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A MULTILINEAR GENERATING FUNCTION FOR THE KONHAUSER SETS OF BIORTHOGONAL POLYNOMIALS SUGGESTED BY THE LAGUERRE POLYNOMIALS

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The polynomial sets $\{Y_n^\alpha(x; k)\}$ and $\{Z_n^\alpha(x; k)\}$, discussed by Joseph D. E. Konhauser, are biorthogonal over the interval $(0, \infty)$ with respect to the weight function $x^\alpha e^{-x}$, where $\alpha > -1$ and k is a positive integer. The object of the present note is to develop a fairly elementary method of proving a general multilinear generating function which, upon suitable specializations, yields a number of interesting results including, for example, a multivariable hypergeometric generating function for the multiple sum:

$$(*) \qquad \sum_{n_1,\ldots,n_r=0}^{\infty} (m + n_1 + \cdots + n_r)! Y_{m+n_1+\cdots+n_r}^{\alpha}(x; k) \cdot \prod_{i=1}^{r} \left\{ \frac{Z_{n_i}^{\beta_i}(y_i; s_i) u_i^{n_i}}{(1 + \beta_i)_{s_i n_i}} \right\},$$

involving the Konhauser biorthogonal polynomials; here, by definition,

$$\alpha > -1; \quad \beta_i > -1; \quad k, s_i = 1, 2, 3, \ldots; \qquad \forall\, i \in \{1, \ldots, r\}.$$

1. Introduction. Joseph D. E. Konhauser ([5]; see also [4]) introduced two interesting classes of polynomials: $Y_n^\alpha(x; k)$ a polynomial in x, and $Z_n^\alpha(x; k)$ a polynomial in x^k, $\alpha > -1$ and $k = 1, 2, 3, \ldots$. For $k = 1$, these polynomials reduce to the classical Laguerre polynomials $L_n^{(\alpha)}(x)$, and for $k = 2$ they were encountered earlier by Spencer and Fano [8] in the study of the penetration of gamma rays through matter and were discussed subsequently by Preiser [7]. Also [5, p. 303]

$$(1) \quad \int_0^\infty x^\alpha e^{-x} Y_m^\alpha(x; k) Z_n^\alpha(x; k)\, dx = \frac{\Gamma(kn + \alpha + 1)}{n!} \delta_{mn}, \qquad \forall\, m, n \in \{0, 1, 2, \ldots\},$$

so that the Konhauser polynomial sets $\{Y_n^\alpha(x;\, k)\}$ and $\{Z_n^\alpha(x;\, k)\}$ are *biorthogonal* over the interval $(0, \infty)$ with respect to the weight function $x^\alpha e^{-x}$, where $\alpha > -1$, k is a positive integer, and δ_{mn} is the Kronecker delta.

The following explicit expression for the polynomials $Z_n^\alpha(x;\, k)$ was given by Konhauser [**5**, p. 304, Eq. (5)]:

$$(2) \qquad Z_n^\alpha(x;\, k) = \frac{\Gamma(kn+\alpha+1)}{n!} \sum_{j=0}^{n} (-1)^j \binom{n}{j} \frac{x^{kj}}{\Gamma(kj+\alpha+1)}.$$

Subsequently, Carlitz pointed out that [**2**, p. 427, Eq. (9)]

$$(3) \qquad Y_n^\alpha(x;\, k) = \frac{1}{n!} \sum_{j=0}^{n} \frac{x^j}{j!} \sum_{l=0}^{j} (-1)^l \binom{j}{l} \left(\frac{l+\alpha+1}{k}\right)_n,$$

where $(\lambda)_n = \Gamma(\lambda+n)/\Gamma(\lambda)$.

