PACIFIC JOURNAL OF MATHEMATICS
Vol. 62, No. 1, 1976

MOORE–POSTNIKOV TOWERS FOR FIBRATIONS IN WHICH π_1(fiber) IS NON-ABELIAN

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When Moore–Postnikov towers for fibrations $p: E \to X$ were first developed, Moore constructed the tower for arbitrary maps p and, when all action on π_n(fiber) were trivial, showed that each stage was induced from the loop-path fibration over a $K(\pi, n)$ and classified by the corresponding k-invariant. Barratt–Gugenheim–Moore showed that without restriction each stage could be induced from suitable universal fibrations. Subsequent authors, including McClendon, Nussbaum, Robinson and Siegal, based on the above and work by Olum, described the classifying map by k-invariant and local coefficients when $\pi_1(X)$ acts and π_1(fiber) is Abelian, and Bousfield and Kan described the case when π_1 acts nilpotently. This note gives a method for handling fibrations requiring only that all spaces be path-connected.

In the previous cases, it is assumed that X is path-connected, F is $(n-1)$-connected, $n \geqq 1$, and that we can compute what a certain class in $H^n(F)$ transgresses to in $H^{n+1}(X)$. In particular, local coefficients may be necessary, and hence it is assumed that we know the action of $\pi_1(X)$, which is completely determined by the action of $\pi_1(E)$. If $n = 1$ and $\pi_1(F)$ is non-Abelian, a direct generalization would require computing with non-Abelian coefficients. We avoid this by showing it is only necessary to know the action of $\pi_1(E)$ on $\pi_1(F)$ in order to build the second stage of a tower which naturally replaces p with a map whose fiber is the universal cover of F. The remaining stages can then be constructed as in the classical case.

The paper is organized as follows: some basic facts are recalled in §2, the construction is given in §3 with the proof of one basic lemma put off until §5, and in §4 we give an example.

2. We recall a few basic facts from algebra and topology. Details for the following may be found in **[3]**, **[16]**, **[6]**, or **[8]**.

For G a group, let Aut G, In G, Out G = Aut G/In G be the group of automorphisms, innerautomorphisms, outerautomorphisms, respectively.

Let $F \xrightarrow{i} E \xrightarrow{p} B$ be a fibration (all spaces are path-connected). Suppose, at first, that F is simply-connected (so $p_*: \pi_1(E) \cong \pi_1(B)$).

Make p into an inclusion. Then $\pi_1(E)$ acts on the long exact homotopy sequence of the pair (B, E). Since $\pi_{n-1}(F) \cong \pi_n(B, E)$, $\pi_1(E)$ acts on $\pi_n(F)$, all n. This, in turn induces an action of $\pi_1(B)$ on $\pi_n(F)$, all n (and is the same as that induced by "dragging" F around loops in B). Suppose, now, that $\pi_1(F) \neq 1$. Then it also acts on $\pi_n(F)$, all n, and this is the same action as induced by i_* and the above action of $\pi_1(E)$ (see **[16]**). Thus, $\pi_1(B)$ only acts now on $\pi_n(F)$ mod the action of $\pi_1(F)$ on $\pi_n(F)$. Letting $n = 1$ and recalling that π_1 acts on itself by innerautomorphisms, p thus induces a homomorphism $\varphi: \pi_1(E) \to \text{Aut}\ \pi_1(F)$ and which, in turn, induces a $\psi: \pi_1(B) \to \text{Out}\ \pi_1(F)$, which we will call a semi-action of $\pi_1(B)$ on $\pi_1(F)$.

Let G be a non-Abelian group and let $K(G, 1)$ be an Eilenberg–MacLane space of type $(G, 1)$. Recall, even though $K(G, 1)$ is not an H-space, there is a universal classifying fibration, hereafter referred to as $K(G, 1) \xrightarrow{i} E_G \xrightarrow{p} B$ (where B is, of course, $B_{AK(G,1)}$, with $AK(G, 1)$ the H-space of homotopy equivalences of $K(G, 1)$).

