PACIFIC JOURNAL OF MATHEMATICS
Vol 70, No 1, 1977

LOCALLY BOUNDED TOPOLOGIES ON $F(X)$

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It is classic that, to within equivalence, the only valuations on the field $F(X)$ of rational functions over a field F that are improper on F are the valuations v_p, where p is a prime of the principal ideal subdomain $F[X]$ of $F(X)$, and the valuation v_∞, defined by the prime X^{-1} of the principal ideal subdomain $F[X^{-1}]$ of $F(X)$ ([1], p. 94, Corollary 2). If $\mathcal{T}$ is the supremum of finitely many of the associated valuation topologies, then $\mathcal{T}$ is a Hausdorff, locally bounded ring topology on $F(X)$ for which F is a bounded set and for which there is a nonzero topological nilpotent a. In this paper we shall show conversely that any topology on $F(X)$ having these properties is the supremum of finitely many valuation topologies.

A subset S of a topological ring R is *bounded* if given any neighborhood V of 0, there exists a neighborhood U of 0 such that $SU \subseteq V$ and $US \subseteq V$. The topology on R is *locally bounded* if there is a bounded neighborhood of 0. A bounded subfield of a Hausdorff topological ring is discrete ([2], p. 119, Exercise 13). It is easy to see that if $\lim_{n\to\infty} x_n = 0$ and if $\{a_n\}_{n=1}^\infty$ is a bounded sequence, then $\lim_{n\to\infty} a_n x_n = 0$.

An element c of a topological ring is a *topological nilpotent* if $\lim_{n\to\infty} c^n = 0$. Let $\mathcal{P} = \{p \in F[X]: p$ is a prime polynomial$\}$, and let $\mathcal{P}' = \mathcal{P} \cup \{\infty\}$. For each $p \in \mathcal{P}'$, we shall denote by $\mathcal{T}_p$ the topology defined by the valuation v_p. Then for any finite subset L of $\mathcal{P}'$, $\sup_{p\in L} \mathcal{T}_p$ has a nonzero topological nilpotent. Indeed, let g be the product of the members of $L \cap \mathcal{P}$. If $\infty \notin L$, g is a nonzero topological nilpotent for $\sup_{p\in L} \mathcal{T}_p$; if $\infty \in L$, let q be a prime polynomial not in L and let $r > 0$ be such that $\deg(q^r) > \deg g$; then $g q^{-r}$ is a nonzero topological nilpotent of $\sup_{p\in L} \mathcal{T}_p$.

We recall that a *norm* $\|\cdot\|$ on a field K is a function to the nonnegative reals satisfying $\|x\| = 0$ if and only if $x = 0$, $\|x - y\| \leqq \|x\| + \|y\|$, and $\|xy\| \leqq \|x\|\,\|y\|$ for all $x, y \in K$. Clearly a subset of K is bounded in norm if and only if it is bounded for the topology defined by the norm; in particular the topology given by a norm is a locally bounded topology. We shall use the following theorem of P. M. Cohn ([4], Theorem 6.1): A Hausdorff, locally bounded ring topology on a field K for which there is a nonzero topological nilpotent is defined by a norm.

THEOREM 1. *Let F be a field and x a transcendental element over F in some field extension. Let $\mathcal{T}$ be a Hausdorff, locally bounded ring* *topology on $F(x)$ for which the subfield F is bounded* (and hence discrete) *and for which there exists a nonzero topological nilpotent $g_0(x) \in F[x]$, and let $J = \{f \in F[X]: \lim_{n\to\infty} f(x)^n = 0\}$. Then*

1. *$F[x]$ is bounded.*
2. *J is a proper ideal of $F[X]$; its monic generator h is the product of a sequence $p_1, \cdots, p_k$ of distinct prime polynomials of $F[X]$.*
3. *$\mathcal{T} = \sup_{1 \leqq i \leqq k} \mathcal{T}_{p_i(x)}$.*

Proof. 1. By Cohn's Theorem there exists a norm $\|\cdot\cdot\|$ defining $\mathcal{T}$. Then $\|g_0(x)^n\| < 1$ for some $n \geqq 1$; let $g = g_0^n$. Then $g(x)$ is a topological nilpotent and $\|g(x)\| < 1$.

