CORRECTION TO "RATIONAL HOMOLOGY AND WHITEHEAD PRODUCTS"

MICHEAL DYER

The definitions of Im_{ij} and Ker_{ij} in §4 of [2] are incorrect. We will supply the appropriate ones here. With these definitions, the statements and proofs of Theorems 4.1 through 4.4 stand as written. Theorem 4.5 needs an additional hypothesis, which is given below. I would like to thank H. J. Baues for pointing out the difficulty in §4; his extension of those results will appear in [1].

Let X be a CW-complex which is rationally $(n - 1)$-connected and let π_i denote $\pi_i(X) \otimes Q$, the rational homotopy of X. Let $S(\pi_i)$ denote the skew-symmetric tensor product $(\pi_i \otimes \pi_i)/R$, where R is the subspace generated by $\{\alpha \otimes \beta - (-1)^i \beta \otimes \alpha \mid \alpha, \beta \in \pi_i\}$. Furthermore, let $\pi_{i,j}$ denote $\pi_i \otimes \pi_j$ if $i \neq j$, or $S(\pi_i)$ if $i = j$. If A is a vector subspace of π_{i-1}, any arrow

$$\pi_{i,j} \to \pi_{i+j-1}/A$$

is the homomorphism induced by the rational Whitehead product.

DEFINITION 1. Let $\text{Im}_{i-1,j} = \text{im}\{\pi_{j,i-1} \to \pi_{i-1}\}$ for $n \leqq j \leqq [i/2]$ and $\text{Im}_{i-1} = \Sigma_{n \leqq j \leqq [i/2]} \text{Im}_{i-1,j}$ (just sum, not necessarily direct).

DEFINITION 2. We define $\text{Ker}_{i,j}$ inductively for $n \leqq j \leqq [i/2]$. First, $\text{Ker}_{i,[i/2]} = \ker\{\pi_{[i/2], i-[i/2]} \to \pi_{i-1}\}$. If $1 \leqq k \leqq [i/2] - n$, then

$$\text{Ker}_{i,[i/2]-k} = \ker\left\{\pi_{[i/2]-k} \otimes \pi_{[i/2]+k+1} \to \pi_{i-1} \Big/ \sum_{j=0}^{k-1} \text{Im}_{i-1,[i/2]-j}\right\}.$$

Let $\text{Ker}_i = \bigoplus_{0 \leqq j \leqq [i/2]-n} \text{Ker}_{i,[i/2]-j}$.

The next lemma easily implies that my results now agree with Baues.

LEMMA. *Let $f: A \to C$ and $g: B \to C$ be homomorphisms of abelian groups such that $\ker f$ is a direct summand of A. Let $g': B \to \text{coker} f$ be the composite of g with the natural projection $C \to \text{coker} f$. Then*

$$\ker\{f + g: A \oplus B \to C\} \cong \ker f \oplus \ker g'.$$

Finally, Theorem 4.5 should read as follows:

THEOREM 4.5. *Let X be rationally $(n-1)$-connected and $i \leqq 3(n-1)$. Consider these statements:*

    (a)   *π_i is generated by Whitehead products.*

    (b)   *For all r such that $n \leqq r \leqq [i/2]$, $\pi_{r,i-r} \rightarrow \pi_{i-1}$ is injective.*

    (c)   *The sum $\Sigma_{n \leqq j \leqq [i/2]} \mathrm{Im}_{i-1,j}$ is a direct sum.*

The following are true.

    (d) $h_i \otimes 1 = 0 \Leftrightarrow$ (a)

    (e) $\mathrm{coker}\, h_i \otimes 1 = 0 \Leftrightarrow$ (b) and (c)

    (f) $H_i(X, Q) = 0 \Leftrightarrow$ (a), (b) and (c).

REFERENCES

1. H. J. Baues, *On the rational Hurewicz homomorphism*, (to appear).

2. M. N. Dyer, *Rational Homology and Whitehead Products*, Pacific J. Math., **40** (1972), 59–71.

UNIVERSITY OF OREGON
EUGENE, OR 97403