PACIFIC JOURNAL OF MATHEMATICS
Vol. 94, No. 2, 1981

ON THE WEIERSTRASS POINTS ON OPEN RIEMANN SURFACES

YUKIO HIRASHITA

The number of Weierstrass points on a compact Riemann surface of finite genus g is at most $(g-1)g(g+1)$ and at least $2(g+1)$. After the Riemann-Roch's theorem for the class of canonical semi-exact differentials, Watanabe considered the number of Weierstrass points on an open Riemann surface of class O_{KD}. In this paper it will be shown that Watanabe's estimate can be proved without any conception of principal operators.

Using the notation and terminology of [1], the following theorem [6, Theorem 2] will be proved without use of the results of Mori [3], Rodin [4] and Royden [5]. Note that a meromorphic function on an open Riemann surface is said to be rational if $\mathrm{Re}\, df$ is distinguished.

THEOREM. *Suppose that R is a Riemann surface of finite genus g on which $\Gamma_{he} \cap \Gamma^*_{hse} \subset \Gamma^*_{he}$ holds. Then the number of Weierstrass points on R is at most $(g-1)g(g+1)$.*

Let S be a compact continuation of R such that the genus of S is g. Suppose P is a Weierstrass points on R and f is a rational function on R which has the only singularity of order at most g at P. Let D be a closed disk with $P \in \mathring{D} \subset D \subset R$. Then the Dirichlet integral of f over $R - D$ is finite. Since $\int_{\partial D} d\,\mathrm{Re}\, f^* = 0$, there exist harmonic functions u on $S - D$ and v on R such that $u - v = \mathrm{Re}\, f$ on $R - D$. Thus we have $dv \in \Gamma_{he} \cap \Gamma^*_h$.

We wish to show that dv^* is semi-exact on R. If c is a dividing cycle on $R - D$, then c is homologous to zero on $S - D$. This gives that

$$\int_c dv^* = \int_c du^* - \int_c d\,\mathrm{Im}\, f = 0 .$$

Since $dv \in \Gamma_{he} \cap \Gamma^*_{hse}$, it follows from the assumption that $dv^* \in \Gamma_{he}$.

We define

$$\lambda = \begin{cases} du & \text{on } S - D \\ dv + d\,\mathrm{Re}\, f & \text{on } R . \end{cases}$$

Then λ and λ^* have no periods along any cycle b on S, where $b \not\ni P$. Therefore $\int \lambda + i\lambda^*$ is a meromorphic function on S. It is easy to see that $\int \lambda + i\lambda^*$ has as its only singularity a pole of order at most g at P. This shows that P is a Weierstrass point on S. Due to the classical result on the Weierstrass points on compact Riemann surfaces our assertion is obtained.

References

1. L. V. Ahlfors and L. Sario, *Riemann Surfaces*, Princeton, 1960.
2. A. Mori, *A remark on the prolongation of Riemann surface of finite genus*, J. Math. Soc. Japan, **4** (1952), 27-30.
3. M. Mori, *Contributions to the theory of differentials on open Riemann surfaces*, J. Math. Kyoto Univ., **4** (1964), 77-97.
4. B. Rodin, *On a paper of M. Watanabe*, J. Math. Kyoto Univ., **6** (1967), 393-395.
5. H. L.Royden, *On a class of null-bounded Riemann surfaces*, Comm. Math. Helv., **34** (1960), 37-51.
6. M. Watanabe, *A remark on the Weierstrass points on open Riemann surfaces*, J. Math. Kyoto Univ., **5** (1966), 185-192.

Received October 26, 1979 and in revised form March 6, 1980.

CHUKYO UNIVERSITY
101 YAGOTOHONMACHI
SHOWA-KU, NAGOYA
466 JAPAN