As F is bounded for $\mathcal{T}$, there exists a positive constant M such that $\|a\| \leqq M$, for all $a \in F$. Let $L = M \sum_{i=0}^{m} \|x\|^i$, where $m = \deg g$. As F is discrete, F contains no nonzero topological nilpotents, so $m \geqq 1$. For any polynomial f such that $\deg f < m$, $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0$ where $n < m$, $a_0, a_1, \cdots, a_n \in F$; so

$$\|f(x)\| \leqq \sum_{i=0}^{n} \|a_i\| \, \|x\|^i \leqq M \sum_{i=0}^{n} \|x\|^i \leqq M \sum_{i=0}^{m} \|x\|^i = L.$$

Therefore, $\{f(x) \in F[X]: \deg f < m\}$ is a bounded subset of $F(x)$.

We shall show next that $\|f(x)\| \leqq S$ for all $f \in F[X]$, where $S = L/(1 - \|g(x)\|)$. Given $f \in F[X]$, there exists an integer $n \geqq 0$ and polynomials $q_0, \cdots, q_n$ such that $f = g^n q_n + \cdots + gq_1 + q_0$ and for each $i \in [0, n]$, $q_i = 0$ or $\deg q_i < m$. Therefore,

$$\begin{aligned} \|f(x)\| &= \|g(x)^n q_n(x) + \cdots + g(x)q_1(x) + q_0(x)\| \\ &\leqq \sum_{i=0}^{n} \|g(x)^i\| \, \|q_i(x)\| \leqq L \sum_{i=0}^{n} \|g(x)^i\| \\ &\leqq L \sum_{i=0}^{\infty} \|g(x)^i\| \leqq L \sum_{i=0}^{\infty} \|g(x)\|^i = S. \end{aligned}$$

Thus $F[x]$ is bounded.

2. Let $f_1, f_2 \in J$, and let $t \in F[X]$. Then

$$\begin{aligned} \|(f_1 + f_2)^{2n}(x)\| &= \left\| \sum_{k=0}^{2n} \binom{2n}{k} f_1^{2n-k}(x) f_2^k(x) \right\| \\ &= \left\| f_1^n(x) \sum_{k=0}^{n} \binom{2n}{k} f_1^{n-k}(x) f_2^k(x) \right. \\ &\qquad \left. + f_2^n(x) \sum_{k=n+1}^{2n} \binom{2n}{k} f_1^{2n-k}(x) f_2^{k-n}(x) \right\| \end{aligned}$$

$$\leqq \|f_1^n(x)\| \left\| \sum_{k=0}^{n} \binom{2n}{k} f_1^{n-k}(x) f_2^k(x) \right\|$$
$$+ \|f_2^n(x)\| \left\| \sum_{k=n+1}^{2n} \binom{2n}{k} f_1^{2n-k}(x) f_2^{k-n}(x) \right\|$$
$$\leqq S\|f_1^n(x)\| + S\|f_2^n(x)\| \to 0.$$

Thus, $(f_1+f_2)^2(x)$ is a topological nilpotent, whence $(f_1+f_2)(x)$ is. Also $\|(f_1t)^n(x)\| \leqq \|f_1^n(x)\|\ \|t^n(x)\| \leqq S\|f_1^n(x)\| \to 0$, so $f_1t \in J$. Hence as $1 \notin J$, J is a proper nonzero ideal of $F[X]$, a principal ideal domain. Let h be its monic generator, and let $h = \Pi_{i=1}^k p_i^{r_i}$ where $p_1, \cdots, p_k$ are distinct prime polynomials. Let $h_0 = \Pi_{i=1}^k p_i$, and let $r = \max\{r_1, \cdots, r_k\}$. Then $h \mid h_0^r$, so $h_0^r \in J$ and hence, $h_0(x)$ is a topological nilpotent. Therefore, h_0 belongs to J. So as $h_0 \mid h$, $h_0 = h$.

3. We shall first show that the topology induced on $F[x]$ by $\mathcal{T}$ is weaker than that induced on $F[x]$ by $\sup_{1\leqq i\leqq k} \mathcal{T}_{p_i(x)}$.

