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# WORD PROBLEMS FOR FREE OBJECTS IN CERTAIN VARIETIES OF COMPLETELY REGULAR SEMIGROUPS

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**Semigroups which are unions of groups are said to be completely regular. They form a variety when considered as semigroups with an operation of inverse. The variety of bands and the variety of completely simple semigroups are subvarieties. The present paper reduces the word problem for the free semigroups in each subvariety $\mathcal{V}$ of the join of the variety of bands and the variety of completely simple semigroups to the word problem for certain groups in $\mathcal{V}$. In particular if the word problems for the latter have a solution so does the word problem in $\mathcal{V}$.**

Semigroups which are unions of groups are said to be *completely regular*. A completely regular semigroup $S$ is provided in a natural way with a unary operation of inverse by defining $a^{-1}$ for $a \in S$ to be the group inverse of $a$ in the maximal subgroup of $S$ to which $a$ belongs. This operation satisfies the identities

$$xx^{-1}x = x, \tag{1}$$

$$xx^{-1} = x^{-1}x, \tag{2}$$

$$(x^{-1})^{-1} = x. \tag{3}$$

In fact a completely regular semigroup can be defined as a semigroup with a unary operation which satisfies these identities. The class of completely regular semigroups is therefore a variety of (universal) algebras with one unary and one binary operation. This variety is denoted by $\mathcal{CR}$. Two important subvarieties are the variety of bands $\mathcal{B}$ and the variety of completely simple semigroups $\mathcal{CS}$. The join of these two varieties has recently been described in [**4**] and [**9**]. It is the subvariety of $\mathcal{CR}$ defined by the identity

$$(xy)^0x^0(zx)^0 = (xyxzx)^0,$$

where as usual $u^0 = uu^{-1}$.

In [**4**] this variety is called the variety of pseudo orthodox bands of groups. Following the terminology of [**6**] it might be called the variety of pseudo orthocrypto groups. Here we will simply refer to it as $\mathcal{B} \vee \mathcal{CS}$.

The present paper gives a solution to the word problem for the free semigroups in each of the subvarieties of $\mathcal{B} \vee \mathcal{CS}$. A solution for the subvarieties of $\mathcal{B}$ is known (see [**3**]). The free objects in $\mathcal{CS}$ have been described (see [1] and [8]) and an algorithm for the word problem given in [8]. Here we present another solution for the word problem for the free objects of $\mathcal{CS}$. It is given in terms of invariants on words in the free unary semigroup. The subvarieties of $\mathcal{CS}$ are described in [8]. Using this description we can adapt our solution of the word problem for $\mathcal{CS}$ to reduce the word problem for the free objects in the subvarieties $\mathcal{V}$ of $\mathcal{CS}$ to the word problem for certain groups in $\mathcal{V}$. Finally by showing that every subvariety of $\mathcal{B} \vee \mathcal{CS}$ is the join of a subvariety of $\mathcal{B}$ and a subvariety of $\mathcal{CS}$, we reduce the solution of the word problem for free objects in all subvarieties $\mathcal{V}$ of $\mathcal{B} \vee \mathcal{CS}$ to the word problem for certain groups in $\mathcal{V}$.

Section 1 gives preliminary results needed in the paper. In particular, it describes the free unary semigroup on a set, and gives results on the structure and congruences of completely simple (Rees matrix) semigroups. The solution of the word problem for free objects in $\mathcal{CS}$ is given in §2. In §3 we derive the Clifford-Rasin model for the free objects in $\mathcal{CS}$. In §4 we describe the word problem for free objects in the subvarieties of $\mathcal{CS}$ and in §5 for the subvarieties of $\mathcal{B} \vee \mathcal{CS}$.

**1. Preliminaries.** The free unary semigroup on a set $S$ provides a natural setting for the study of word problems in semigroups with an added unary operation (including groups). Let $F$ be the free semigroup on $X \cup \{(\ ,\ )^{-1}\}$ where ( and $)^{-1}$ are treated as two new variables. The free unary semigroup $U(X)$ is just the smallest subset of $F$ satisfying the following properties.