In a recent paper [**10**] we derived various properties of (for example) the Konhauser biorthogonal polynomials $Y_n^\alpha(x;\, k)$ by suitably specializing those of the Srivastava-Singhal polynomials $G_n^{(\alpha)}(x, h, p, k)$ which are defined by the generalized Rodrigues formula [**14**, p. 75, Eq. (1.3)]

$$(4) \qquad G_n^{(\alpha)}(x, h, p, k) = \frac{x^{-kn-\alpha}\exp(px^h)}{n!} \cdot \left(x^{k+1}D_x\right)^n \left\{x^\alpha \exp(-px^h)\right\}, \qquad D_x = \frac{d}{dx},$$

and given explicitly by [**14**, p. 77, Eq. (2.1)]

$$(5) \qquad G_n^{(\alpha)}(x, h, p, k) = \frac{k^n}{n!} \sum_{j=0}^{n} \frac{(px^h)^j}{j!} \sum_{l=0}^{j} (-1)^l \binom{j}{l} \left(\frac{hl+\alpha}{k}\right)_n,$$

where the parameters α, h, k and p are unrestricted, in general. In fact, by comparing (5) with Carlitz's result (3), we at once deduce the known relationship [**13**, p. 315, Eq. (83)]

$$(6) \qquad Y_n^\alpha(x;\, k) = k^{-n} G_n^{(\alpha+1)}(x, 1, 1, k), \qquad \alpha > -1;\, k = 1, 2, 3, \ldots,$$

which was of fundamental importance in our paper [**10**].

The object of the present note is first to give a rather elementary proof of a general multilinear generating function for the Srivastava-Singhal polynomials $G_n^{(\alpha)}(x, h, p, k)$. We then show how this multilinear generating function can be further generalized and applied to derive a number of interesting results including, for example, a multivariable hypergeometric generating function for the multiple sum (*) involving the product of several Konhauser biorthogonal polynomials. Our main result is contained in the following

THEOREM. *For a bounded multiple sequence* $\{\Lambda(n_1,\ldots,n_r)\}$ *of arbitrary complex numbers, let*

$$(7)\quad \mathscr{H}[n_1,\ldots,n_r;y_1,\ldots,y_r] = \sum_{j_1=0}^{[n_1/m_1]}\cdots\sum_{j_r=0}^{[n_r/m_r]}\frac{(-n_1)_{m_1j_1}}{j_1!}\cdots\frac{(-n_r)_{m_rj_r}}{j_r!}\cdot\Lambda(j_1,\ldots,j_r)\,y_1^{j_1}\cdots y_r^{j_r},$$

where $m_1,\ldots,m_r$ *are positive integers. Also let* Δ_r *be defined by*

$$(8)\qquad \Delta_r = 1-\sum_{i=1}^{r}u_i, \qquad r=1,2,3,\ldots.$$

Then, for every nonnegative integer m,

$$(9)\quad \sum_{n_1,\ldots,n_r=0}^{\infty}(m+n_1+\cdots+n_r)!\,G^{(\alpha)}_{m+n_1+\cdots+n_r}(x,h,p,k)\cdot\mathscr{H}[n_1,\ldots,n_r;y_1,\ldots,y_r]\frac{(u_1/k)^{n_1}}{n_1!}\cdots\frac{(u_r/k)^{n_r}}{n_r!}$$
$$= k^m\exp(px^h)\,\Delta_r^{-m-\alpha/k}\cdot\sum_{n,n_1,\ldots,n_r=0}^{\infty}\left(\frac{hn+\alpha}{k}\right)_{m+m_1n_1+\cdots+m_rn_r}\left(\frac{1}{n!}\right)\Lambda(n_1,\ldots,n_r)\left(-\frac{px^h}{\Delta_r^{h/k}}\right)^n\cdot\prod_{i=1}^{r}\left\{\frac{[(-u_i/\Delta_r)^{m_i}y_i]^{n_i}}{n_i!}\right\},\qquad k\neq 0,$$

provided that the multiple series on the right-hand side of (9) *has a meaning, and*

