of $L^1(F)$ into $M(G)$ with $\bar{\psi} = A_\alpha^{-1}\bar{\varphi}$. Since α extends ρ we have for each x in F, $\bar{\psi}(\delta_x) = A_\alpha^{-1}(\lambda_{x,g}\delta_g * \rho m_K) = (\lambda_{x,g}\overline{\alpha(g)})\delta_g * m_K$. Thus, $\bar{\varphi}$ and $\bar{\psi}$ determine the same compact K, subgroup H_0 in G, and maps ζ and θ. Let $\eta_{x,g} = \lambda_{x,g}\overline{\alpha(g)}$ so that $\bar{\psi}(\delta_x) = \eta_{x,g}\delta_g * m_K$ where $\pi_K(g) = \zeta(x)$. Since $\delta_g * m_K$ does not depend on the representative g in the coset gK, $\gamma(x) = \eta_{x,g}$ is a well defined function on F to S^1. It follows easily from the continuity properties of $\bar{\psi}$ that $\gamma \in \hat{F}$; moreover, γ extends γ_ψ. Now, the map $\pi_K^* \zeta_* A_\gamma$ agrees with $\bar{\psi}$ on $\{\lambda\delta_x : x \in F, |\lambda| = 1\} = S^1\xi_F$. As $\pi_K^* \zeta_* A_\gamma$ and $\bar{\psi}$ are ((so), (σ))-continuous on norm bounded sets and as $\operatorname{co}[S^1\xi_F : (\mathrm{so})]$ is the unit ball in $M(F)$ ([3, Lemma 1.1.3]), we have $\pi_K^* \zeta_* A_\gamma = \bar{\psi}$ and $\varphi = A_\alpha \pi_K^* \zeta_* A_\gamma$. The converse of (ii) is easily seen from the relation $A_\alpha \pi_K^* \zeta_* A_\gamma(\delta_e) = A_\alpha(m_K) = (\alpha|_K)m_K$.

Proof of Theorem 2. As the adjoint map of the homomorphism ψ_* is $\hat{\psi}: (G/J)^\wedge \to \hat{H}$ when H and G are Abelian, the equivalence of

(ii) and (v) is due to P. Cohen [1, Theorem 1]. As it is well known that (i) implies (ii), and clearly (ii) implies (iii), it is necessary only to prove (iii) implies (iv) and (iv) implies (i).

Assume (iii). We first show $\psi_*\pi_K^*(L^1(H/K)) \subset L^1(G/J)$. Let f be in $\mathscr{K}(H/K)$, and let $C = \pi_K^{-1}(\operatorname{supp} f)$, a compact subset of H. Let $\beta^\sim$ denote the bounded Borel function on G/J defined by $\beta^\sim = 0$ outside of $\psi(C)$ and $\beta^\sim(x) = \overline{\beta(y)}$ for $x \in \psi(C)$ and $\psi(y) = x$, $y \in H$. Since $\operatorname{Ker}\psi \subset \operatorname{Ker}\beta$, $\beta^\sim$ is well defined on the compact subset $\psi(C)$ and it is continuous on $\psi(C)$. Let $A_{\beta^\sim}: L^1(G/J) \to L^1(G/J)$ denote the bounded linear map of pointwise multiplication by $\beta^\sim$. Now, since $\beta^\sim \circ \psi = \bar{\beta}$ on C, $\pi_K^*(f) = f \circ \pi_K$ is supported on C, and $\psi_* A_\beta \pi_K(f) \in L^1(G/J)$, we have $\langle A_{\beta^\sim}\psi_* A_\beta \pi_K^*(f), g\rangle = \langle \pi_K^*(f), \beta(\beta^\sim \circ \psi)(g \circ \psi)\rangle = \langle \pi_K^*(f), g \circ \psi\rangle = \langle \psi_*\pi_K^*(f), g\rangle$ for all g in $C_0(G/J)$. Therefore, $A_{\beta^\sim}\psi_* A_\beta \pi_K^*(f) = \psi_*\pi_K^*(f)$ and it is in $L^1(G/J)$. Since f in $\mathscr{K}(H/K)$ is arbitrary and $\psi_*\pi_K^*$ is continuous, we have that $\psi_*\pi_K^*$ maps $L^1(H/K)$ into $L^1(G/J)$.

To prove (iv), first consider any Borel $m_{G/J}$-null set B in G/J, and let C be any compact subset of H with $C \cdot K = C$. Now, as $\chi_C = \pi_K^*(\chi_{\pi_K(C)})$, as $\psi_*\pi_K^*(\chi_{\pi_K(C)})$ is in $L^1(G/J)$, and as B is a null set, we have $m_H(C \cap \psi^{-1}(B)) = \chi_C dm_H(\psi^{-1}(B)) = \psi_*\pi_K^*(\chi_{\pi_K(C)})(B) = 0$. Thus, $C \cap \psi^{-1}(B)$ is an m_H-null set. Since every compact subset of H is contained in a compact set C where $C \cdot K = C$, it follows that $\psi^{-1}(B)$ is m_H-locally null and hence $\psi^{-1}(M)$ is m_H-locally null for $\underline{\text{any}}$ $m_{G/J}$-null set M in G/J. Finally, let N be any $m_{G/J}$-locally null set in G/J and let C be any compact subset of H. Since $C \cap \psi^{-1}(\psi(C) \cap N) = C \cap \psi^{-1}(N)$, and since $\psi(C) \cap N$ is $m_{G/J}$-null, we have that $C \cap \psi^{-1}(N)$ is m_H-null. Thus, $\psi^{-1}(N)$ is m_H-locally null and (iv) holds.

Assume (iv). As every locally compact group is the union of σ-compact open subgroups, it suffices (in order to prove (i)) to show $\psi(S)$ is open in G/J and $\psi|_S$ is an open map for any σ-compact open subgroup S in H. A theorem of Pontryagin (cf., [5, (5.29)]) shows that any continuous epimorphism between σ-compact locally compact groups is an open map. Therefore, it suffices to show $\psi(S)$ is open in G/J and hence a (σ-compact) locally compact group with the relative topology. It is clear that $\psi(S)$ is at least σ-compact and therefore measurable. Since the restriction of the Haar measure m_H to the open subgroup S is the Haar measure on S, it is clear that $(\psi|_S)^{-1}$ satisfies property N_{loc}. Thus $\psi(S)$ is not $m_{G/J}$-locally null. Let $L_S = \pi_J^{-1}(\psi(S))$, an F_σ-subgroup of G. By [6, § 3.9, p. 66 and § 2.2, p. 165] L_S is m_G-locally null iff $\psi(S)$ is $m_{G/J}$-locally null. Therefore L_S is not m_G-locally null and thereby contains a Borel subset A of positive finite measure. Then AA^{-1}, a subset of L_S, is a neighborhood of the identity of G [5, (20.17)], and so L_S and $\pi_J(L_S) = \psi(S)$ are open in G and G/J, respectively. The proof is complete.

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