(i) $$X \subseteq U(X),$$

(ii) $$w \in U(X) \text{ implies } (w)^{-1} \in U(X),$$

(iii) $$u, v \in U(X) \text{ implies } uv \in U(X).$$

This description can be thought of as an inductive definition of $U(X)$ and many of our proofs will be by induction on the number of operations occurring in $w$, that is the number of times (ii) or (iii) is used to build $w$ from variables (elements of $X$).

The class of completely simple semigroups is essentially the same as the class of Rees matrix semigroups. A Rees matrix semigroup $\mathcal{M}(I, G, \Lambda; P)$ is given by two index sets $I$, $\Lambda$, a group $G$ and a matrix $P = (p_{\lambda i})$, $i \in I$, $\lambda \in \Lambda$ of elements of $G$. Elements of the semigroup are triples $(i, g, \lambda)$, $i \in I$, $g \in G$, $\lambda \in \Lambda$, and multiplication follows the rule

$$(i, g, \lambda)(j, h, \nu) = (i, gp_{\lambda j}h, \nu).$$

The matrix $P$ can be taken to be *normalized* at $(i, \lambda)$ for any $i \in I$, $\lambda \in \Lambda$ which means that $p_{\mu i} = p_{\lambda j} = 1$, the identity of $G$, for all $\mu \in \Lambda, j \in I$.

Let $M = \mathfrak{M}(I, G, \Lambda; P)$ be a Rees matrix semigroup. Let $H$ be a normal subgroup of $G$, and let $r$ and $\pi$ be equivalence relations on $I$ and $\Lambda$ respectively. The triple $(r, H, \pi)$ is *admissible* if and only if $irj$ (respectively $\lambda\pi\mu$) implies $p_{\lambda i}p_{\mu i}^{-1}p_{\mu j}p_{\lambda j}^{-1} \in H$ for all $\lambda, \mu \in \Lambda$ (respectively for all $i, j \in I$). With each admissible triple there is associated a congruence relation $\theta = \theta(r, H, \pi)$ defined by $(i, f, \lambda)\theta(j, g, \mu)$ if and only if $irj$, $\lambda\pi\mu$ and $p_{\nu i}fp_{\lambda k}H = p_{\nu j}gp_{\mu k}H$ for some $k \in I$, $\nu \in \Lambda$. In fact every congruence on $M$ is the congruence associated with a unique triple. For background on Rees matrix semigroups and, in particular, for a description of their congruences we refer the reader to **[5]** or **[6]**.

**2. The word problem for completely simple semigroups.** Let $X$ be a set (of variables) and let $z \in X$ be a distinguished element. For every $x, y \in X$, let $p_{xy}$ be an element with the properties $p_{xz} = p_{zx} = 1$ for all $x \in X$ and otherwise the $p_{xy}$ are distinct from each other and from the elements of $X$. Let $P$ be the matrix $(p_{xy})$. Let $G$ be the free group on $X \cup \{p_{xy} \mid x, y \in X\}$ where 1 is taken to be the identity of $G$. For $u \in U(X)$, the free unary semigroup on $X$, let $h$: $U(X) \to X$ and $t$: $U(X) \to X$ be defined by taking $h(u)$ to be the first variable in $u$ and $t(u)$ to be the last variable in $u$. Note that $h$ and $t$ can be defined more formally by induction. Let $m$: $U(X) \to G$ be defined by induction on the number of operations occurring in $u$ as follows.

$$m(x) = x \quad \text{for all } x \in X,$$
$$m(uv) = m(u)p_{t(u)h(v)}m(v),$$
$$m\left((u)^{-1}\right) = p_{t(u)h(u)}^{-1}[m(u)]^{-1}p_{t(u)h(u)}^{-1}.$$

It is necessary to show that $m$ is well defined, that is to show that $uv = u'v'$ implies $m(uv) = m(u'v')$. By symmetry we may assume that $u'w = u$ and $wv = v'$ for some $w$. Also by induction we can assume that $m(u)$, $m(v)$, $m(u')$, $m(v')$ and $m(w)$ are defined. Then

$$\begin{aligned} m(uv) &= m(u)p_{t(u)h(v)}m(v) \\ &= m(u')p_{t(u')h(w)}m(w)p_{t(w)h(v)}m(v), \quad \text{since } u = u'w \\ &= m(u')p_{t(u')h(v')}m(v'), \quad \text{since } v' = wv \\ &= m(u'v'). \end{aligned}$$

DEFINITION 2.1. Let $w, w' \in U(X)$. Then $w \sim w'$ if and only if $h(w) = h(w')$, $m(w) = m(w')$ and $t(w) = t(w')$.

