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ON THE RADON-NIKODYM PROPERTY IN A CLASS OF LOCALLY CONVEX SPACES

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In an earlier paper we studied the Radon-Nikodym property (RNP) for Fréchet spaces. D. Gilliam continued the study by examining the RNP for locally convex spaces with the strict Mackey convergence property. The aim of this paper is to take one more step by studying the RNP for the class of locally convex spaces in which every bounded subset is metrizable. Although this class strictly includes the class of spaces with the strict Mackey convergence property, our goal is not a generalization for the sake of generalization. Indeed, we shall prove a theorem that reduces the study of the RNP for this class of spaces directly to the study of the RNP for Banach spaces. This will provide a quick and simultaneous extension of many of the basic Radon-Nikodym theorems in Banach spaces to this class of locally convex spaces. We hope that our technique will eliminate some of the mystery that seems to surround the RNP for locally convex spaces.

1. Definitions and preliminaries. Throughout this paper (E,τ) will always be a quasi-complete locally convex Hausdorff space in which every bounded subset is metrizable and τ will denote its topology.

Let (T,Σ,P) be a probability space and $m:\Sigma\to E$ be a vector measure. For every continuous semi-norm q on E, the q-variation of m over X in Σ is defined to be

$$|m|_q(X)=\sup\left\{\sum_{i=1}^{n} q(m(X_i));\{X_i\}_{i=1}^{n} \text{ disjoint, } X_i\subset X \text{ and } X_i\in\Sigma \text{ for } 1\leqq i\leqq n\right\}.$$

The function $|m|_q$ is an extended real-valued measure. The vector measure m is said to be of bounded variation if $|m|_q(T)<+\infty$ for every continuous semi-norm q on E. Also m is said to be P-continuous (denoted $m\ll P$) if $m(X)=0$, whenever $P(X)=0$ and $X\in\Sigma$. It is clear that $m\ll P$ if and only if for every continuous semi-norm q on E we have $|m|_q\ll P$. The set

$$\mathrm{Am}(\Sigma)=\left\{\frac{m(X)}{P(X)};X\in\Sigma,P(X)>0\right\}$$

is called the *P-average range of m*.

DEFINITION 1.1. A function $f\colon T \to E$ is said to be P-integrable if and only if there exists a sequence f_n of simple functions such that:

(i) $\lim_n f_n(t) = f(t)$ P-almost everywhere (P.a.e.)

(ii) $\lim_n \int_T q(f_n(t) - f(t))dP = 0$ for every continuous semi-norm q on E.

This definition allows us to define $\int_X fdP = \lim_n \int_X f_n dP$ for each X in Σ, using the fact that E is quasi-complete.

It can be verified that this definition is independent of the choice of the sequence (f_n), and if F is another quasi-complete locally convex Hausdorff space and $U\colon E \to F$ is a continuous linear operator, then $U \circ F$ is also P-integrable and $U\left(\int_X fdP\right) = \int_X U \circ fdP$ for all X in Σ.

We adopted this definition because all the P-integrable functions we will be dealing with take their values in a bounded metrizable set. This definition is equivalent to the one used in **[15]** when the space E is a Fréchet space.

DEFINITION 1.2. Let C be a closed bounded convex subset of E. The set C is said to *have the RNP* if for every probability space (T, Σ, P) and every vector measure $m\colon \Sigma \to E$ whose P-average range is contained in C there exists a P-integrable function $f\colon T \to C$ such that $m(X) = \int_X fdP$ for every X in Σ.

If every bounded closed convex subset of E has the RNP, then E is said to have RNP.

Note that in this definition the boundedness of the set C insures that any vector measure whose P-average range is contained in C is of finite variation and is P-continuous.

For each subset B of E, let $\overline{\text{conv}}(B)$ denote the closed convex hull of B and define $s(B)$ to be the set

$$s(B) = \left\{ \sum_{n=1}^{\infty} \lambda_n b_n ; \lambda_n > 0, \sum_{n=1}^{\infty} \lambda_n = 1, (b_n) \subset B \text{ and } \sum_{n=1}^{\infty} \lambda_n b_n \text{ converges} \right\}.$$

The set $s(B)$ is called the s-convex hull of B. It is clear that $B \subset s(B) \subseteq \overline{\text{conv}}(B)$.

DEFINITIONS 1.3. A subset B of E is said to be *dentable* (*s-dentable*) if and only if for every zero-neighborhood V in E there exists $b \in B$ such that $b \notin \overline{\text{conv}}(B \backslash (b + V))$ ($b \notin s(B \backslash (b + V))$).