THEOREM 2.1. (a) $\pi_1(B) \cong \text{Out}\ G$, $\pi_2(B) \cong C$, *the center of G, and* $\pi_i(B) = 0$, *otherwise.*

(b) *E is a* $K(\text{Aut}\ G, 1)$, *the homotopy sequence for p reduces to the natural* $0 \to C \to G \to \text{Aut}\ G \to \text{Out}\ G \to 1$, *and the (above) semi-action of* $\pi_1(B)$ *on* $\pi_1(K(G, 1))$ *is the identity.*

Part (a) was proved by Gottlieb **[6]** and (b) is proved in **[8]**.

Thus B has a single k-invariant in $H^3(K(\text{Out}\ G, 1); \{C\})$, which we briefly describe. Let H and K be groups and let $G \to H \to K$ be an extension of G by K. Then the extension induces, by innerautomorphisms in H, a semi-action of K on G, $\rho: K \to \text{Out}\ G$. (See MacLane **[12]**.) Given an arbitrary semi-action $\rho: K \to \text{Out}\ G$, there may not be an extension of G by K which induces it. By Eilenberg–MacLane **[5]**, there is an extension inducing ρ if and only if a certain obstruction $k \in H^3(K; C)$ is zero. Restricting to the case $K = \text{Out}\ G$ and $\rho = \text{id}$. yields an element $U \in H^3(\text{Out}\ G; C)$. By **[8]**, U is the universal example for k, and it corresponds to the k-invariant for B (under the natural isomorphism between group cohomology and the (singular) cohomology of a $K(\ \ , 1)$).

3. Statements and proofs of the main results. Let G be a group, C its center, and In G, Aut G, Out G as in §2. Then $C \to G \to \text{In}\ G$ is a central extension of C by In G, and it has a characteristic class $c \in H^2(\text{In}\ G; C)$. Let Φ be the natural isomorphism between group cohomology and (singular) cohomology of $K(\ \ , 1)$. We abuse notation by denoting also by c the element $\Phi(c) \in H^2(K(\text{In}\ G, 1); C)$ and, as is standard practice, also denoting by c a map $c: K(\text{In}\ G, 1) \to K(C, 2)$ (which is unique up to homotopy) such that c^* (fundamental class) $= c$.

LEMMA 3.1. *If* $c: K(\text{In } G, 1) \to K(C, 2)$ *is an inclusion, the homotopy sequence of the pair* $(K(C, 2), K(\text{In } G, 1))$ *reduces to*

$$0 \to \pi_2(K(C,2)) \to \pi_2(K(C,2),\ K(\text{In } G, 1)) \to \pi_1(K(\text{In } G, 1)) \to 1$$

and this is $0 \to C \to G \to \text{In } G \to 1$.

Proof. The pull-back by c of the loop-path fibration over $K(C, 2)$ gives a fibration $K(C, 1) \to T \xrightarrow{q} K(\text{In } G, 1)$. By [**7**, Theorem 1], T is a $K(G, 1)$ and the homotopy sequence for q is $0 \to C \to G \to \text{In } G \to 1$. Since T is the fiber of c, the lemma follows.

We now assume $c: K(\text{In } G, 1) \to K(C, 2)$ is an inclusion.

LEMMA 3.2. *Suppose that* (X, A) *is* (*of the homotopy type of*) *a* 1-*connected CW pair, that* X *is* 1-*connected, and that* $\tau: \pi_1(A) \to \text{In } G$ *and* $\varphi: \pi_2(X, A) \to G$ *are homomorphisms such that the diagram*

$$\begin{array}{ccc} \pi_2(X, A) & \to & \pi_1(A) \\ \varphi \downarrow & & \tau \downarrow \\ G & \to & \text{In } G \end{array} \qquad \textit{commutes.}$$

Then there is a unique homotopy class of maps of pairs $g: (X, A) \to (K(C, 2), K(\text{In } G, 1))$ *such that*

$$\tau = (g \mid A)_*: \pi_1(A) \to \pi_1(K(\text{In } G, 1))$$

and $\varphi = g_*: \pi_2(X, A) \to \pi_2(K(C, 2), K(\text{In } G, 1))$.

The proof of 3.2 is given in §5.

The universal fibration $K(G, 1) \to E_G \xrightarrow{p} B$ was described in §2. Now make p into an inclusion.