For all $n \geqq 1$, let $U_n = \{f(x) \in F[x]\colon p_i^n \mid f, 1 \leqq i \leqq k\}$. Then $(U_n)_{n\geqq 1}$ is a fundamental system of neighborhoods of zero for the topology induced on $F[x]$ by $\sup_{1\leqq i\leqq k} \mathcal{T}_{p_i(x)}$. But clearly $p_i^n \mid f$ for all $i \in [1, k]$ if and only if $h^n \mid f$ as $h = p_1 \cdots p_k$. Thus $U_n = F[x]h^n(x)$.

For each $\epsilon > 0$, let $B_\epsilon = \{y \in F(x)\colon \|y\| < \epsilon\}$. It suffices to show that there exists $N \in \mathbf{N}$ such that $F[x]h^N(x) \subseteq B_\epsilon \cap F[x]$.

Let N be such that $\|h(x)^N\| < \epsilon/S$. Then for any $g(x) \in F[x]$,

$$\|g(x)h^N(x)\| \leqq \|g(x)\|\ \|h^N(x)\| < S \cdot \frac{\epsilon}{S} = \epsilon.$$

Hence $F[x]h^N(x) \subseteq B_\epsilon \cap F[x]$.

We next show that $\mathcal{T} \subseteq \sup_{1\leqq i\leqq k} \mathcal{T}_{p_i(x)}$, that is, that given $\epsilon > 0$, there exists $R \geqq 0$ such that for every $y \in F(x)$, if $v_{p_i(x)}(y) > R$ for all $i \in [1, k]$, then $\|y\| < \epsilon$.

Let $\epsilon > 0$. As the mapping $(y, z) \to yz$ is continuous at $(1, 0)$, there exists $\delta, \gamma > 0$ such that $(B_\delta + 1)B_\gamma \subseteq B_1$. By 2, there exists k_0 such that $\|h(x)^{k_0}\| < \gamma$. Then $(B_\delta + 1) \subseteq h(x)^{-k_0}B_1$ and so $h(x)^{-k_0}B_1$ is a neighborhood of 1. As the topology $\mathcal{T}$ is given by a norm, $y \to y^{-1}$ is continuous for $\mathcal{T}$ on the set of nonzero elements of $F(x)$ (the proof is the same as for normed algebras found in **[3]**, p. 75, Proposition 13), so there exists η such that $0 < \eta < 1$ and $(B_\eta + 1)^{-1} \subseteq h(x)^{-k_0}B_1$. Since the topology induced on $F[x]$ by $\mathcal{T}$ is weaker than that induced on $F[x]$ by $\sup_{1\leqq i\leqq k} \mathcal{T}_{p_i(x)}$, there exists N such that $F[x]h(x)^N \subseteq B_\eta$. Then for all $t \in F[X]$,

$$\left\| \frac{1}{t(x)\,h(x)^N + 1} \right\| \leqq \|h(x)^{-k_0}\|.$$

Choose $R \in \mathbf{N}$ such that $\| h(x)^R \| < \epsilon / \| h(x)^{-k_0} \| S$. Suppose $v_{p_i}(y) \geqq R$ for all $i = 1, 2, \cdots, k$. Then

$$y = p_1^R(x) \cdots p_k^R(x)\, r(x) \frac{f(x)}{q(x)} = h^R(x) r(x) \frac{f(x)}{q(x)}$$

where $r, f, q \in F[X]$, and in $F[X]$, $p_i \nmid f$, $p_i \nmid q$, $i = 1, 2, \cdots, k$. Thus h^N and q are relatively prime, so there exists polynomials $s, t \in F[X]$ such that $q(x)s(x) = h^N(x)t(x) + 1$.
Thus,

$$\begin{aligned} y &= h^R(x) r(x) \frac{f(x)}{q(x)} \\ &= h^R(x) \frac{r(x)\, s(x) f(x)}{h^N(x)\, t(x) + 1} \end{aligned}$$

and therefore

$$\begin{aligned} \| y \| &\leqq \| h^R(x) \| \, \| r(x)\, s(x) f(x) \| \, \| (h^N(x)\, t(x) + 1)^{-1} \| \\ &< \frac{\epsilon}{\| h(x)^{-k_0} \| S} S \| h(x)^{-k_0} \| = \epsilon. \end{aligned}$$