THEOREM 2.2. *The semigroup $U(X)/\sim$ is the free completely simple semigroup on the set $X$.*

*Proof.* It is straightforward to check that $\sim$ is an equivalence relation on $U(X)$ and that $w \sim w'$ implies $wu \sim w'u$, $uw \sim uw'$ and $w^{-1} \sim (w')^{-1}$. The relation $\sim$ is therefore a congruence relation on $U(X)$.

To show that the unary semigroup $U(X)/\sim$ is completely simple, we show that it satisfies the defining identities (1)–(3) given in the introduction, plus the identity $x^0 = (xyx)^0$, where $x^0 = xx^{-1}$, which defines $\mathcal{CS}$ as a subvariety of $\mathcal{CR}$. This is of course equivalent to checking that

$$a \sim aa^{-1}a,$$
$$aa^{-1} \sim a^{-1}a,$$
$$a \sim (a^{-1})^{-1},$$
$$aa^{-1} \sim (aba)(aba)^{-1}$$

for all $a, b \in U(X)$.

The following arguments are shorter to write if we let $h(a) = h$ and $t(a) = t$. If $w \sim w'$ is one of the above expressions then $h(w) = h(w') = h$ and $t(w) = t(w') = t$ and it is enough to check that $m(w) = m(w')$, for each of the cases.

$$\begin{aligned} m(aa^{-1}a) &= m(a)p_{th}m(a^{-1})p_{th}m(a) \\ &= m(a)p_{th}p_{th}^{-1}[m(a)]^{-1}p_{th}^{-1}p_{th}m(a) \\ &= m(a). \end{aligned}$$

$$\begin{aligned} m(aa^{-1}) &= m(a)p_{th}m(a^{-1}) \\ &= m(a)p_{th}p_{th}^{-1}[m(a)]^{-1}p_{th}^{-1} \\ &= p_{th}^{-1}. \end{aligned}$$

A similar calculation shows that $m(a^{-1}a) = p_{th}^{-1}$ and so $m(aa^{-1}) = m(a^{-1}a)$.

$$\begin{aligned} m\big((a^{-1})^{-1}\big) &= p_{th}^{-1}[m(a^{-1})]^{-1}p_{th}^{-1} \\ &= p_{th}^{-1}\big[p_{th}^{-1}[m(a)]^{-1}p_{th}^{-1}\big]^{-1}p_{th}^{-1} \\ &= m(a). \end{aligned}$$

$$\begin{aligned} m\big((aba)(aba)^{-1}\big) &= m(aba)p_{th}m\big((aba)^{-1}\big) \\ &= m(aba)p_{th}p_{th}^{-1}[m(aba)]^{-1}p_{th}^{-1} \\ &= p_{th}^{-1} \\ &= m(aa^{-1}) \quad \text{as shown above.} \end{aligned}$$

In order to show that $U(X)/\sim$ is free we show that any homomorphism $\chi\colon U(X) \to M = \mathfrak{M}(I, H, \Lambda; Q)$ can be factored through $U(X)/\sim$. It is in fact enough to show that $w\chi = (h(w)\alpha, m(w)\gamma, t(w)\beta)$ for suitable maps $\alpha\colon X \to I$, $\beta\colon X \to \Lambda$, $\gamma\colon G \to H$ and for all $w \in U(X)$.