$$(10)\qquad |u_1+\cdots+u_r|<1.$$

2. Proof of the theorem. For convenience, let $\Omega(u_1,\ldots,u_r)$ denote the left-hand side of (9), and set

$$(11)\qquad N=n_1+\cdots+n_r \quad\text{and}\quad J=m_1j_1+\cdots+m_rj_r.$$

Applying the explicit representation (5) and the definition (7), we find that

$$(12)\qquad \Omega(u_1,\dots,u_r) = k^m \sum_{n_1,\dots,n_r=0}^{\infty} u_1^{n_1}\cdots u_r^{n_r}$$

$$\cdot \sum_{j=0}^{m+N} \frac{(px^h)^j}{j!} \sum_{l=0}^{j} (-1)^l \binom{j}{l} \left(\frac{hl+\alpha}{k}\right)_{m+N}$$

$$\cdot \prod_{i=1}^{r} \left\{ \sum_{j_i=0}^{[n_i/m_i]} \frac{[(-1)^{m_i} y_i]^{j_i}}{j_i!(n_i - m_i j_i)!} \right\} \Lambda(j_1,\dots,j_r)$$

$$= k^m \sum_{j_1,\dots,j_r=0}^{\infty} \Lambda(j_1,\dots,j_r) \prod_{i=1}^{r} \left\{ \frac{[(-u_i)^{m_i} y_i]^{j_i}}{j_i!} \right\}$$

$$\cdot \sum_{n_1,\dots,n_r=0}^{\infty} \frac{u_1^{n_1}}{n_1!} \cdots \frac{u_r^{n_r}}{n_r!} \sum_{j=0}^{m+N+J} \frac{(px^h)^j}{j!}$$

$$\cdot \sum_{l=0}^{j} (-1)^l \binom{j}{l} \left(\frac{hl+\alpha}{k}\right)_{m+N+J}.$$

Now we appeal to the series identity [**9**, p. 4, Eq. (12)]

$$(13)\qquad \sum_{n_1,\dots,n_r=0}^{\infty} f(n_1 + \cdots + n_r) \frac{u_1^{n_1}}{n_1!} \cdots \frac{u_r^{n_r}}{n_r!}$$

$$= \sum_{n=0}^{\infty} f(n) \frac{(u_1 + \cdots + u_r)^n}{n!},$$

and (12) becomes

$$(14)\qquad \Omega(u_1,\dots,u_r) = k^m \sum_{n,j_1,\dots,j_r=0}^{\infty} \frac{(u_1 + \cdots + u_r)^n}{n!}$$

$$\cdot \prod_{i=1}^{r} \left\{ \frac{[(-u_i)^{m_i} y_i]^{j_i}}{j_i!} \right\} \sum_{j=0}^{m+n+J} \frac{(px^h)^j}{j!}$$

$$\cdot \sum_{l=0}^{j} (-1)^l \binom{j}{l} \left(\frac{hl+\alpha}{k}\right)_{m+n+J},$$

where J is defined, as before, by (11).

The innermost sum in (14) is the jth difference of a polynomial of degree $m + n + J$ in α; it is nil when $j > m + n + J$. Thus we have

$$\sum_{j=0}^{m+n+J} \frac{(px^h)^j}{j!} \sum_{l=0}^{j} (-1)^l \binom{j}{l} \left(\frac{hl+\alpha}{k}\right)_{m+n+J}$$
$$= \sum_{l=0}^{\infty} \left(\frac{hl+\alpha}{k}\right)_{m+n+J} \frac{(-px^h)^l}{l!} \sum_{j=0}^{\infty} \frac{(px^h)^j}{j!}$$
$$= \exp(px^h) \sum_{l=0}^{\infty} \left(\frac{hl+\alpha}{k}\right)_{m+n+J} \frac{(-px^h)^l}{l!},$$

and substituting this expression in (14), and applying the binomial expansion to sum the resulting n-series, we finally obtain

$$(15)\quad \Omega(u_1,\ldots,u_r) = k^m \exp(px^h)\Delta_r^{-m-\alpha/k}$$
$$\cdot \sum_{l,j_1,\ldots,j_r=0}^{\infty} \left(\frac{hl+\alpha}{k}\right)_{m+J} \left(\frac{1}{l!}\right) \Lambda(j_1,\ldots,j_r) \left(-\frac{px^h}{\Delta_r^{h/k}}\right)^l$$
$$\cdot \prod_{i=1}^{r} \left\{ \frac{[(-u_i/\Delta_r)^{m_i} y_i]^{j_i}}{j_i!} \right\}, \qquad k \neq 0,$$

where Δ_r and J are given by (8) and (11), respectively, and the inequality in (10) is assumed to hold.