To complete the proof of the theorem it remains to show that $\sup_{1 \leqq i \leqq k} \mathcal{T}_{p_i(x)} \subseteq \mathcal{T}$. For this, as both $\mathcal{T}$ and $\sup_{1 \leqq i \leqq k} \mathcal{T}_{p_i(x)}$ are locally bounded topologies, it suffices to show that $B_1 \subseteq \{ y \in F(x) \colon v_{p_i(x)}(y) \geqq 1, 1 \leqq i \leqq k \}$. Let $y \in B_1$ and let $y = p_1(x)^{e_1} \cdots p_k(x)^{e_k} f(x)/q(x)$ where $e_i \in \mathbf{Z}$, $f, q \in F[X]$, and $p_i \nmid f$, $p_i \nmid q$ for all $i \in [1, k]$, and f and q are relatively prime. Let $I = \{ i \in [1, k] \colon e_i \leqq 0 \}$. Then $yq(x) \Pi_{i \in I} p_i(x)^{-e_i}$ is a topological nilpotent in $\mathcal{T}$ as $F[x]$ is bounded. But

$$yq(x) \prod_{i \in I} p_i(x)^{-e_i} = f(x) \prod_{i \notin I} p_i(x)^{e_i} \in F[x],$$

so $f \Pi_{i \notin I} p_i^{e_i} \in J = (h)$. Hence as $h = p_1 \cdots p_k$, $[1, k] \sim I = [1, k]$, i.e., $I = \phi$. So $e_i \geqq 1$ for all $i \in [1, k]$, and thus $v_{p_i(x)}(y) \geqq 1$ for all $i \in [1, k]$.

COROLLARY. *Let $\mathcal{T}$ be a Hausdorff ring topology on $F(x)$ for which the subfield F is bounded and let p be a prime polynomial in $F[X]$. Then $\mathcal{T} = \mathcal{T}_{p(x)}$ if and only if $\mathcal{T}$ is locally bounded and* $\lim_{n \to \infty} p^n(x) = 0$.

THEOREM 2. *Let F be a field, x a transcendental element over F. Let $\mathcal{T}$ be a Hausdorff, locally bounded ring topology on $F(x)$ for which the subfield F is bounded and for which there is a nonzero topological* *nilpotent y. Then there exists a finite subset S of* $\mathcal{P}'$ *such that* $\mathcal{T} = \sup_{s\in S}\mathcal{T}_s$.

Proof. As each valuation v on $F(x)$ that is improper on F is equivalent to v_s for exactly one $s \in \mathcal{P}'$ (**[1]**, p. 94, Corollary 2) it suffices to show that $\mathcal{T}$ is the supremum of finitely many valuation topologies where each valuation is improper on F.

Let $y = g(x)/h(x) \neq 0$ where $(g(x), h(x)) = 1$. By Cohn's Theorem, there is a norm $\|\cdot\cdot\|$ defining the topology on $F(x)$.

Case 1. $\deg h < \deg g$. Let $n = \deg h$, $n + r = \deg g$. We shall show that

$$F[x] \subseteq F[y] + F[y]x + \cdots + F[y]x^{n+r-1},$$

that is for each $f \in F[X]$ there exists $Q \in F[y][X]$ of degree $< n + r$ such that $f(x) = Q(x)$. For this it clearly sufficies to show that for each $k \geqq 0$, $x^{n+r+k} = Q_k(x)$ for some $Q_k \in F[Y][X]$ of degree $< n + r$.

Let $g(x) = a_{n+r}x^{n+r} + \cdots + a_1x + a_1x + a_0$, where each $a_i \in F$, and let $h(x) = b_nx^n + \cdots + b_1x + b_0$, where each $b_i \in F$. Then $yb_nx^n + yb_{n-1}x^{n-1} + \cdots + yb_0 = a_{n+r}x^{n+r} + \cdots + a_1x + a_0$. So $x^{n+r} = Q_0(x)$ where

$$Q_0(X) = \sum_{j=0}^{n} a_{n+r}^{-1}(b_jy - a_j)X^j - \sum_{j=n+1}^{n+r-1} a_{n+r}^{-1}b_jX^j.$$