Let $x\chi = (x\alpha, x\gamma, x\beta)$. This defines $\alpha$, $\beta$ and $\gamma$ on $X$. Extend $\gamma$ by defining $p_{xy}\gamma = q_{x\beta, y\alpha}$. Then $\gamma$ is defined on the generators of $G$ and so can be extended uniquely to a homomorphism on $G$. Clearly $x\chi = (h(x)\alpha, m(x)\gamma, t(x)\beta)$. The proof that $w\chi = (h(w)\alpha, m(w)\gamma, t(w)\beta)$ for all $w \in U(X)$ is by induction. If $w = uv$ then

$$\begin{aligned} w\chi = (u\chi)(v\chi) &= (h(u)\alpha, m(u)\gamma, t(u)\beta)(h(v)\alpha, m(v)\gamma, t(v)\beta) \\ &= \big(h(w)\alpha, m(u)\gamma q_{t(u)\beta, h(v)\alpha} m(v)\gamma, t(w)\beta\big) \\ &= (h(w)\alpha, m(w)\gamma, t(w)\beta). \end{aligned}$$

If $w = u^{-1}$ then

$$\begin{aligned} w\chi = (u\chi)^{-1} &= (h(u)\alpha, m(u)\gamma, t(u)\beta)^{-1} \\ &= \big(h(w)\alpha, q^{-1}_{t(w)\beta, h(w)\alpha}[m(u)\gamma]^{-1} q^{-1}_{t(w)\beta, h(w)\alpha}, t(w)\beta\big) \\ &= \big(h(w)\alpha, m(u^{-1})\gamma, t(w)\beta\big) \\ &= (h(w)\alpha, m(w)\gamma, t(w)\beta). \end{aligned}$$

**3. The Clifford-Rasin model.** In **[1]** and **[8]** it is shown that the free completely simple semigroup is isomorphic to $M = \mathfrak{M}(X, G, X; P)$ normalized at $z$. A proof of this result can be obtained from our work as follows. Let $\psi\colon U(X) \to M$ be defined by

$$w\psi = (h(w), m(w), t(w)).$$

Clearly $\psi$ is a (unary semigroup) homomorphism with kernel $\sim$. To establish the isomorphism of $U(X)/\sim$ with $M$ it is therefore enough to show that $\psi$ is onto. Let $z^0 = zz^{-1}$.

$$\begin{aligned} (x, g, y) &= (x, 1, z)(z, g, z)(z, 1, y), \\ (x, 1, z) &= \big(x(z^0xz^0)^{-1}\big)\psi, \\ (z, 1, y) &= \big((z^0yz^0)^{-1}y\big)\psi. \end{aligned}$$

It remains to recognize $(z, g, z)$ as an image under $\psi$ for any $g \in G$. Since $G$ is generated by $X$ and $\{p_{xy}\}$, it is enough to show that $(z, x, z)$ and

$(z, p_{xy}, z)$ are images under $\psi$. Indeed,

$$(z, x, z) = (z^0 x z^0)\psi,$$
$$(z, p_{xy}, z) = \left((z^0 x z^0)^{-1} xy (z^0 y z^0)^{-1}\right)\psi.$$

These calculations are essentially those used by Clifford in the proof of ([1], Lemma 7.3).

**4. The word problem for varieties of completely simple semigroups.** As is well known (see [2] page 163) there is a one-to-one correspondence between the subvarieties of a variety and the fully invariant congruences on the free object of the variety on countably many generators. The description of the fully invariant congruences on free completely simple semigroups given in [8] can therefore be interpreted as a description of all subvarieties of $\mathcal{CS}$.

Let $FCS(X) = \mathfrak{M}(X, G, X; P)$ be the free semigroup in $\mathcal{CS}$ on the set $X$ (as described in §3). In order to simplify notation let $[x, y]$ stand for $p_{xy}$. Let $\mathcal{E}$ be the set of all endomorphisms $\omega$ of $G$ for which there exist transformations $\varphi$ and $\psi$ of $X$ such that

$$[x, y]\omega = [z\psi, z\varphi][x\psi, z\varphi]^{-1}[x\psi, y\varphi][z\psi, y\varphi]^{-1}.$$

A normal subgroup $H$ of $G$ is *$\mathcal{E}$-invariant* if $H\omega \subseteq H$ for all $\omega \in \mathcal{E}$.

