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ON CAUCHY'S THEOREM FOR REAL ALGEBRAIC CURVES WITH BOUNDARY

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On a real algebraic curve with a nonempty boundary, one must orient the several boundary components in order to pose the question considered in Cauchy's theorem for analytic differentials. It is proved that the conclusion of Cauchy's theorem is true, in this context, if and only if the orientation in question is induced by an orientation of the interior of the curve.

Let $\mathfrak{Y}$ be a real algebraic curve (i.e., a compact Klein surface [3, 4]), whose boundary ∂Y has r components, where $r > 0$. Let g be the algebraic genus of $\mathfrak{Y}$: i.e., the genus of the field E of meromorphic "functions" on $\mathfrak{Y}$ that are real valued on ∂Y; then g is the first Betti number of Y, the underlying space of $\mathfrak{Y}$, and the Euler characteristic χ of Y is $1 - g$ [4, 2]. Y is—of course—characterised topologically by knowing g, r and whether or not Y is orientable. In [2] the author investigated some sheaves that arise from analytic problems on $\mathfrak{Y}$, whose cohomology groups reflect the orientability of Y; however these sheaves and groups seemed rather remote from analytic function theory on $\mathfrak{Y}$. This paper is an outgrowth of the search for a simple analytic question which could be posed on $\mathfrak{Y}$, whose answer would reflect the orientability of Y. What analytic question on $\mathfrak{Y}$ is, after all, more basic than Cauchy's theorem?

In order to pose the question considered in Cauchy's Theorem on $\mathfrak{Y}$, we must orientate the r-components of ∂Y; there are 2^r ways to do this. If Y is orientable, then two of these 2^r orientations are engendered by the two possible orientations of Y; these will be called *indigenous orientations* of ∂Y. If Y is nonorientable, then ∂Y has no indigenous orientations. Let $\mathscr{O}$ be an orientation of ∂Y.

Next we must have a space of analytic differentials on $\mathfrak{Y}$ to integrate along ∂Y, as orieted by $\mathscr{O}$. A space $\Omega_{\mathfrak{Y}}$ of analytic "differentials" on $\mathfrak{Y}$ was defined in [4] which is the natural generalization of the space of Schottky differentials on a bordered Riemann surface, in that they are real on ∂Y. (The space of meromorphic differentials on $\mathfrak{Y}$ [4, 1.10] is also very natural from the point of view of the algebraic geometry of E.) Even though $\omega \in \Omega_{\mathfrak{Y}}$ is called a "differential" its integral along an oriented Jordan curve, or arc Γ, need not have an invariant meaning! If $\Gamma \subset \partial Y$ then it does, and $\int_\Gamma \omega \in \boldsymbol{R}$. The real part of $\int_\Gamma \omega$ always has an invariant meaning. If Γ is contained in an orientable tubular neighborhood, then $\int_\Gamma \omega$ is always invariantly defined, up to complex conjugation. The real dimension of $\Omega_{\mathfrak{Y}}$ is g [2]. (The reason it is so small is that the condition that ω be real on ∂Y allows it to be extended to the complex double.)

$\lambda_{\mathscr{O}}: \omega \in \Omega_{\mathfrak{Y}} \mapsto \int_{\partial Y, \mathscr{O}} \omega \in \boldsymbol{R}$ is a well defined $\boldsymbol{R}$-linear functional on $\Omega_{\mathfrak{Y}}$ whose image is either of dimension 0 or 1. Accordingly we will say that *the conclusion of Cauchy's theorem holds on* $\mathfrak{Y}$ *for* $\mathscr{O}$, or *does not hold on* $\mathfrak{Y}$ *for* $\mathscr{O}$.

THEOREM 1. *The conclusion of Cauchy's theorem holds on* $\mathfrak{Y}$ *for an orientation* $\mathscr{O}$ *of* $\partial Y (\neq \emptyset)$ *if and only if* $\mathscr{O}$ *is an indigenous orientation of* ∂Y (i.e., one induced by an orientation of Y). *Thus* Y *is orientable* (resp. nonorientable) *if and only if there exists* 2 (resp. 0) *orientations* $\mathscr{O}$ *of* ∂Y *for which the conclusion of Cauchy's theorem holds on* $\mathfrak{Y}$.