A set B is said to be *subset dentable* (*subset s-dentable*) if every subset of B is dentable (*s-dentable*).

If A is a bounded subset of E, a *slice* of A is a subset of A defined by

$$S(f, r, A) = \{x \in A\,; f(x) \geqq \sup_A f - r\}$$

where f is in $E^*, f \neq 0$ and $r > 0$.

DEFINITION 1.4. A point x in A is said to be *denting* if for every zero-neighborhood V in E $x \notin \overline{\text{conv}}(A \backslash (x + V))$.

DEFINITION 1.5. A point x in A is said to be *exposed* if there exists $f \in E^*$ such that $f(x) = \sup_A f$ and $f(z) < f(x)$ for all $z \in A, z \neq x$.

DEFINITION 1.6. A point x in A is said to be *strongly exposed* if there exists $f \in E^*$ such that for every zero-neighborhood V there exists $r > 0$ such that $x \in S(f, r, A) = S$ and $S - S \subset V$.

Before proving the main theorem, we are going to give some examples of locally convex spaces in which every bounded subset is metrizable.

Obviously every Fréchet space and every locally convex space with the strict Mackey convergence property **[10]** belong to this class. The space l^1 with its w^*-topology belongs to this class but does not have the strict Mackey convergence property.

It can be shown that this class is sequentially closed under strict inductive limits: in particular every LF-space belongs to this class.

The results of this paper were announced in **[16]**.

2. The space $(\hat{E}_M, N)$: properties and consequences.

Let C be a closed bounded convex subset of (E, τ), let $M = \overline{\text{conv}}(C \cup -C)$ and let $E_M = \bigcup_{n=1}^{\infty} nM$. Then we have the following theorem.

THEOREM 2.1. *There exists a norm N on E_M such that the topology induced by (E_M, N) on M coincides with the topology induced by (E, τ) on M.*

Proof. There exists a sequence V_n of closed absolutely convex zero-neighborhoods in (E, τ) such that

(1) $V_{n+1} + V_{n+1} \subset V_n$ for every $n \geqq 1$.

(2) $\{V_n \cap (M - M)\}_{n \geqq 1}$ forms a fundamental system of zero-neighborhoods in $(M - M, \tau)$.

Let τ_1 be the topology on E that has $\{V_n\}_{n \geqq 1}$ as a fundamental system of zero-neighborhoods. The topology τ_1 is not in general Hausdorff but the restriction of τ_1 on E_M is Hausdorff and τ_1 induces on M the same topology as τ. To see this, let $x \in M$ and let V be a τ-zero-neighborhood in E; it is enough to show that $(x+V) \cap N$ contains $(x+V_n) \cap M$ for some n.

To this end, note that there exists n such that

$$V_n \cap (M-M) \subset V \cap (M-M).$$

Let $y \in (x+V_n) \cap M$, then one has $y - x \in V_n \cap (M-M)$, accordingly $y - x \in V \cap (M-M)$; hence $y \in (x+V) \cap M$.

Thus $(x+V_n) \cap M \subset (x+V) \cap M$. This proves that the restriction of τ to M is coarser than the restriction of τ_1 to M. On the other hand, it is clear that τ restricted to M is finer than τ_1 restricted to M. Thus τ and τ_1 agree on M.

We now turn to the construction of the norm N. Since M is bounded, for every n there exists $a_n \geqq 1$ such that $M \subset a_n V_n$. Let p_n be the gauge functional of V_n. For every $x \in E_M$ define

$$N(x) = \sum_{n=1}^{\infty} \frac{1}{a_n 2^n} p_n(x).$$

It is clear that $N(x) < +\infty$ for every $x \in E_M$. If $N(x) = 0$ then $p_n(x) = 0$ for every n, this implies that $x = 0$ because τ_1 is Hausdorff on E_M. It follows that N is a norm on E_M, let τ_2 be the topology defined by N on E_M.

