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AN INVARIANCE PRINCIPLE FOR ASSOCIATED RANDOM FIELDS

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Applying known tightness criteria to Poisson cluster random measures, it is shown that if the total member size has a finite $2+\delta$ moment, then the random measure satisfies an invariance principle.

I. Introduction. Let $\{X_{\underline{k}} \mid \underline{k} \in \mathbf{Z}^d\}$ be a random field that is centered, stationary, associated and has a summable covariance function. C. Newman [10] showed that, when viewed as an element in d-dimensional Skorohod space, the renormalizations of $\{X_{\underline{k}} \mid \underline{k} \in \mathbf{Z}^d\}$ converge to a Wiener measure in the sense of finite dimensional distributions. Newman and Wright [11] showed that this may be improved to an invariance principle if $d = 1$ or 2. Analogous results hold in the case of random measures. A tightness criterion of Bickel and Wichera [1] is applicable in the case of general d. This criterion is applied to Poisson center cluster random measures. It is shown that if the total member size has a finite $2+\delta$ moment then the random measure satisfies an invariance principle.

II. Random fields and random measures. A *random field* is a collection of nondegenerate random variables indexed by $\mathbf{Z}^d$ and is denoted $\{X_{\underline{k}} \mid \underline{k} \in \mathbf{Z}^d\}$. All random fields in this section are assumed centered and stationary, i.e. $E[X_{\underline{k}}] = 0$ and the distribution is invariant with respect to translations of the indices by the group $\mathbf{Z}^d$. A random field is *associated* if whenever $A \subseteq \mathbf{Z}^d$ is a finite subset and $f, g: \mathbf{R}^A \to \mathbf{R}$ are coordinatewise increasing then $\mathrm{Cov}[f(X_{\underline{k}}: \underline{k} \in A), g(X_{\underline{k}}: \underline{k} \in A)]$ is nonnegative whenever the covariance is defined. Association is a strong positive dependence property implying, in particular, nonnegative correlations of the random variables $X_{\underline{k}}$ (if they exist). For details concerning association see Esary, Proschan and Walkup [4].

A random field may be interpolated and rescaled to form a random element of d-dimensional Skorohod space $\mathfrak{D}([0,1]^d)$ by setting

$$W_n(\underline{t}) = n^{-d/2} \sum_{j_1=1}^{[nt_1]} \cdots \sum_{j_d=1}^{[nt_d]} X_{\underline{j}}$$

where $\underline{t} = (t_1, \ldots, t_d) \in [0, 1]^d$ and $[\cdot]$ is the greatest integer function. $\{X_{\underline{k}}\}$ is said to satisfy an *invariance principle* or *functional central limit theorem* if $W_n(\underline{t})$ converges weakly in $\mathfrak{D}([0, 1]^d)$ to a d-dimensional Wiener process with some finite diffusion constant σ^2. This is equivalent to convergence of the finite dimensional distributions together with tightness. The following condition arises naturally when investigating invariance principles.

DEFINITION 2.1. $\{X_{\underline{k}}\}$ has *finite susceptibility* equal to $\sigma^2 < \infty$ if $\sum_{\underline{k} \in \mathbf{Z}^d} \mathrm{Cov}[X_{\underline{0}}, X_{\underline{k}}] = \sigma^2$.

REMARK. For $\underline{k} \in \mathbf{Z}^d$ with positive components let $S_{\underline{k}} = \sum_{\underline{0} < \underline{j} \leq \underline{k}} X_{\underline{j}}$. Assume that the covariances of the $X_{\underline{k}}$ are nonnegative. Then $\{X_{\underline{k}}\}$ has finite susceptibility if and only if the following expectation is bounded in $\underline{k}$.

$$E[|S_{\underline{k}}/|\underline{k}|^{1/2}|^2] \leq C$$

where $|\underline{k}| = k_1 \cdots k_d$.

The following theorem is due to Newman **[10]** and Newman and Wright **[11]**.

THEOREM 2.2. *Let $\{X_{\underline{k}}\}$ be an associated random field and have finite susceptibility equal to σ^2 then*

(1) *(Newman) the finite dimensional distributions of $W_n(\underline{t})$ converge to those of the Wiener process with diffusion σ^2;*

(2) *(Newman and Wright) further if $d = 1$ or $d = 2$ then $\{X_{\underline{k}}\}$ satisfies an invariance principle.*

Whether an invariance principle holds for $d > 2$ is still open. A result of Bickel and Wichura **[1]** allows us to conclude tightness if we strengthen the hypothesis of finite susceptibility.

DEFINITION 2.3. $\{X_{\underline{k}}\}$ has *finite δ-susceptibility* if there is a constant C so that for all $\underline{k}$ with positive components

$$E[|S_{\underline{k}}/|\underline{k}|^{1/2}|^{2+\delta}] \leq C.$$

The above results may be combined to get the following.