Proof. Assume first that $\mathscr{O}$ is an indigenous orientation of ∂Y; then, by definition, $\mathscr{O}$ is induced by an orientation of Y. In the dianalytic structure on Y that gives $\mathfrak{Y}$ we may choose an analytic structure and thus consider the bordered Riemann surface structure $\mathfrak{Y}_1$ on Y that engenders $\mathscr{O}$ on ∂Y [4]. Then $\Omega_{\mathfrak{Y}}$ becomes the space of Schottky differentials on $\mathfrak{Y}_1$. In this context the Cauchy theorem is known to hold. (This can be shown directly by triangulating Y and using the Cauchy theorem in $\boldsymbol{C}$.) *Assume, henceforth that* $\mathscr{O}$ *is nonindigenous.*

It is well known (see e.g., [6]), that a topological model of Y can be built up from a closed unit disc D, by adjoining various strips and handles to it. Adopting a different construction suited to our purposes, first adjoin to D $r-1$ untwisted strips—glued to the boundary of D—to form D' so that $\partial D'$ has r components. Let D' be imbedded in $\boldsymbol{C}$ (in some way), let $\Gamma_1, \cdots, \Gamma_{r-1}$ be the components of $\partial D'$ that bound bounded components of $\boldsymbol{C} - D'$, and let them be positively oriented relative to D' (as oriented by $\boldsymbol{C}$). The Euler characteristic of D', $\chi(D')$, is $2-r$, which is—necessarily—not smaller than $\chi(Y)$. Next choose the largest integer h such that $2-r-2h \geqq \chi(Y)$, in the orientable case or $2-r-2h \geqq \chi(Y)+1$ in the nonorientable. Adjoin h handles to D'—by removing $2h$ open discs from D', whose boundaries do not meet $\partial D'$, and attaching h handles (each on the same side of D'), to some choice of h-pairs of these circular boundaries—to form D_0. If Y is orientable, then Y and D_0 are homeomorphic. In this event let Y and D_0 be identified. If Y is nonorientable, let D_1 be formed by adjoining a half twisted strip to D_0; then $\chi(D_1) = 2 - r - 2h - 1$, which is either $\chi(Y)$ or is $\chi(Y) + 1$. In the first case Y and D_1 are homeomorphic, and are to be identified. In the second let one more half twisted strip be adjoined to D_1 to form D_2, a space that is homeomorphic to Y; then let D_2 and Y be identified. In general, $\chi(Y) = 2 - r - 2h - m$, where $m = 0, 1$, or 2, and Y and D_m are identified. Let Γ_r be $\partial Y - (\Gamma_1 \cup \cdots \cup \Gamma_{r-1})$, and let it be oriented in such a way that Γ_j and Γ_r have opposing orientations on the untwisted strip adjoined to D to form D', which gives rise to Γ_j, for eace $1 \leqq j < r$.

The main analytic technique we will use, that of doubling $\mathfrak{Y}$, goes back—essentially to Schottky and Schwarz—and explicitly, in this context, to Klein. Let $(\mathfrak{X}, \tau, p)$ be the complex double of $\mathfrak{Y}$ [4, 1.6]: i.e., $\mathfrak{X}$ is a compact Riemann surface (without boundary), τ is an anti-analytic involution of $\mathfrak{X}$, and p is an analytic map of $\mathfrak{X}$ onto $\mathfrak{Y}$ (i.e., a morphism [4, 1.4]). $p^{-1}(y)$ has one (resp. two) points in it, for $y \in Y$, if and only if $y \in \partial Y$ (resp. $y \in Y - \partial Y$). For $1 \leqq j \leqq r$, let Δ_j be the pullback of Γ_j to X, endowed with the orientation induced on it by the orientation of Γ_j; thus Δ_j is an oriented Jordan curve in X. For $1 \leqq j < r$, let $a_j \equiv \Delta_j$. It is easy to see that these, regarded as elements in $H_1(X, Z)$, are part of the usual a-paths (see e.g., [7, Chapt. 10] for details), which arise from doubling the $r - 1$ untwisted strips that were adjoined to D to form D'. τ induces an involution σ on $\Omega_{\mathfrak{x}}$, the $\boldsymbol{C}$-space of analytic differentials on $\mathfrak{X}$, which is $\boldsymbol{R}$-linear, such that $\sigma(i\omega) = -i\sigma(\omega)$, for each $\omega \in \Omega_{\mathfrak{x}}$. $\Omega_{\mathfrak{x}}$ is then the direct sum of the $\boldsymbol{R}$-space, $\Omega_{\mathfrak{x},s}$, of symmetric elements of $\Omega_{\mathfrak{x}}$, and the $\boldsymbol{R}$-space $\Omega_{\mathfrak{x},a}$ of anti-symmetric elements of $\Omega_{\mathfrak{x}}$. Further $\Omega_{\mathfrak{x},a} = i\Omega_{\mathfrak{x},s}$; thus the real dimensions of $\Omega_{\mathfrak{x},a}$ and $\Omega_{\mathfrak{x},s}$ are the same, namely g, the genus of $\mathfrak{X}$—which is also the algebraic genus of $\mathfrak{Y}$. Given $\omega \in \Omega_{\mathfrak{x}}$, let $\omega = \rho + \zeta$ where ρ is symmetric and ζ is anti-symmetric. This convention will hold throughout the paper. Further $\Omega_{\mathfrak{Y}}$ can be naturally identified with $\Omega_{\mathfrak{x},s}$ (see [4], [1], and [2] for more details).