Assume $x^{n+r+k} = Q_k(x)$ where Q_k is a polynomial of degree $\leqq n + r - 1$ over $F[y]$; let $Q_k = g_0(y)x^{n+r-1} + P_k$ where $g_0 \in F[X]$ and P_k is a polynomial in $F[y][X]$ of degree $\leqq n + r - 2$. Then

$$\begin{aligned} x^{n+r+k+1} &= g_0(y)\,x^{n+r} + x\,P_k(x) \\ &= g_0(y)\,Q_0(x) + x\,P_k(x) = Q_{k+1}(x) \end{aligned}$$

where $Q_{k+1}(X) = g_0(y)\,Q_0(X) + XP_k(X)$, a polynomial over $F[y]$ of degree $\leqq n + r - 1$. Therefore

$$F[x] \subseteq F[y] + F[y]x + \cdots + F[y]x^{n+r-1}.$$

By 1 of Theorem 1, applied to $F(y)$ with its induced nondiscrete topology which is given by the induced norm, $F[y]$ is bounded in norm, and hence $F[x]$ is also. Thus $F[x]$ is a bounded subset of $F(x)$. Consequently, $h(x)y$ is a topological nilpotent; but $h(x)y = g(x)$, so $g(x)$ is a topological nilpotent. Then by Theorem 1, $\mathcal{T}$ is the supremum of p_i-adic topologies for some finite sequence of primes in $F[x]$.

Case 2. $\deg h = \deg g$.

Choose N such that

$$\left\|\left(\frac{g(x)}{h(x)}\right)^N\right\| < \frac{1}{\|x\|}. \quad \text{Then} \quad x\left(\frac{g(x)}{h(x)}\right)^N$$

is a topological nilpotent. Since $\deg g = \deg h$, $\deg(Xg^N) > \deg h^N$. By Case 1, $\mathcal{T} = \sup_{1\leqq i\leqq k} \mathcal{T}_{p_i}$ for some sequence $p_1, \cdots, p_k$ of primes in $F[x]$.

Case 3. $\deg g < \deg h$ and there exists $a_0 \in F$ such that $X - a_0 \nmid g$. Since the substitution mapping $f \to f(x - a_0)$ is an automorphism of $F[x]$, let $g(x) = g_1(x - a_0)$, $h(x) = h_1(x - a_0)$ where $g_1, h_1 \in F[X]$. Then $\deg g_1 < \deg h_1$ and as $X - a_0 \nmid g$, $X \nmid g_1$. Let $g_1 = C_nX^n + \cdots + C_0$, $h_1 = b_{n+r}X^{n+r} + \cdots + b_0$. Then $C_0 \neq 0$ as $X \nmid g_1$. Hence

$$\begin{aligned}\frac{g(x)}{h(x)} &= \frac{C_n(x-a_0)^n + \cdots + C_0}{b_{n+r}(x-a_0)^{n+r} + \cdots + b_0}\\ &= \frac{(x-a_0)^{n+r}}{(x-a_0)^{n+r}} \frac{C_n(x-a_0)^{-r} + \cdots + C_0(x-a_0)^{-(n+r)}}{b_{n+r} + \cdots + b_0(x-a_0)^{-(n+r)}}\\ &= \frac{g_0((x-a_0)^{-1})}{h_0((x-a_0)^{-1})}\end{aligned}$$

where $g_0 = C_0X^{n+r} + \cdots + C_nX^r$, a polynomial of degree $n + r$ as $C_0 \neq 0$, and $h_0 = b_0X^{n+r} + \cdots + b_{n+r}$, a polynomial of degree $\leqq n + r$. Let $z = (x - a_0)^{-1}$. Then $F(z) = F(x)$ and $g_0(z)/h_0(x)$ is a topological nilpotent. By Cases 1 and 2, $\mathcal{T} = \sup_{1\leqq i\leqq k} \mathcal{T}_{p_i}$ for some sequence of primes in $F[(x - a_0)^{-1}]$.

Case 4. $\deg g < \deg h$ and for all $a \in F$, $X - a \mid g$.

In this case F is a finite field; let p be the characteristic of F and let q be its order.

By the corollary to Theorem 1 applied to the prime polynomial Y of $F[Y]$, the topology induced on $F(y)$ is the y-adic topology. Moreover, $[F(x): F(y)] \leq \deg h$.