THEOREM 2.4. *Let $\{X_{\underline{k}}\}$ be a centered, stationary, finite variance, associated random field that satisfies δ-susceptibility for some $\delta > 0$. Then $\{X_{\underline{k}}\}$ satisfies the invariance principle.*

REMARK. The above theorem applies to models in mathematical physics. Random variables X, Y, Z, W are said to satisfy the *Lebowitz inequality* if

$$E[XYZW] \leq E[XY]E[ZW] + E[XZ]E[YW] + E[XW]E[YZ].$$

A random field $\{X_{\underline{k}}\}$ satisfies the Lebowitz inequalities if any four coordinate random variables satisfy the above inequality. If $\{X_{\underline{k}}\}$ is also stationary and has finite susceptibility then it has finite δ-susceptibility with $\delta = 2$. This computation appears in Wood **[18]**. Ferromagnetic Ising models often satisfy the Lebowitz inequality and a fortiori satisfy the invariance principle. We have not been able to find this fact in the literature.

A corresponding theory exists for random measures. We let M be the set of all nonnegative Borel measures on $\mathbf{R}^d$ that are finite on compact sets [i.e. Radon measures]. Let $N \subseteq M$ be the set of counting measures, i.e. B Borel and $\mu \in N$ implies $\mu(B) \in \{0, 1, 2, \dots, \infty\}$. There is a one-to-one correspondence between $\mu \in N$ and unordered sequences $\{x_i\}$ of points in $\mathbf{R}^d$ with no limit points because each such μ must be a sum of Dirac point masses. M is a Polish space with the vague topology and N is closed in M. Let the Borel σ-fields of M and N be $\mathscr{M}$ and $\mathscr{N}$ respectively.

DEFINITION 2.5. A *random measure* X is a measurable map from a fixed probability space $(\Omega, \mathscr{F}, P)$ to $(M, \mathscr{M})$. X is called a *point random field* if $P[X \in N] = 1$.

If X is a random measure and B is a Borel subset of $\mathbf{R}^d$ then $X(B)$ denotes the mass the random measure gives to B. All random measures will be assumed to be stationary, i.e. with a translation invariant distribution. The most well known random measure is the *Poisson point random field* with parameter ρ. X has this distribution if whenever $B_1, \dots, B_n$ are disjoint bounded Borel sets then $X(B_1), \dots, X(B_n)$ are independent Poisson random variables with respective parameters $\rho|B_1|, \dots, \rho|B_n|$ where $|\cdot|$ denotes Lebesgue measure. M has a partial ordering defined by $\mu \leq \nu$ if for each bounded Borel set B, $\mu(B) \leq \nu(B)$. See Kallenberg **[6]** for a more complete discussion of random measures.

DEFINITION 2.6. A random measure X is *associated* if whenever $F, G\colon M \to \mathbf{R}$ is measurable and increasing with respect to the partial ordering on M then $\mathrm{Cov}[F(X), G(X)]$ is nonnegative whenever the covariance is defined. It follows from work of the first author and Waymire [2, 3] that X is associated if and only if the family of random variables $\{X(B) \mid B \text{ bounded Borel}\}$ is associated.

DEFINITION 2.7. If X is a random measure we define the λ-renormalization of X to be the signed random measure X_λ where $X_\lambda(B) = \lambda^{-d/2}[X(\lambda B) - E[X(\lambda B)]]$. We consider X_λ as a random element of $\mathfrak{D}([0,1]^d)$ by setting $X_\lambda(\underline{t}) = X_\lambda([\underline{0}, \underline{t}])$ where $[\underline{0}, \underline{t}]$ is the rectangle $[0, t_1] \times \cdots \times [0, t_d]$.

DEFINITION 2.8. X satisfies the *invariance principle* with parameter σ^2 if as $\lambda \to \infty$, X_λ converges weakly to the d-dimensional Wiener measure on $\mathfrak{D}([0,1]^d)$ with diffusion constant σ^2.

DEFINITION 2.9. Let I be the unit cube in $\mathbf{R}^d$ and X be associated. X has *finite susceptibility* σ^2 if

$$\sum_{\underline{k} \in \mathbf{Z}^d} \operatorname{Cov}[X(I), X(I + \underline{k})] = \sigma^2.$$

X has *finite δ-susceptibility* if there is a constant $K < \infty$ depending only on δ and X so that for all rectangular boxes $B \supseteq I$ we have

$$(*) \qquad E[|X(B) - E[X(B)]|^{2+\delta}] \leq K|B|^{1+\delta/2}.$$

It would be a more pleasing definition to require $(*)$ in Definition 2.9 to hold for all rectangular boxes B but this is asking too much. For example such a condition is untrue in the case where X is a Poisson point random field for any $\delta > 0$. A simple argument using Chebyshev's inequality allows us to extend the invariance principle for associated random fields to random measures.