LEMMA. *Let a be an oriented Jordan curve* (arc) *in X such that $\tau(a) = a$, and assume that $\int_a \omega \equiv t$ is real; then $\int_a \rho = t$ and $\int_a \zeta = 0$.*

Proof. Using [1, 3.1], we see that $\int_a \rho = \kappa \circ \int_{\tau(a)} \rho = \kappa \circ \int_a p$, so $\int_a \rho \in \boldsymbol{R}$. $\zeta = i\eta$ for some $\eta \in \Omega_{\mathfrak{x},s}$, so $\int_a \zeta = i \int_a \eta \in i\boldsymbol{R}$, proving the lemma.

Returning now to the proof of Theorem 1, first let us treat the (trivial) case in which Y is orientable, or equivalently in which $m =$

0. Then Y and D_0 are identified, and $\Gamma_1 + \cdots + \Gamma_r = 0$ in $H_1(Y, Z)$. The orientation $\mathscr{O}$, of ∂Y, can be given by choosing $e_j \in \{\pm 1\}$, $1 \leqq j \leqq r$, and by assigning each e_j to Γ_j. Since $\mathscr{O}$ is nonindigenous, not all the e_j's are alike. $\{a_1, \cdots, a_{r-1}\}$ is contained in a basis $B_0 \equiv \{a_1, b_1, \cdots, a_g, b_g\}$ of $H_1(X, Z)$ such that

a) $$a_1 \cdot b_1 \cdot a_1^{-1} \cdot b_1^{-1} \cdot \cdots \cdot a_g \cdot b_g \cdot a_g^{-1} \cdot b_g^{-1} = 1, \text{ in } \pi_1(X) .$$

Let $\{\omega_1, \cdots, \omega_g\}$ be a basis of $\Omega_{\mathfrak{X}}$ over $\boldsymbol{C}$ such that $\int_{a_j} \omega_k = \delta_{jk}$, $1 \leqq j, k \leqq g$. (See e.g., [7, Chapt. 10] for details.) As noted above there exists k, $1 \leqq k < r$, such that $e_k \neq e_r$. Using, among other things, the lemma, we find that

$$\int_{\partial Y, \mathscr{O}} \rho_k = \sum_{j=1}^{r} e_j \int_{\Delta_j} \rho_k = e_k \int_{\Delta_k} \rho_k + e_r \int_{\Delta_r} \rho_k .$$

Since $\Gamma_1 + \cdots + \Gamma_r = 0$ in $H_1(Y, Z)$, $\Delta_r = -\Delta_1 - \cdots - \Delta_{r-1}$ in $H_1(X, Z)$; thus $e_r \int_{\Delta_r} \rho_k = -e_r \int_{\Delta_k} \rho_k$, by Cauchy's theorem, and thus is $-e_r$, and so $\int_{\partial Y, \mathscr{O}} \rho_k = e_k - e_r \neq 0$, disposing of the proof, if Y is orientable. Before going on to the nonorientable cases, let us consider a useful example.