Let K be a maximal subfield of $F(x)$ such that K contains $F(y)$ and the topology induced on K by $\mathcal{T}$ is the supremum of finitely many valuation topologies, each of which is discrete on F. Let $K_0 = \{z \in F(x)$: z is separable over $K\}$. Then by Theorem 5 of (**[5]**), the topology induced on K_0 is the supremum of finitely many valuation topologies. Hence $K = K_0$, that is $F(x)$ is a purely inseparable extension of K. Let n be such that $x^{p^n} \in K$.

Let $v_1, \cdots, v_r$ be valuations on K such that $\mathcal{T}|_K = \sup_{1 \leqq i \leqq r} \mathcal{T}_{v_i}$. Since every valuation on K admits an extension to $F(x)$ (**[1]**, p. 105, Proposition 5), we may assume each $v_i = v_{s_i}|_K$ for some $s_i \in \mathcal{P}'$.

By Theorem 4 of (**[5]**) there exist ring topologies $\mathcal{T}_1, \cdots, \mathcal{T}_r$ on $F(x) = K[x]$ such that $\mathcal{T}_i|_K = \mathcal{T}_{v_i}$ and $\mathcal{T} = \sup_{1 \leqq i \leqq r} \mathcal{T}_i$. Furthermore, as $\mathcal{T}$ is a locally bounded topology, by the proof of Theorem 4 of (**[5]**), each $\mathcal{T}_i$ is locally bounded. Therefore it suffices to show that for $1 \leqq i \leqq r$, $\mathcal{T}_i$ is the supremum of finitely many valuation topologies, each of which is discrete on F.

Let $1 \leqq i \leqq r$. Suppose that $v_i = v_s|_K$ where s is a prime polynomial of $F[X]$. As $g(x)/h(x)$ is a topological nilpotent for the given topology it is also a topological nilpotent for the weaker topology induced on K by v_i, and hence $v_i(g(x)/h(x)) > 0$. Furthermore by our assumption on v_i, $v_i(x) \geqq 0$. Let m be such that $p^n m > \deg h$. Then $v_i(x^{p^n m} g(x)/h(x)) > 0$, so $x^{p^n m} g(x)/h(x)$ is a nonzero topological nilpotent for $\mathcal{T}_{v_i}$ and hence for $\mathcal{T}_i$. Furthermore, $\deg X^{p^n m} g > \deg h$. As F is a finite field, F is a bounded set for $\mathcal{T}_i$ and therefore by Case 1, $\mathcal{T}_i$ is the supremum of finitely many valuation topologies on $F(x)$, each of which is discrete on F.

So we may assume that $v_i = v_\infty|_K$. Let t be the highest power of X occuring in the factorization of the polynomial g. Choose m such that $mp^n > t$. Let g_0, h_0 be relatively prime polynomials such that $g_0/h_0 = g/h\ 1/X^{p^n m}$. Since $v_\infty(g(x)/h(x)) > 0$ and $v_\infty(1/X^{p^n m}) > 0$, $g_0(x)/h_0(x)$ is a nonzero topological nilpotent for $\mathcal{T}_{v_i}$ and hence for $\mathcal{T}_i$. Moreover, $X \nmid g_0$. Therefore by Case 3, $\mathcal{T}_i$ is the supremum of finitely many valuation topologies, each of which is discrete on F. This completes the proof of the theorem.

COROLLARY. *Let F be a field, x a transcendental element over F. Let $\mathcal{T}$ be a Hausdorff locally bounded ring topology on $F(x)$ for which the subfield F is bounded and for which there is a nonzero topological nilpotent. Assume further that the completion of $F(x)$ for $\mathcal{T}$ is a field. Then $\mathcal{T}$ is either the p-adic topology for some prime p in $F[x]$ or $\mathcal{T}$ is $\mathcal{T}_\infty$.*

Proof. By Theorem 2, there exists a nonempty finite subset S of $\mathcal{P}'$ such that $\mathcal{T} = \sup_{s \in S} \mathcal{T}_s$. As the completion of $F(x)$ for $\mathcal{T}$ is a field, the cardinality of S is 1 by the Approximation Theorem (**[1]**, p. 136, Theorem 2).

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Received October 29, 1976 and in revised form January 7, 1977 The results presented here are part of a doctoral dissertation, written under the supervision of Seth Warner of Duke University.

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