To complete the proof it is enough to show that τ_2 restricted to M is the same as τ_1 restricted to M. Evidently τ_1 restricted to M is coarser than τ_2 restricted to M. Conversely let $x \in M$, let

$$B_N(x, \epsilon) = \{y \in M; N(x-y) \leqq \epsilon\}$$

and let

$$B_k(x, \epsilon) = \{y \in M; p_k(x-y) \leqq \epsilon\}.$$

It is enough to prove that:

$$B_k\left(x, \frac{1}{2^k}\right) \subset B_N\left(x, \frac{3}{2^k}\right).$$

To this end, let $y \in B_k(x, 1/2^k)$. Note that $p_1(x-y) \leqq p_2(x-y) \leqq \cdots \leqq p_k(x-y) \leqq 1/2^k$. From this we obtain

$$
\begin{aligned}
N(x-y) &= \sum_{n=1}^{k} \frac{1}{a_n 2^n} p_n(x-y) + \sum_{n=k+1}^{\infty} \frac{1}{a_n 2^n} p_n(x-y) \\
&\leqq \frac{1}{2^k} \sum_{n=1}^{k} \frac{1}{2^n} + \sum_{n=k+1}^{\infty} \frac{2a_n}{a_n 2^n} \\
&= \frac{1}{2^k} \sum_{n=1}^{k} \frac{1}{2^n} + \frac{1}{2^k} \sum_{n=0}^{\infty} \frac{1}{2^n} \\
&\leqq \frac{1}{2^k}[1+2] = \frac{3}{2^k}.
\end{aligned}
$$

One can also easily check that the uniform structure induced by N on M coincides with the uniform structure induced by τ on M and consequently M is complete in (E_M, N) because it is complete in (E, τ). Let $(\hat{E}_M, N)$ be the completion of (E_M, N).

As a corollary of Theorem 2.1 we have:

COROLLARY 2.2. *Let C and M be as in Theorem* 2.1. *Then*:

(i) *The set C is dentable (s-dentable) in (E, τ) if and only if C is dentable (s-dentable) in $(\hat{E}_M, N)$.*

(ii) *A point $x \in C$ is denting in (E, τ) if and only if x is denting in $(\hat{E}_M, N)$.*

Before establishing the relations between dentability, s-dentability and the Radon–Nikodym property we need the following theorem.

THEOREM 2.3. *Let (T, Σ, P) be a probability space and let C and M be as above. Then*:

A function $f: T \to C$ is P-integrable in $(\hat{E}_M, N)$ if and only if f is P-integrable in (E, τ).

In this case $\int_X fdP$ in $(\hat{E}_M, N)$ is the same as $\int_X fdP$ in (E, τ) for every X in Σ.

Proof. Suppose that f is P-integrable in $(\hat{E}_M, N)$, then there exists a sequence $f_n: T \to C$ of simple functions such that

(i) $\lim_n N(f_n(t) - f(t)) = 0$ P.a.e. and

(ii) $\lim_n \int_T N(f_n(t) - f(t))dP = 0$.

By Theorem 2.1 $f_n(t) \to f(t)$ P.a.e. in (E, τ). Thus to complete the proof we must show that $\lim_n \int_T q(f_n(t) - f(t))dP = 0$ for every continuous seminorm q on (E, τ). For note that although the injection $(E_M, N) \to (E, \tau)$ is not necessarily continuous, its restriction to M is continuous by Theorem 2.1. Consider the sequence $h_n(t) = q(f_n(t) - f(t))$. This sequence is a real valued sequence of uniformly bounded integrable functions which tends to zero P.a.e. By an appeal to the bounded convergence theorem, we have $\lim_n \int_T q(f_n(t) - f(t))dP = 0$.

Conversely, suppose that $f\colon T \to C$ is P-integrable in (E, τ). Consider the sequence p_n which defines the topology τ_1 on E_M (see Theorem 2.1) with the help of ([**9**], p. 241), choose for every $n \geqq 1$ a sequence $(\phi_k^n)_{k \geqq 1}$ of simple functions from T to C such that

$$\lim_k p_n(\phi_k^n(t) - f(t)) = 0 \ \ P\text{.a.e.}$$

By the bounded convergence theorem, we have

$$\lim_k \int_T p_n(\phi_k^n(t) - f(t))dP = 0$$

for every $n \geqq 1$.

By ([**9**], p. 254), one can find a sequence $f_k\colon T \to C$ of simple functions such that $\lim_k \int_T p_n(f_k(t) - f(t))dP = 0$ for every $n \geqq 1$. Now use the diagonal process to choose a sequence $g_n\colon T \to C$ of simple functions that converges to f P.a.e. for the topology τ_1. This proves that g_n converges to f P.a.e. in $(\hat{E}_M, N)$, and thus proves that f is P-measurable in $(\hat{E}_M, N)$. Since f is bounded in $(\hat{E}_M, N)$ this proves that f is P-integrable in $(\hat{E}_M, N)$.