THEOREM 4.1 (*Theorem 3 of* [8]). *Let $\theta = \theta(r, H, \pi)$ be a congruence on $FCS(X)$. Then $\theta$ is fully invariant if and only if*

(i) *$H$ is $\mathcal{E}$-invariant and*

(ii) *$r, \pi \in \{\nabla, \Delta\}$*

*where $\Delta$ is equality and $\nabla$ is the equivalence which identifies every element of $X$.*

Let $\mathcal{V}$ be a subvariety of $\mathcal{CS}$. Let $\theta = \theta(r, H, \pi)$ be the fully invariant congruence on $FCS(X)$, $X$ countable, such that $FV(X) = FCS(X)/\theta$ is the free semigroup on $X$ in $\mathcal{V}$. Let $m_{\mathcal{V}}\colon U(X) \to G/H$ be defined by

$$m_{\mathcal{V}}(w) = m(w)H.$$

There are four different situations depending on the values of $r$ and $\pi$.

*Case* 1. $r = \pi = \Delta$.

In this case $(i, f, \lambda)\theta(j, g, \mu)$ if and only if $i = j$, $\lambda = \mu$ and $fH = gH$. It follows that

$$(h(w), m(w), t(w)) \to (h(w), m(w)H, t(w))$$

is the unique homomorphism of $FCS(X)$ onto $FV(X)$. Consequently if we define $\sim$ on $U(X)$ by $w \sim w'$ if and only if

$$h(w) = h(w')$$
$$t(w) = t(w') \quad \text{and}$$
$$m_{\mathcal{V}}(w) = m_{\mathcal{V}}(w')$$

it follows that $U(X)/{\sim} \cong FV(X)$ and the invariants $h$, $t$ and $m_{\mathcal{V}}$ give a solution to the word problem for $FV(X)$ in terms of the solution of the word problem for $G/H$.

*Case* 2. $r = \nabla$, $\pi = \Delta$.

In this case $(i, f, \lambda)\theta(j, g, \mu)$ if and only if $\lambda = \mu$ and $p_{\nu i} f p_{\lambda k} H = p_{\nu j} g p_{\lambda k} H$ for some $\nu \in M$, $k \in I$. Since $(r, H, \pi)$ is admissible it follows, in this case, that $p_{\lambda i} p_{\mu i}^{-1} p_{\mu j} p_{\lambda j}^{-1} \in H$ for all $i$, $j$, $\lambda$, $\mu$. Setting $\lambda = i = z$ shows that $p_{\mu j} \in H$ for all $\mu$, $j$. Since $H$ is normal it follows that $(i, f, \lambda)\theta(i, g, \mu)$ if and only if $\lambda = \mu$ and $fH = gH$. Therefore

$$(h(w), m(w), t(w)) \to (m(w)H, t(w))$$

is the unique homomorphism of $FCS(X)$ onto $FV(X)$. Consequently, if we define $\sim$ on $U(X)$ by $w \sim w'$ if and only if

$$t(w) = t(w') \quad \text{and}$$
$$m_{\mathcal{V}}(w) = m_{\mathcal{V}}(w')$$

then $U(X)/{\sim} \cong FV(X)$ and the invariants $t$ and $m_{\mathcal{V}}$ give a solution to the word problem $FV(X)$ in terms of the solution of the word problem for $G/H$.

Since $p_{\mu j} \in H$ for all $\mu$, $j$, it follows that $m_{\mathcal{V}}(w) = wH$ in this case. Thus the invariant $m_{\mathcal{V}}$ is the same as for groups. In fact, each variety $\mathcal{V}$ with $r = \nabla$ and $\pi = \Delta$ is a variety of "right groups", that is a variety in which every semigroup is a product of a group and a right semigroup.

*Case* 3. $r = \Delta$, $\pi = \nabla$.

This is similar to case 2 and each variety is a variety of left groups.

*Case* 4. $r = \pi = \nabla$,

In this case we have a variety of groups.