LEMMA 3.3. *Let* (X, A) *be* (*of the homotopy type of*) *a* 1-*connected CW pair, where* X *is path-connected. Let* $\chi: \pi_1(A) \to \text{Aut } G$ *and assume* $\varphi: \pi_2(X, A) \to G$ *is* χ-*equivariant. Then there is a unique homotopy class of maps* $f: (X, A) \to (B, E_G)$ *such that* $\chi = (f \mid A)_*: \pi_1(A) \to \pi_1(E)$ *and* $\varphi = f_*: \pi_2(X, A) \to \pi_2(B, E)$.

Proof. (Compare with the proof of 1.2 in [**17**].) Note that φ and χ induce a unique $\psi: \pi_1(X) \to \text{Out } G$. Assume that (X, A) has no (relative) cells in dimensions < 2. Let X^*, B^* be the universal cover of X, B, respectively, and let A^*, E_G^* be the restrictions to A, E_G, respectively. The homotopy classes $(X, A) \to (B, E)$ which induce ψ correspond (exactly) to the based ψ-equivariant homotopy classes $(X^*, A^*) \to (B^*, E_G^*)$. Now $\pi_2(X^*, A^*) \cong \pi_2(X, A)$, $\pi_2(B^*, E_G^*) \cong \pi_2(B, E_G)$ and χ induces $\tau\colon \pi_1(A^*) \to \text{In } G \cong \pi_1(E_G^*)$. So we have only to show there is a unique ψ-equivariant homotopy class which induces τ and φ.

By 3.2, there is a unique homotopy class $g\colon (X^*, A^*) \to (B^*, E_G^*)$ inducing τ and φ. The map $g \mid A^*$ is completely determined (up to homotopy) by the requirement $(g \mid A^*)_* = \tau$ and we can pick g so that $g \mid A^*$ is ψ-equivariant (and is unique up to ψ-equivariant homotopy). (Indeed, just take $g \mid A^*$ to cover a map $A \to E_G$ which induces χ.) Now g is homotopic to a ψ-equivariant map. For the facts that $\pi_1(X)$ acts freely on the cells of $X^* - A^*$ and that φ is χ-equivariant make it possible to construct, skeleton by skeleton, a homotopy from g to an equivariant f. If f_1 is another equivariant map, it is homotopic to g, by 3.2, and hence to f, and there is no obstruction to deforming the homotopy into an equivariant one. This completes the proof.

Suppose now that $F \to E \xrightarrow{q} X$ is a fibration, with $\varphi\colon \pi_1(F) \cong G$ and all spaces are path-connected. By §2, q induces a $\chi\colon \pi_1(E) \to \text{Aut } G$. Then 3.3 and the usual arguments yield:

PROPOSITION 3.4. *There is a map of fibrations, unique up to homotopy,*

$$\begin{array}{ccc} F & \xrightarrow{f|F} & K(G,1) \\ \downarrow & & \downarrow \\ E & \xrightarrow{f} & E_G \\ \downarrow q & & \downarrow p \\ X & \xrightarrow{f'} & B \end{array}$$

where $f_* = \chi\colon \pi_1(E) \to \pi_1(E_G)$, $(f \mid F)_* = \varphi\colon \pi_1(F) \to \pi_1(K(G, 1))$.

Let $K(G, 1) \to E_2 \to X$ be the fibration induced from p by f', so that q factors as $E \xrightarrow{q_2} E_2 \to X$. The fiber of q_2 is the seme as the fiber of $f \mid F$, which is F^*, the universal cover of F. Making q_2 into a fibration , we have thus constructed the first stage of a tower for q by factoring it through a fibration with a simply-connected fiber. Observe that $\pi_1(E_2) \cong \pi_1(E)$, $\pi_2(E_2) \cong \text{im } q_*$, and $\pi_i(E_2) = \pi_i(X)$, $i > 2$, and that the coefficient system $H^*(E_2; \{\pi_2(F^*)\})$ is completely given by the action of $\pi_1(E)$. We can complete the Moore–Postnikov tower in the classical way for q_2.