COROLLARY 2.4. *Let C and M be as above. Then C has the RNP in (E, τ) if and only if C has the RNP in (E, N).*

Now Corollary 2.2 and Corollary 2.4 together with results of Rieffel [**14**], Maynard [**12**], Davis–Phelps [**4**] and Huff [**11**] (see [**5**] and [**6**]) for Banach spaces prove the following result:

THEOREM 2.5. *Let C be a closed bounded convex subset of E, then the following assertions are equivalent*:

(i) *The set C has the RNP.*

(ii) *The set C is subset dentable.*

(iii) *The set C is subset s-dentable.*

We now pass to the discussion of the existence of denting points in a closed bounded convex subset of E.

Phelps **[13]** showed that if F is a Banach space such that every subset of F is dentable then every closed bounded convex subset of F is the closed convex hull of its strongly exposed points. Phelps's argument is global in nature and does not seem to give local information about subset dentable closed bounded convex sets in arbitrary Banach spaces. J. Johnson and J. Bourgain have independently shown that the following theorem is a consequence of a recent paper of Bourgain **[1]**.

THEOREM 2.6. *Let F be a Banach space and C be a closed convex bounded subset of E having the RNP then C is the closed convex hull of its strongly exposed points.*

Now using this theorem together with Corollary 2.2 and Corollary 2.4 we can prove the following theorem.

THEOREM 2.7. *Let C be a closed bounded convex subset of (E, τ). Then the following assertions are equivalent:*

(i) *The set C has the RNP.*

(ii) *Every closed convex subset of C is the closed convex hull of its denting points.*

Proof. (i) $\Rightarrow$ (ii) Let $M = \overline{\text{conv}}(C \cup -C)$ and consider $M \subset (\hat{E}_N, N)$. Let C_1 be a closed convex subset of C. Then C_1 has the RNP in (E, τ) and therefore C_1 has the RNP in $(\hat{E}_M, N)$. By Theorem 2.6, C_1 is the closed convex hull of its strongly exposed points in $(\hat{E}_M, N)$ and in particular of its denting points in $(\hat{E}_M, N)$. An appeal to Theorem 2.1 and Corollary 2.2 finishes the proof.

The other implication is immediate from the definitions and Theorem 2.5.

It is natural to ask whether one can replace denting points by strongly exposed points in Theorem 2.7.

The answer is no. Consider

$$C = [-1, 1]^N.$$

The set C is a convex compact set in the Fréchet space F, but from the fact that F^* consists of the finitely nonzero sequences, it is easily seen that C does not even have any exposed points.

3. The Radon–Nikodym theorem, Dunford–Pettis–Phillips theorem, Liaponouv–Uhl's theorem and Edgar's theorem. Now we will use the well known results in Banach spaces and what we did before to deduce the following Radon-Nikodym theorem. Before doing this let us recall one definition.

DEFINITION 3.1. Let (T, Σ, P) be a probability space and $m : \Sigma \to E$ be a vector measure. The measure m is said to have a locally relatively compact (relatively weakly compact,...) P-average range if and only if for every $\epsilon > 0$ there exists $T_\epsilon \subset T$ such that $P(T \backslash T_\epsilon) \leqq \epsilon$ and the set

$$\left\{\frac{m(X)}{P(X)};\ X \in \Sigma, X \subset T_\epsilon, P(X) > 0\right\}$$

is relatively compact (relatively weakly compact,...).

THEOREM 3.2. *Let (T, Σ, P) be a probability space and $m : \Sigma \to (E, \tau)$ be a vector measure with bounded P-average range then the following assertions are equivalent*:

(i) *The measure m has a locally relatively compact P-average range.*

(ii) *The measure m has a locally relatively weakly compact P-average range.*

(iii) *The measure m has a locally dentable P-average range.*

(iv) *The measure m has a locally s-dentable P-average range.*

(v) *There exists $f: T \to E$ P-integrable such that $m(X) = \int_X f dP$ for every $X \in \Sigma$.*

Proof. We reduce the proof to the case of Banach spaces by considering

$$M = \overline{\text{conv}}(\text{Am}(\Sigma) U - \text{Am}(\Sigma))$$

and everything can be studied inside M considered as a subset of the Banach space $(\hat{E}_M, N)$. With this in mind apply Theorem 2.1, Corollary 2.2, Theorem 2.3 and the results in Banach spaces **[6]** to complete the proof.

Before proving a theorem of Dunford–Pettis–Phillips type we need the following proposition which can be proved using Smulian's theorem, ([7] p. 433) and Theorem 2.1.