So far we have referred only to the word problem for semigroups on countably many generators. However, the algorithm can be applied to all the free semigroups without change. If there are only finitely many generators we can add more variables to make up countably many. The words we are interested in will not use any of these new variables. On the other hand, even if there are more than countably many variables, it remains true that each word only involves finitely many variables and the equality or inequality of words can be described with reference to countably many (or even finitely many) variables.

## 5. The word problem for subvarieties of $\mathfrak{B} \vee \mathcal{CS}$.

LEMMA 5.1. *If* $\mathcal{V} \subseteq \mathfrak{B} \vee \mathcal{CS}$, *then* $\mathcal{V} = (\mathcal{V} \cap \mathfrak{B}) \vee (\mathcal{V} \cap \mathcal{CS})$.

*Proof*. It is shown in [4] (Corollary 5.5(ii)) that if $S \in \mathcal{V}$, then $S$ is a subdirect product of a band and a normal band of groups. Consequently $\mathcal{V} = (\mathcal{V} \cap \mathfrak{B}) \vee (\mathcal{V} \cap \mathfrak{N}\mathfrak{B}\mathfrak{G})$, where $\mathfrak{N}\mathfrak{B}\mathfrak{G}$ is the variety of normal bands of groups. Since $\mathcal{V} \cap \mathfrak{N}\mathfrak{B}\mathfrak{G}$ is a subvariety of $\mathfrak{N}\mathfrak{B}\mathfrak{G}$, it follows from [7] (Theorem 4.7) that $\mathcal{V} \cap \mathfrak{N}\mathfrak{B}\mathfrak{G} = (\mathcal{V} \cap \mathfrak{N}\mathfrak{B}\mathfrak{G} \cap \mathfrak{Y}) \vee (\mathcal{V} \cap \mathfrak{N}\mathfrak{B}\mathfrak{G} \cap \mathcal{CS})$ $= (\mathcal{V} \cap \mathfrak{Y}) \vee (\mathcal{V} \cap \mathcal{CS})$ where $\mathfrak{Y}$ is the variety of semilattices. Therefore $\mathcal{V} = (\mathcal{V} \cap \mathfrak{B}) \vee (\mathcal{V} \cap \mathfrak{Y}) \vee (\mathcal{V} \cap \mathcal{CS}) = (\mathcal{V} \cap \mathfrak{B}) \vee (\mathcal{V} \cap \mathcal{CS})$.

This lemma enables us to discuss the word problem in each subvariety $\mathcal{V}$ of $\mathfrak{B} \vee \mathcal{CS}$. It was shown at the end of §4 that we can restrict ourselves to $FV(X)$ with $X$ countable. Let $FV(X) = U(X)/\approx$. The congruence $\approx$ can be thought of as giving the equations which hold in $\mathcal{V}$. As pointed out in [2] (page 47), the Galois connection between varieties and equations shows that if $\mathcal{V} = \mathcal{V}_1 \vee \mathcal{V}_2$ then $\mathfrak{E}q(\mathcal{V}) = \mathfrak{E}q(\mathcal{V}_1) \cap \mathfrak{E}q(\mathcal{V}_2)$ where $\mathfrak{E}q(\mathcal{V})$ is the system of equations holding in $\mathcal{V}$. The congruence $\approx$ therefore is just the intersection of the congruence describing the free object in $\mathcal{V} \cap \mathfrak{B}$ and the congruence describing the free object in $\mathcal{V} \cap \mathcal{CS}$. The latter congruence is described in §4. The former congruence is essentially given in [3]. The solution of the word problem in each subvariety of $\mathfrak{B}$ is given there but in terms of a congruence on $S(X)$, the free semigroup on $X$ and not in terms of a congruence on $U(X)$. Of course $S(X)$ is a homomorphic image of $U(X)$, the homomorphism being given by simply removing all ( and $)^{-1}$ from the expression in $U(X)$. Thus the congruence on $U(X)$ which describes the free object in a subvariety of $\mathfrak{B}$ can be described by saying that two words in $U(X)$ are congruent if after removing all occurrences of ( and $)^{-1}$ they are congruent modulo the congruence on $S(X)$ which describes the subvariety of $\mathfrak{B}$.

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