4. The following simple example illustrates various aspects of the theory. Let S^3 = the topological group of unit quaternions = $\{w + xi + yj + zk \mid w, x, y, z \in R,\ ij = k,\ i^2 = -1, \text{ etc.}\}$ and let n be a product of primes, each prime $\equiv 1$ (4) (so that $\sqrt{-1} \in Z_n$). Let $G = \langle e^{\pi i/n} = a, j\rangle \subset S^3$ (where $\langle\ \ \rangle$ denotes "subgroup generated by"). Let $F = S^3/G$, so $\pi_1(F) \cong G$. If $p\colon E \to RP^k$, $k > 1$, is a fibration with fiber F, then $\pi_1(F) \to \pi_1(E) \to \pi_1(RP^k)$ is an extension of G by Z_2.

There are three possible actions, ρ, of $\pi_1(E)$ on $\pi_1(F)$. For, let $\alpha, \beta \in \operatorname{Aut} G$ be given by $\alpha(a) = a^k$, where $(a^k)^2 = a^{-1}$, $\alpha(j) = j$, $\beta(a) = a$, $\beta(j) = -j$, and let α^*, β^* be their images in, and which generate, $\operatorname{Out} G \cong Z_2 \times Z_2$. It turns out that the universal obstruction in $H^3(\operatorname{Out} G; Z_2)$, mentioned in §2, is non-zero, and in fact any extension of G by Z_2 inducing $\rho^*\colon Z_2 \to \operatorname{Out} G$ must have $\rho^*(\text{generator}) \neq \alpha^*\beta^*$. (These facts follow from computations in [**9**, section 3].) The result follows from this and the fact $\rho \mid G$ must be the natural $G \to \operatorname{Aut} G$.

We now restrict to a case for which $\rho^* \neq 0$. Let $H = \langle e^{\pi i/2n} = b, j\rangle \subset S^3$. Then $G \subset H$, $G/H \cong Z_2$, and the induced $\rho^*\colon Z_2 \to \operatorname{Out} G$ is non-zero. For $k = 2, 3, \cdots, \infty$, let H act on S^k by $b(s) = -s$ and $j(s) = s$, on S^3 naturally, and on $S^3 \times S^k$ diagonally. Let $E = S^3 \times S^k/H$, so that $S^3/G \to S^3 \times S^k/H \xrightarrow{p} RP^k$ is a fibration $F \to E \xrightarrow{p} RP^k$, and $\pi_1(F) \to \pi_1(E) \to \pi_1(RP^k)$ is $G \to H \to Z_2$, as above.

Proceeding as in §3, we get a map of p into the universal $K(G, 1)$ fibration, take the pull-back $K(G, 1)$-fibration over RP^k, q_k, and obtain a map of fibrations, p_2, and the following diagram:

$$\begin{array}{ccccccc}
S^3 \to & S^3/G & \xrightarrow{p_2 \mid F} & K(G,1) & \to & K(G,1) \\
\| & \downarrow & & \downarrow & & \downarrow \\
S^3 \to & S^3 \times S^k/H & \xrightarrow{p_2} & E_1 & \to & K(H,1) \\
& \downarrow p & & \downarrow q_k & & \downarrow q_\infty \\
& RP^k & = & RP^k & \to & K(Z_2, 1) = RP^\infty.
\end{array}$$

As q_∞ is the case $k = \infty$, q_k can be considered as induced from q_∞ by the inclusion $RP^k \to RP^\infty = K(Z_2, 1)$. We compute the second k-invariant, k_2.