PROPOSITION 3.3. *Let C and M be as in the Theorem* 2.1. *Then C is weakly compact in (E, τ) if and only if C is weakly compact in $(\hat{E}_M, N)$.*

The following result shows that the Dunford–Pettis–Phillips theorem is valid in the class of spaces E under consideration in this paper.

PROPOSITION 3.4. *For every weakly compact operator $W: L^1[0,1] \to (E, \tau)$ there exists $g: [0,1] \to (E, \tau)$ λ-integrable (λ. the*

Lebesgue measure of $[0,1]$) *such that* $W(f)=\int_0^1 fgd\lambda$ *for every* f *in* $L^1[0,1]$, *and in particular* W *sends weakly relatively compact sets into relatively* τ*-compact sets.*

Proof. Let M be the τ-closure of the image of the unit ball of $L^1[0,1]$ by W, now M is weakly compact in (E,τ) and therefore it is weakly compact in $(\hat{E}_M, N)$ by Proposition 3.4. By the Dunford–Pettis–Phillips theorem there exists $g:[0,1]\to M$ λ-Bochner integrable in $(\hat{E}_M, N)$ such that $W(f)=\int_0^1 fgd\lambda$ for every f in $L^1[0,1]$. It is easy to see that the function $t\to f(t)g(t)$ is λ-integrable from $[0,1]\to(E,\tau)$ and $W(f)=\int_0^1 fgd\lambda$ in (E,τ).

The following theorem was proven by Uhl **[18]** in the case of Banach space. It is a Liapounov type theorem.

THEOREM 3.5. *Let* E *have the RNP and let* $m:\Sigma\to E$ *be a non atomic vector measure with bounded* P*-average range, then the closure of the range of* m *is convex and compact.*

Proof. Let $M=\overline{\text{conv}}(\text{Am}(\Sigma)U-\text{Am}(\Sigma))$. As usual we consider M as a subset of $(\hat{E}_M, N)$. Note that $m:\Sigma\to M$ is a vector measure when M is considered as a subset of $(\hat{E}_M, N)$. Since M has the RNP in (E,τ), then it has the RNP in $(\hat{E}_M, N)$ by the Corollary 2.4. Therefore there exists $f: T\to M$ P-integrable such that $m(X)=\int_X fdP$ for every X in Σ. As in Uhl **[18]** we obtain that the closure of $m(\Sigma)$ is convex and compact in $(\hat{E}_M, N)$. But this closure is a subset of M. Thus it is also compact in (E,τ) by Theorem 2.1.

In **[8]** Edgar established a representation theorem of Choquet type **[3]** for a bounded convex separable subset C of a Banach space when C has the RNP. We are going to show that Edgar's theorem is also valid in the locally convex spaces under consideration.

We refer the reader to **[8]**, for the notations and terminology used in the sequel.

THEOREM 3.6. *Let* C *be a bounded closed convex separable subset of* (E,τ) *having the RNP. Then for every* $a\in C$ *there exists a probability measure* μ *on the universally measurable subsets of* C *such that* $\mu(\text{Ext}(C))=1$ *and* $\int_C xd\mu=a$ *in* (E,τ) ($\text{Ext}(C)$ *is the set of extreme points of* C).

Proof. Let $M = \overline{\text{conv}}(C \cup -C)$ and consider C in $(\hat{E}_M, N)$. Since C has the RNP in (E, τ), the set C has the RNP in $(\hat{E}_M, N)$. Now by Edgar's theorem there exists a probability measure μ defined on the universally measurable subsets of C such that $\mu(\text{Ext}(C)) = 1$ and $\int_C x d\mu = a$ in $(\hat{E}_M, N)$. Therefore by Theorem 2.3 $\int_C x d\mu = a$ in (E, τ).

The uniqueness theorem (see **[17]** and **[2]**) can also be deduced using the space $(\hat{E}_M, N)$ to obtain:

THEOREM 3.7. *Under the same hypothesis as the above theorem: the following assertions are equivalent:*

(i) *The set C is a simplex.*

(ii) *For every $a \in C$ there exists a unique probability measure μ on the universally measurable subsets of C, such that $\int_C x d\mu = a$ and* $\mu(\text{Ext}(C)) = 1$.

We finish by asking the following:

Problem. Let F be a locally convex Hausdorff space and let C be a bounded closed convex metrizable subset of F, is $M = \overline{\text{conv}}(C \cup -C)$ metrizable?

If the answer is yes, then Theorem 2.1 and consequently Theorem 2.5, Theorem 2.7 , Theorem 3.6 and Theorem 3.7 will be true if we suppose only that C is a metrizable subset of an arbitrary quasi-complete locally convex Hausdorff space.

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