The right-hand sides of (9) and (15) are essentially the same. This evidently completes the proof of our theorem under the hypothesis that the various interchanges of the order of summation are permissible by absolute convergence of the series involved. Thus, in general, our theorem holds true whenever each member of (9) has a meaning.

REMARK. Our method of derivation can be applied *mutatis mutandis* in order to prove the following generalization of the multilinear generating function (9):

$$(16)\quad \sum_{n_1,\ldots,n_r=0}^{\infty} (m + n_1 + \cdots + n_r)!\mathscr{F}^{(\alpha)}_{m+n_1+\cdots+n_r}(x, h, p, k)$$
$$\cdot \mathscr{H}[n_1,\ldots,n_r; y_1,\ldots,y_r] \frac{(u_1/k)^{n_1}}{n_1!} \cdots \frac{(u_r/k)^{n_r}}{n_r!}$$
$$= k^m \exp(px^h)\Delta_r^{-m-\alpha/k} \sum_{n,n_1,\ldots,n_r=0}^{\infty} \left(\frac{hn+\alpha}{k}\right)_{m+m_1n_1+\cdots+m_rn_r}$$
$$\cdot \frac{\xi_n}{n!} \Delta(n_1,\ldots,n_r) \left(-\frac{px^h}{\Delta_r^{h/k}}\right)^n \prod_{i=1}^{r} \left\{ \frac{[(-u_i/\Delta_r)^{m_i} y_i]^{n_i}}{n_i!} \right\}, \qquad k \neq 0,$$

where, in terms of the bounded sequence $\{\xi_n\}$ of arbitrary complex numbers,

$$(17)\quad \mathscr{F}_n^{(\alpha)}(x, h, p, k) = \frac{k^n}{n!}\sum_{j=0}^{\infty}\frac{(px^h)^j}{j!}\sum_{l=0}^{j}(-1)^l\binom{j}{l}\xi_l\left(\frac{hl+\alpha}{k}\right)_n,$$

which obviously reduces to the Srivastava-Singhal equation (5) when $\xi_l = 1, l \geq 0$.

3. Applications. By assigning suitable special values to the arbitrary coefficients $\Lambda(j_1,\ldots,j_r)$, the multiple sum in (7) can indeed be expressed in terms of the generalized Lauricella hypergeometric function of r variables [**11**, p. 454]. Thus, following the various notations and conventions explained fairly fully by Srivastava and Daoust ([**11**, p. 545 *et seq.*]; see also [**12**]), we obtain from our theorem the multivariable hypergeometric generating function:

$$(18)\quad \sum_{n_1,\ldots,n_r=0}^{\infty}(m+n_1+\cdots+n_r)!\,G_{m+n_1+\cdots+n_r}^{(\alpha)}(x, h, p, k)$$

$$\cdot F^{A:1+B';\cdots;1+B^{(r)}}_{C:\;D';\cdots;D^{(r)}}\left(\begin{matrix}[(a):\theta',\ldots,\theta^{(r)}]:[-n_1:m_1],\;[(b'):\phi'];\cdots;[-n_r:m_r],[(b^{(r)}):\phi^{(r)}];\\ [(c):\psi',\ldots,\psi^{(r)}]:\qquad [(d'):\delta'];\cdots;[(d^{(r)}):\delta^{(r)}];\end{matrix}\; y_1,\ldots,y_r\right)\left(\frac{u_1}{k}\right)^{n_1}\cdots\left(\frac{u_r}{k}\right)^{n_r}$$

