idempotent, this condition is equivalent to $\phi(d(I+afM))(I+afM) = \phi(d(I+afM))$ for all $d \in D$. That is,

$$(*)\quad \phi(dI)I + a[\phi(dI)fM + \phi(dfM)I] + a^2\phi(dfM)fM = \phi(dI) + a\phi(fM).$$

If $(*)$ holds for three values of a, then (c) is satisfied; and (c) clearly implies $(*)$ for all choices of $a \in \hat{Q}_p$.

LEMMA 5. *If* $\phi \in \mathrm{End}_Q(D)$ *and* $f \in \hat{F}$ *are such that* $N_\phi(1) = N_\phi(f) = \hat{Q}_p$, *then* $\phi f = f\phi$ *(considering* f *as the left translation* $\lambda(f)$*).*

Proof. Let $d \in D$ be arbitrary. By Lemma 4, $N_\phi(1) = \hat{Q}_p$ yields $\phi(dfI)M + \phi(dfM)I = \phi(dfM)$, and $N_\phi(f) = \hat{Q}_p$ implies $\phi(dI)fM + \phi(dfM)I = \phi(dfM)$. Thus, $[\phi(dfI) - f\phi(dI)]IM = [\phi(dfI) - \phi(dI)f]M = 0$, since f centralizes $\hat{D}$. Consequently, $[\phi(dfI) - f\phi(dI)]I = 0$ because the rows of γ are linearly independent over $K \otimes_F D$. By Lemma 4(c)(i), $(\phi f - f\phi)dI = 0$. Since $d \in D$ is arbitrary, $(\phi f - f\phi)\hat{D}I = 0$; and $\phi f = f\phi$ by Lemma 3.

PROPOSITION. *There exists* $\alpha \in \hat{F}$, *transcendental over* K', *such that* $QE(G(\alpha)) = D$.

Proof. To simplify notation, write Φ for $\mathrm{End}_Q(D)\backslash \mathrm{End}_F(D)$. Choose $f \in F$ so that $F = Q(f)$. If $\phi \in \mathrm{End}_Q(D)$ satisfies $\phi f = f\phi$, then $\phi \in \mathrm{End}_F(D)$. Hence, by Lemmas 4 and 5, $\phi \in \Phi$ implies $|N_\phi(1)| \le 2$ or $|N_\phi(f)| \le 2$. Assume that $b \in \hat{Q}_p$ is such that $|N_\phi(b+f)| \le 3$. By Lemma 4,

$$\phi(dI)I = \phi(dI), \quad \phi(dM)M = 0,$$

and

$$b(\phi(dI)M + \phi(dM)I - \phi(dM)) + (\phi(dI)fM + \phi(dfM)I - \phi(dfM)) = 0$$

for all $d \in D$. If also $b \ne c \in \hat{Q}_p$ and $|N_\phi(c+f)| \le 3$, then $\phi \in \mathrm{End}_F(D)$ by Lemmas 4 and 5. Hence, if $\phi \in \Phi$ and $|N_\phi(b+f)| \ge 3$, then $|N_\phi(c+f)| \le 2$ for all $c \ne b$ in $\hat{Q}_p$. Since $\mathrm{End}_\phi(D)$ is countable and $\hat{Q}_p$ is uncountable, there exists $c \in \hat{Q}_p$ such that $|N_\phi(c+f)| \le 2$ for all $\phi \in \Phi$. The countability of K' then implies the existence of $a \in \hat{Q}_p$, transcendental over $K'(c)$, such that $a \notin N_\phi(c+f)$ for all $\phi \in \Phi$. By the definition of $N_\phi(c+f)$, this means that $\Phi \cap QE(G(a(c+f))) = \varnothing$. Hence, $QE(G(\alpha)) \subseteq \mathrm{End}_F(D)$, where $\alpha = a(c+f)$ is transcendental over K'. By Lemma 2, $QE(G(\alpha)) = D$.

REFERENCES

[1] R. S. Pierce and C. Vinsonhaler, *Realizing central division algebras*, Pacific J. Math., **109** (1983), 165–177.

ACKNOWLEDGMENTS

The editors gratefully acknowledge the service of the following persons who have been consulted concerning the preparation of volumes one hundred twenty-three through one hundred twenty-eight of the Pacific Journal of Mathematics.

H. Amann, D. D. Anderson, H. Araki, W. Arendt, J. Auslander, D. Babbitt, A. Baernstein, II, M. Bestvina, C. Borges, G. Brauer, K. S. Brown, R. F. Brown, R. Bruck, G. W. Brumfiel, E. Buchman, D. Burghelea, L. Caffarelli, A. Carbery, G. E. Carlsson, C. Castaing, L. Charlap, E. W. Cheney, S. Y. Cheng, P. Chernoff, R. Cohen, P. Concus, C. B. Croke, D. Curtis, F. Dashiell, R. J. Daverman, R. de Laubenfels, F. DeMeyer, J. Diestel, W. F. Donaghue, I. Ekeland, R. Elman, T. Enright, P. Fife, R. Finn, M. Fried, J. B. Fugate, T. Gamelin, J. Garnett, B. Goldman, W. M. Goldman, D. Goldschmidt, I. Graham, E. E. Granirer, M. Green, U. Haagerup, J. Hass, H. Helson, J. P. Holmes, R. Howe, T. Husain, S. Y. Husseini, W. B. Johnson, D. Kahn, M. Kalka, S. Kantorovitz, Y. Katznelson, L. Kauffman, R. Kirby, W. A. Kirk, S. Kleiman, J. Kleinstein, G. Ladas, C. Lanski, D. G. Larman, D. R. Larson, F. W. Lawere, J. D. Lawson, R. Lazarsfeld, C. R. Lebrun, Jr., D. B. Leep, M. Lerman, P. Leroux, P. W. K. Li, L. Lipschitz, C. Long, A. T. Lundell, R. J. Lyndon, M. A. Marshall, K. McCrimmon, W. H. Meeks, E. Miller, H. Miller, J. Milnor, M. Miranda, K. Murasugi, M. R. Murty, W. Neumann, A. Ocneanu, E. Odell, B. Oksendal, L. G. Oversteegen, R. Penney, U. Persson, R. R. Phelps, P. Rejto, A. Rosenberg, J. Rosenberg, J. Rosenblatt, H. Rossi, D. J. Rudolf, P. Sally, P. Sarnak, C. Schochet, R. Schultz, J. Shapiro, T. Shifrin, J. Simon, L. Simon, L. Small, L. Stout, E. Straus, R. Strong, M. Takesaki, G. Takeuti, F. D. Tall, M. Taylor, M. L. Teply, V. S. Varadarajan, P. Veldkamp, A. Wadsworth, M. Walter, R. B. Warfield, S. Warner, M. Weisfeld, H. C. Williams, O. Wyler, S. S. T. Yau, J. J. Yeh.

CONTENTS

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Vol. 130, No.2 December 1987