EXAMPLE. Let $\alpha \equiv 1/2 + bi$, where $b > 0$, and consider $G \equiv Z \oplus \alpha Z$ in $\boldsymbol{C}$. Let G act as a set of conformal maps of $\boldsymbol{C}$, by translation. Note that G is invariant under κ, where κ is complex conjugation. Let $\mathfrak{X} \equiv \boldsymbol{C}/G$, and let τ be the anti-analytic involution on $\mathfrak{X}$ induced by κ. The parallelogram whose vertices are $0, \bar{\alpha}, 1$, and α can be taken as a fundamental domain for $\mathfrak{X}$. The interval $[0, 1]$ is the set of fixed points of this domain under the action of κ. Let $\mathfrak{Y} \equiv \mathfrak{X}/\{1, \tau\}$; then $\mathfrak{Y}$ is a Möbius strip, the image of $[0, 1]$ in Y being its boundary. The isosceles triangle, whose vertices are $0, 1$ and α, may be taken as the fundamental domain for $\mathfrak{Y}$. Given β in the straight line segment $[0, \alpha]$, it will be identified—when passing to $\mathfrak{Y}$—with $\bar{\beta} + \alpha$ in $[\alpha, 1]$. dz induces a basis $\{dz\}$ of $\Omega_{\mathfrak{Y}}$ over $\boldsymbol{R}$. Since $\int_0^1 dz = 1$, we see that the conclusion of Cauchy's theorem is never true for $\mathfrak{Y}$. (It is also not hard to show that all dianalytic Möbius strips occur in this way.)

Returning again to the proof of Theorem 1, note that the triangle, described in the above example and identified as indicated, is a Möbius strip. Assume now that Y is nonorientable: i.e., $m > 0$. We can modify the construction of D_m from D_0 as follows. Let one end of a strip be glued to $[1/3, 2/3]$ in the Möbius strip above. The resulting space will be referred to as a *Christmas tree*. Glue the other end of the strip that forms the *trunk* of the Christmas tree to the edge of D_0 to form D_1. Repeat the procedure on D_1 to form D_2. Let Λ_r and Λ_{r+m-1} be the boundaries of the Möbius strip—in the form given in the example—before the trunk is glued on, and then glued to D_0, and D_{m-1}. Let them be oriented to agree with the orientation of Γ_r. In doubling $\mathfrak{Y}$ to form $\mathfrak{X}$, the Christmas tree doubles to a torus with a tube running from a hole in it, back to the rest of X. Λ_j lifts to a (nonunique) path Δ_j in X, $r \leqq j \leqq r+m-1$. Let these paths be oriented by the paths onto which they map. Let $a_r \equiv \Delta_r$ and let $a_{r+m-1} \equiv \Delta_{r+m-1}$. $\{a_1, \cdots, a_r, a_{r+m-1}\}$ is contained in a basis $B_m \equiv \{a_1, b_1, \cdots, a_g, b_g\}$ of $H_1(X, Z)$ that satisfies condition a) above. Thus there exists a basis $\{\omega_1, \cdots, \omega_g\}$ of $\Omega_{\mathfrak{X}}$ over $\boldsymbol{C}$ such that $\int_{a_j} \omega_k = \delta_{jk}$, $1 \leqq j, k \leqq g$. (See e.g., [7, Chapt. 10] again.) Let ρ_k be the symmetric component of ω_k, and let $\rho \equiv \sum_{k=1}^{m} \rho_{r+k-1}$; then $\rho \in \Omega_{\mathfrak{Y}}$. There exist $e_j \in \{\pm 1\}$, $1 \leqq j \leqq r$, such that $\int_{\partial Y, \mathcal{O}} \rho = \sum_{j=1}^{r} e_j \int_{\Gamma_j} \rho$. Since—for $1 \leqq j < r$—$\tau(a_j) = a_j$, and since $\int_{a_j} \omega_{r+k-1} = 0$, $k = 1$ or 2, for such j's, we may apply the lemma and conclude that $\int_{a_j} \rho = 0$; thus $\sum_{j=1}^{r} e_j \int_{\Gamma_j} \rho = e_r \int_{\Gamma_r} \rho$. Assume now that $m = 1$. Since we may invoke Cauchy's theorem on an orientable sub-domain of $\mathfrak{Y}$, we find that $\int_{\Gamma_r} \rho = \pm \int_{a_r} \rho$. Let a_r be re-oriented so that positive sign above holds. Since $\int_{a_r} \omega_r = 1$, we may reason as we did above. Since $\int_{\Gamma_r} \omega_r = \int_{a_r} \omega_r = 1$, and since $\tau(\Gamma_r) = \Gamma_r$, we may apply the lemma and conclude that $\int_{\Gamma_r} \rho = 1$. Thus $\int_{\partial Y, \mathcal{O}} \rho = e_r \neq 0$. Assume lastly that $m = 2$. Reasoning as above $\int_{\Gamma_r} \omega_r + \omega_{r+1} = \int_{\pm a_r \pm a_{r+1}} \omega_r + \omega_{r+1}$, the signs being independent, one of the other. If necessary, re-orient a_r or a_{r+1} so that the plus sign holds twice above; thus $\int_{\Gamma_r} \omega_r + \omega_{r+1} = 2$. Since $\tau(\Gamma_r) = \Gamma_r$, we may apply the lemma and conclude that $\int_{\partial Y, \mathcal{O}} \rho = e_r \int_{\Gamma_r} \rho = 2e_r \neq 0$, proving the theorem.