$$= k^m\left(\frac{\alpha}{k}\right)_m \exp(px^h)\,\Delta_r^{-m-\alpha/k}\,F^{1+A:0;B';\cdots;B^{(r)}}_{C:1;D';\cdots;D^{(r)}}$$

$$\left(\begin{matrix}[m+\alpha/k:h/k, m_1,\ldots,m_r],[(a):0,\theta',\ldots,\theta^{(r)}]:\underline{\qquad};[(b'):\phi'];\cdots;[(b^{(r)}):\phi^{(r)}];\\ [(c):0,\psi',\ldots,\psi^{(r)}]:[\alpha/k:h/k];[(d'):\delta'];\cdots;[(d^{(r)}):\delta^{(r)}];\end{matrix}\;\Xi_0,\Xi_1,\ldots,\Xi_r\right),\qquad k\neq 0,$$

where $h/k > 0$, Δ_r is given by (8), and

$$(19)\qquad \Xi_0 = -\frac{px^h}{\Delta_r^{h/k}},\qquad \Xi_i = y_i\left(-\frac{u_i}{\Delta_r}\right)^{m_i},\quad i = 1,\ldots,r.$$

Next we set $A = C = 0$ in (18) and, for convenience, let each of the positive coefficients $\phi_j^{(i)}, j = 1,\ldots,B^{(i)}$; $\delta_j^{(i)}, j = 1,\ldots,D^{(i)}$ $(i = 1,\ldots,r)$ equal 1. Denoting the array of parameters

$$(-n_i+j-1)/m_i,\qquad j = 1,\ldots,m_i,$$

by $\Delta(m_i;\ -n_i)$, $i = 1,\dots,r$, we thus find from (18) that

$$(20)\quad \sum_{n_1,\dots,n_r=0}^{\infty} (m + n_1 + \cdots + n_r)!\,G^{(\alpha)}_{m+n_1+\cdots+n_r}(x, h, p, k)$$

$$\cdot \prod_{i=1}^{r}\left\{ {}_{m_i+B^{(i)}}F_{D^{(i)}}\left[\begin{matrix}\Delta(m_i;\ -n_i), & (b^{(i)}); \\ & (d^{(i)}); \end{matrix}\ y_i m_i^{m_i}\right]\left(\frac{u_i}{k}\right)^{n_i}\right\}$$

$$= k^m\left(\frac{\alpha}{k}\right)_m \exp(px^h)\,\Delta_r^{-m-\alpha/k}$$

$$\cdot F^{1\,:\,0;\,B';\,\cdots;\,B^{(r)}}_{0\,:\,1;\,D';\,\cdots;\,D^{(r)}}\left(\begin{matrix}[m + \alpha/k\colon h/k, m_1,\dots,m_r]\colon\text{———}; \\ \text{————}\ :\ [\alpha/k\colon h/k];\end{matrix}\right.$$

$$\left.\begin{matrix}[(b')\colon 1];\cdots;[(b^{(r)})\colon 1]; \\ [(d')\colon 1];\cdots;[(d^{(r)})\colon 1];\end{matrix}\ \Xi_0, \Xi_1,\dots,\Xi_r\right),\qquad k \neq 0,$$

where $h/k > 0$, Δ_r is given by (8), and $\Xi_0, \Xi_1,\dots,\Xi_r$ are defined by (19).

Obviously, this last formula (20) generates the product of r generalized hypergeometric polynomials; it is a generalization of several known results due to Srivastava and Singhal [**15**].