Considering p_2 to be a fibration, the fiber of p_2 = the fiber of $p_2 \mid F = S^3$. As H acts trivially on $\pi_*(S^3)$, $k_2 \in H^4(E_1; Z)$ and k_2 is the Euler class of p_2. The Euler class of p_2 for arbitrary k restricts from the class for the case $k = \infty$, so consider that case first. Then, p_2 is equivalent to the fibration $S^3 \to S^3/H \to K(H, 1)$ (up to homotopy type) where, as a corollary to the usual spectral sequence argument that groups which act freely on spheres have periodic cohomology, $H^*(K(H, 1); Z)$ is periodic (of degree 4 by Swan [**20**]), given by cup product with the Euler class. To compute the Euler class, observe that H is a semi-direct product of Z_n by $Q = \langle i, j \rangle \subset S^3$, and using $H^*(Q; Z)$ (see Atiyah [**1**, p. 61]) and the spectral sequence for $Z_n \to H \to Q$ we compute $H^*(H; Z) \cong Z_{8n}(\chi) \otimes E_{Z_2}(\delta, \epsilon)$ with relations $\delta\epsilon = 4n\chi$, where $\dim \delta = 2 = \dim \epsilon$, $\dim \chi = 4$, and E_{Z_2} means exterior algebra over Z_2. Using Wall [**21**] we can see $H^*(G; Z) \cong Z_{4n}(\gamma) \otimes Z_4(\eta)$ with relations $\eta^2 = n\gamma$, $\dim \eta = 2$, $\dim \gamma = 4$. Using the spectral sequence for p_∞, which is the spectral sequence for $G \to H \to Z_2$ (see [**9**]), and then seeing how it restricts to p_k, $k \geqq 2$, it is then easy to see that the (minimal) generator of $H^4(E; Z) \cong Z_{8n}$ is $\chi = k_2$.

5. Proof of 3.2. Uniqueness. Suppose $g, g' : (X, A) \to (K(C, 2), K(\text{In } G, 1))$ both satisfy the hypotheses. Then $g|A \sim g'|A$, since they both induce the same homomorphism on π_1 (and the range is a $K(\ \ , 1)$). This homotopy can be extended first to the 2-skeleton of X, since they both induce the same homomorphism on π_2, and then to the rest of X since all obstructions are zero.

Existence. Let $i: A \to X$ and $j: X \to (X, A)$ be the inclusions. By exactness and commutivity, $\varphi j_* \pi_2(X) \subset C$ (see diagram 5.2 below) so let $\sigma: \pi_2(X) \to C$ be the induced homomorphism. Maps from $(n-1)$-connected spaces into $K(\ \ , n)$'s are completely determined (up to homotopy) by their induced homomorphisms on π_n. Let $g: X \to K(C, 2)$, $f: A \to K(\text{In } G, 1)$ be maps which induce σ on π_2, τ on π_1, respectively.

LEMMA 5.1. *The maps cf, gi: $A \to K(C, 2)$ are homotopic.*

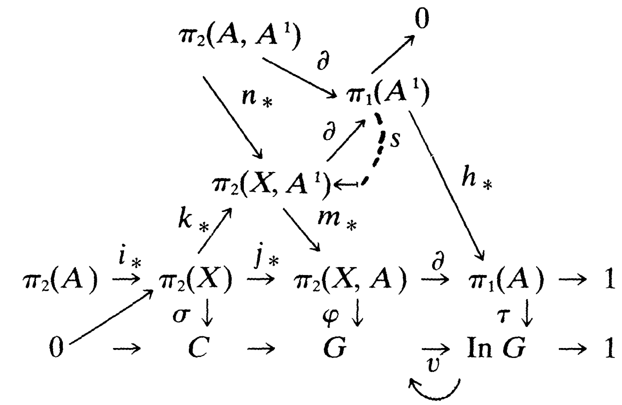
This lemma completes the proof of 3.2, since relative CW complexes have the homotopy extension property so we can actually take g to extend cf.

Proof of 5.1. The homotopy classes of maps $A \to K(C, 2)$ are in one-to-one correspondence with $H^2(A; C)$, so it is sufficient to show $(cg)^*$ (fundamental class) $= (fi)^*$ (fundamental class). We will do this by describing their corresponding CW cochains.