Greenleaf and Read considered a related question in [5]. Given an orientation $\mathcal{O}$ of ∂Y they defined the notion of an analytic differential ρ as being *positive at* y (in ∂Y), *relative to* $\mathcal{O}$ as follows: if given $f \in E$—a local uniformizer at y—that is increasing near y, relative to $\mathcal{O}$ then $0 < h(y)$, where $\rho = hdf$ for a (unique) $h \in E$. (This definition is independent of the choice of f, as may easily be seen.) ρ is said to be *positive relative to* $\mathcal{O}$ if it is positive at y, relative to $\mathcal{O}$, for each $y \in \partial Y$. Greenleaf and Read proved that if $\mathcal{O}$ is indigenous, then no positive analytic differentials exist in $\Omega_{\mathfrak{Y}}$, that if $\mathcal{O}$ is non-indigenous and $\mathfrak{Y}$ is *elliptic* or *hyperelliptic* then positive analytic differentials always exist; and then they went on to conjecture that the condition that $\mathfrak{Y}$ be elliptic or hyperelliptic can be dropped while preserving the conclusion above. If the Greenleaf-Read conjecture is correct, then it would imply our theorem; thus our theorem may lend additional credence to their conjecture.

Having set up this machinery, let us use it to draw some additional conclusions. As noted before, the real dimension of $\Omega_{\mathfrak{Y}}$ is g and $g = r - 1 + m + 2h$. Given $\rho \in \Omega_{\mathfrak{Y}}$, let

$$\varphi_1(\rho) \equiv \int_{\Gamma_1} \rho, \cdots, \varphi_{r-1}(\rho) \equiv \int_{\Gamma_{r-1}} \rho .$$

Clearly these maps are real valued. If $m \geqq 1$, let $\varphi_r(\rho) \equiv \int_{\Lambda_r} \rho$. If $m = 2$, let $\varphi_{r+1}(\rho) \equiv \int_{\Lambda_{r+1}} \rho$. Let $r \leqq k \leqq r + m - 1$. By [1, 3.1], $\int_{\tau(\Delta_k)} \rho = \kappa(\phi_k(\rho))$. Since Δ_k and $\tau(\Delta_k)$ are homologous, $\int_{\tau(\Delta_k)} \rho = \phi_k(\rho)$, proving that $\phi_k(\rho)$ is real. $\phi_k(\rho)$ can also be computed by integrating allong Λ_k. Note that Λ_k lifts to Δ_k and to $\tau(\Delta_k)$, and that the integral of ρ allong each of these paths is the same. Let Θ_j be an oriented a-path about the j^{th} handle adjoined to D' to form D_0, for $1 \leqq j \leqq h$.

For each such path we may choose an analytic structure in a tubular neighborhood; then integration of ρ along each such is well defined. As remarked before the real parts of these integrals are a priori well defined, whereas the imaginary parts are a priori well defined only up to sign. If $h > 0$, let $\varphi_g(\rho) \equiv \operatorname{Re} \int_{\Theta_1} \rho$, $\varphi_{g-1}(\rho) \equiv \pm \operatorname{Im} \int_{\Theta_1} \rho, \cdots, \varphi_{g-2h+2}(\rho) \equiv \operatorname{Re} \int_{\Theta_h} \rho$, and $\varphi_{g-2h+1}(\rho) \equiv \pm \operatorname{Im} \int_{\Theta_h} \rho$, the signs above being independent of one another.