For special values of the parameters, the Srivastava-Singhal polynomials $G_n^{(\alpha)}(x, h, p, k)$ can be reduced to the classical Hermite and Laguerre polynomials and their various generalizations studied in the literature (*cf.* [**14**, p. 76]). Furthermore, the generalized hypergeometric polynomials occurring in (20) can be specialized to several important classes of hypergeometric polynomials including, for example, the classical Hermite polynomials and their such generalizations as those considered by Gould and Hopper [**3**, p. 58]

$$(21)\qquad g_n^m(x, \lambda) = \sum_{j=0}^{[n/m]} \frac{n!}{j!(n - mj)!}\lambda^j x^{n-mj}$$

$$= x^n\,{}_mF_0\left[\begin{matrix}\Delta(m;\ -n); \\ \text{———};\end{matrix}\ \lambda\left(-\frac{m}{x}\right)^m\right],$$

and by Brafman [**1**, p. 186]

$$(22)\qquad \mathscr{B}_n^m[\alpha_1,\dots,\alpha_r;\ \beta_1,\dots,\beta_s\colon x]$$

$$= {}_{m+r}F_s\left[\begin{matrix}\Delta(m;\ -n), & \alpha_1,\dots,\alpha_r; \\ & \beta_1,\dots,\beta_s;\end{matrix}\ x\right],$$

where, as in (20), $\Delta(m;\ -n)$ abbreviates the array of m parameters

$$(-n + j - 1)/m,\qquad j = 1,\dots,m,$$

m being an arbitrary positive integer. The details involved in these derivations of known or new multilinear generating functions from (20) may be left as an exercise to the interested reader.

Yet another interesting application of our theorem would result when in (18) we set

$$\begin{cases} h = p = 1, \quad A = B^{(i)} = C = D^{(i)} - 1 = 0, \\ d_1^{(i)} = 1 + \beta_i, \quad \delta_1^{(i)} = s_i, \quad m_i = 1, \qquad i = 1,\dots,r, \end{cases}$$

replace α by $\alpha + 1$, and y_i by $y_i^{s_i}$, $i = 1,\dots,r$, and appeal to the relationship (6) and to the explicit representation (2). We thus obtain our desired multilinear generating function for the Konhauser biorthogonal polynomials in the form:

$$(23) \quad \sum_{n_1,\dots,n_r=0}^{\infty} (m + n_1 + \cdots + n_r)!\, Y^{\alpha}_{m+n_1+\cdots+n_r}(x;k) \cdot \prod_{i=1}^{r} \left\{ Z^{\beta_i}_{n_i}(y_i;s_i) \frac{u_i^{n_i}}{(1+\beta_i)_{s_i n_i}} \right\}$$
$$= \left(\frac{\alpha+1}{k}\right)_m e^x \Delta_r^{-m-(\alpha+1)/k} \cdot F^{1:0;\,\cdots;\,0}_{0:1;\,\cdots;\,1}\left(\begin{matrix} [m+(\alpha+1)/k : 1/k, 1,\dots,1] : \\ \overline{} : \end{matrix} \right.$$
$$\begin{matrix} \overline{}; & \overline{}; \cdots; \\ [(\alpha+1)/k : 1/k]; & [1+\beta_1 : s_1]; \cdots; \end{matrix}$$
$$\left. \begin{matrix} \overline{}; \\ [1+\beta_r : s_r]; \end{matrix} \; -\frac{x}{\Delta_r^{1/k}}, -\frac{u_1 y_1^{s_1}}{\Delta_r}, \dots, -\frac{u_r y_r^{s_r}}{\Delta_r} \right),$$

where, by definition,

$$(24) \quad \alpha > -1; \quad \beta_i > -1; \quad k, s_i = 1,2,3,\dots; \qquad \forall\, i \in \{1,\dots,r\}.$$

A seriously erroneous version of a *special* case of the multilinear generating function (23), when $s_1 = \cdots = s_r = s$, was proven earlier by Patil and Thakare [**6**] who incidentally used a markedly different method. In fact, (23) with $k = s_1 = \cdots = s_r = 1$ is a well-known result (involving the classical Laguerre polynomials) due to Srivastava and Singhal [**15**, p. 1239, Eq. (5)].

Since $s_1,\dots,s_r$ are, by definition, positive integers, the multilinear generating function (23) would follow also as an obvious special case of (20).

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