Let $\Pi = \pi_1(A)$ and let $K(\Pi)$ be the $K(\Pi, 1)$ constructed by geometrically realizing the bar construction on Π. (A proof that $K(\Pi)$ is as described below may be found in [**7**].) Assume that $A = K(\Pi) \cup \vee_\lambda S^2_\lambda \cup$ higher dimensional cells, and that (X, A) has no (relative) cells in dimensions < 2 (the case of an arbitrary (X, A) following by homotopy equivalence). Thus the 2-skeleton of A is $e^0 \cup \bigcup_\alpha e^1_\alpha \cup \bigcup_\beta e^2_\beta \cup \vee_\lambda S^2_\lambda$, where there is a single 1-cell e^1_α for each $1 \neq \alpha \in \pi_1(A)$, and a single 2-cell e^2_β for each $\beta = (\alpha, \alpha_1)$ where $1 \neq \alpha, \alpha_1 \in \pi_1(A)$. The 1-cells e^1_α represent α in $\pi_1(A, e^0)$, the 2-cells e^2_β for $\beta = (\alpha, \alpha_1)$ attach the relation $e^1_\alpha \cdot e^1_{\alpha_1} = e^1_{\alpha\alpha_1}$ in $\pi_1(A, e^0)$, and all of $\pi_2(A)$ is generated by

$\vee_\lambda S^2_\lambda$. Let $A^1 = e^0 \cup \bigcup_\alpha e^1_\alpha$ be the 1-skeleton of A, so the following diagram is commutative:

(5.2)



all five rows and diagonals are exact, and h, i, j, k, m, and n are inclusions. Since $\pi_1(A^1)$ is free (generated by $\{\alpha \mid 1 \neq \alpha \in \Pi\}$), there is an (algebraic) splitting $s\colon \pi_1(A^1) \to \pi_2(X, A^1)$.

We first describe the cocycle corresponding to cg. Since $\pi_2(K(\text{In } G, 1)) = 0$, its values on the cells S^2_λ are zero. Thus the cocycle is completely determined by its restriction to $K(\Pi)$. There, it represents the characteristic class ρ in $H^2(K(\Pi); C)$ of the extension of C by Π which is pulled-back from $C \to G \to \text{In } G$ by τ, and is described in [7]. For each $x \in \text{In } G$, pick $v(x) \in G$ which projects back to x, but pick $v(1) = 1$. By exactness, for each $x, x_1 \in \text{In } G$, there is a $e(x, x_1) \in C$ such that $v(x)v(x_1) = e(x, x_1)v(xx_1)$.

LEMMA 5.3. *The cochain in* $C^2(K(\Pi); C)$ *given by* $e^2_{\alpha,\alpha_1} \to e(\tau(\alpha), \tau(\alpha_1))$ *is a cocycle, and it represents* ρ.

Proof. This follows immediately from [7, §2, 3].

We will need another cocycle which represents ρ. For each $\alpha \in \pi_1(A)$, pick a $y \in \pi_1(A^1)$ such that $h_*(y) = \alpha$, but pick 1 for 1. Then $m_*s(y) \in \pi_2(X, A)$ and it projects to α under ∂. Let $w(\alpha) = \varphi m_* s(y) \in G$, which projects to $\tau(\alpha)$. Define CW cochains $b \in C^1(K(\Pi); C)$ and $d \in C^2(K(\Pi); C)$ by $b(e^1_\alpha) = w(\alpha)(v\tau(\alpha))^{-1}$ and $d(e^2_{\alpha,\alpha_1}) = w(\alpha)w(\alpha_1)w(\alpha\alpha_1)^{-1}$. Easily, the cochains d and e differ by δb, so we have proven:

LEMMA 5.4. *The cochain d is a cocycle and also represents* ρ.

We now describe the cocycle corresponding to fi. Observe first that $\sigma i_* \pi_2(A) = 0$, since $C \to G$ is a monomorphism and by commutivity and exactness. Thus we can take a representative cocycle to be zero on the cells S^2_λ. We can consider the cells e^2_{α,α_1} to represent elements of $\pi_2(A, A^1)$. By commutivity, $n_*(e^2_{\alpha,\alpha_1}) = (k_*z, s(r))$, for some $z \in \pi_2(X)$ and $r = \partial e^2_{\alpha,\alpha_1} =$ the relation $e^1_\alpha e^1_{\alpha_1} e^1_{\alpha\alpha_1} = 1$. Thus $j_*(-z) = m_*s(r)$ in $\pi_2(X, A)$ and $-z$ is the element in $\pi_2(X)$ determined by e^2_{α,α_1}. But, by commutivity, $\varphi m_*s(r)$ is exactly $d(e^2_{\alpha,\alpha_1})$ as described above, so by 5.4 we are done.

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Received February 18, 1975.

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