THEOREM 2. *Given* $\rho \in \Omega_{\mathfrak{Y}}, \rho = 0$ *if and only if* $\varphi_1(\rho) = \cdots = \varphi_g(\rho) = 0$.

Proof. Since each of the φ_j's is an $\boldsymbol{R}$-linear functional, $\rho = 0$ implies that each $\varphi_j(\rho) = 0$. Conversely, let $\rho \in \Omega_{\mathfrak{Y}}$ such that $\varphi_j(\rho) = 0$, for each $j, 1 \leqq j \leqq g$. As noted in the proof of Theorem 1, Γ_j lifts to $a_j \in H_1(X, Z)$ for $1 \leqq j \leqq r - 1$. If $m > 0$, Λ_r lifts to a_r, and if $m = 2$, Λ_{r+1} lifts to a_{r+1}. For $1 \leqq j \leqq h$, Θ_j lifts to two oriented a-paths on X, a_{g-2j+2} and a_{g-2j+1}, which are permuted by τ. Finally, $\{a_1, \cdots, a_g\}$ is contained in a basis $B \equiv \{a_1, b_1, \cdots, a_g, b_g\}$ of $H_1(X, Z)$ such that condition a) holds. By [1, 3.1], the integral of ρ about a_{g-2j+2} and about a_{g-2j+1} are complex conjugates of each other. Because of this symmetry, to know that ρ is zero it suffices to know that its periods with respect to $a_1, \cdots, a_{r-1}, \cdots, a_{r-1+m}, a_g, a_{g-2}, \cdots, a_{g-2h+4}$, and a_{g-2h+2} are all zero: i.e., the periods of ρ with respect to $\Gamma_1, \cdots, \Gamma_{r-1}, \cdots, \Lambda_{r-1+m}, \Theta_1, \Theta_2, \cdots, \Theta_{h-1}\Theta_h$ are all zero; but this is implied by the condition that $\varphi_1(\rho) = \cdots = \varphi_g(\rho) = 0$, proving the theorem.

COROLLARY. *There exists a unique basis* $\{\rho_1, \cdots, \rho_g\}$ *in* $\Omega_{\mathfrak{Y}}$ *of* $\Omega_{\mathfrak{Y}}$ *over* $\boldsymbol{R}$ (resp. in $\Omega_{\mathfrak{Y}}$ of $\Omega_{\mathfrak{X}}$ over $\boldsymbol{C}$), *such that* $\varphi_j(\rho_j) = \delta_{jk}$, *for all* $1 \leqq j, k \leqq g$.

Bibliographic note. See also [8, 9] for related results on the period matrix of a symmetric Riemann surface.

REFERENCES

1. N. L. Alling, *Analytic and harmonic obstruction on nonorientable Klein surfaces*, Pacific J. Math., **36** (1971), 1-19.

2. ———, *Analytic qeometry on real algebraic curves*, Math. Ann., **207** (1974), 23-46.

3. N. L. Alling and N. Greenleaf, *Klein surfaces and real algebraic function fields*, Bull. Amer. Math. Soc., **75** (1969), 869-872.

4. ———, *Foundations of the Theory of Klein Surfaces and Real Algebraic Function Fields*, Springer-Verlag Lecture Notes in Mathematics, vol. **219** (1971).

5. N. Greenleaf and W. Read, *Position holomorphic differentials on Klein surfaces*, Pacific J. Math., **32** (1970), 711-713.

6. W. S. Massey, *Algebraic Topology: An Introduction*, Harcourt, Brace and World, 1967.

7. G. Springer, *Introduction to Riemann Surfaces*, Addison-Wesley, 1957.

8. G. Weichold, *Über symmetrische Riemannsche Flächen und die Periodizitätsmoduln der zugchörigen Abelschen Normalintegrale erster Gettung*, Zeitschrift f. Math. u. Phys., **28** (1883), 321-351.

9. R. Zarrow, *Symmetric and quasi-symmetric Riemann surfaces*, unpublished dissertation, Cornell University, 1972.

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