THEOREM 2.10. *Let X be a stationary associated random measure with finite δ-susceptibility for some $\delta > 0$. Then X satisfies the invariance principle.*

III. Cluster random measures. In this section we apply Theorem 2.10 to Poisson center cluster random measures. These have been used as models of infinite divisibility and self-similarity [**14**, **15**] as well as models of natural phenomena such as storm systems and galaxies [**12**, **16**, **17**]. These are constructed as follows. Let U be a stationary Poisson point random field with parameter ρ. Let $V = \{V_{\underline{x}} \mid \underline{x} \in \mathbf{R}^d\}$

be a collection of iid random measures with $E[V_{\underline{x}}(\mathbf{R}^d)] = \xi < \infty$. Then we say that X is a cluster process with centers U and members V if

$$X(B) = \sum_{\underline{x}:\ U(\underline{x})>0} V_{\underline{x}}(B - \underline{x})$$

for each bounded Borel set B. We denote X by $[U, V]$. It is natural to hope that moment conditions on V will imply moment conditions on X regardless of the "shape" of V in $\mathbf{R}^d$. This is made precise in the following theorem.

THEOREM 3.1. *Let $X = [U, V]$ as above. Let B be a rectangular box in $\mathbf{R}^d$ and $0 \le \delta \le 2$; then there is a constant K depending only on δ and $|B|$ so that*

(1) $E[|X(B)|^{2+\delta}] \le KE[(V_{\underline{x}}(\mathbf{R}^d))^{2+\delta}]$.

(2) *If $E[(V_{\underline{x}}(\mathbf{R}^d))^{2+\delta}] < \infty$ then X has finite δ-susceptibility.*

The first part of the next theorem appears in joint work of the first author and Waymire [2] and the second part is immediate from the first part of Theorem 3.1.

THEOREM 3.2. *Let $X = [U, V]$ as above.*

(1) *X is associated.*

(2) *If $E[(V_{\underline{x}}(\mathbf{R}^d))^{2+\delta}] < \infty$ then X satisfies the invariance principle.*

We note that the second part of Theorem 3.2 improves a theorem of Ivanoff [5] where it was assumed that $\delta = 4$ and that the V was a point random field with cumulant density functions.

IV. Proof of Theorem 3.1. First two lemmas.

LEMMA 4.1. (1) *If $Y_1, Y_2, \ldots$ are centered iid random variables and $E|Y_i|^p < \infty$ for some $p \ge 1$ then there is a constant C depending only on p so that*

$$E\left|\sum_{i=1}^n Y_i\right|^p \le CE\left|\sum_{i=1}^n Y_i^2\right|^{p/2}.$$

(2) *If $Y_1, Y_2, \ldots$ are iid random variables and $E|Y_i|^{2+\delta} < \infty$ for some $0 \le \delta \le 2$ then there is a constant C' depending only on δ so that*

$$E\left|\sum_{i=1}^n Y_i\right|^{2+\delta} \le C'(nE|Y_1|^{2+\delta} + |nEY_1^2|^{1+\delta/2} + |nEY_1|^{2+\delta}).$$

Proof. (1) is a standard square inequality attributed to Marcinkiewicz and Zygmund **[9]** (see also **[13]**, p. 59). (2) follows from two applications of (1). Let $\mu = E[Y_1]$ and $\sigma^2 = \mathrm{Var}[Y_1]$ and $\tilde{\leq}$ denote "bounded by a constant multiple of". Then

$$\begin{aligned}
E\left|\sum_{i=1}^n Y_i\right|^{2+\delta} &\tilde{\leq} E\left|\sum_{i=1}^n (Y_i-\mu)\right|^{2+\delta} + |n\mu|^{2+\delta} \\
&\tilde{\leq} E\left|\sum_{i=1}^n (Y_i-\mu)^2\right|^{1+\delta/2} + |n\mu|^{2+\delta} \\
&\tilde{\leq} E\left|\sum_{i=1}^n [(Y_i-\mu)^2-\sigma^2]\right|^{1+\delta/2} + |n\sigma^2|^{1+\delta/2} + |n\mu|^{2+\delta} \\
&\tilde{\leq} E\left|\sum_{i=1}^n [(Y_i-\mu)^2-\sigma^2]^2\right|^{1/2+\delta/4} \\
&\quad + |n\sigma^2|^{1+\delta/2} + |n\mu|^{2+\delta} \\
&\tilde{\leq} E\sum_{i=1}^n |[(Y_i-\mu)^2-\sigma^2]^2|^{1/2+\delta/4} \\
&\quad + |n\sigma^2|^{1+\delta/2} + |n\mu|^{2+\delta} \\
&= nE|[(Y_1-\mu)^2-\sigma^2]|^{1+\delta/2} + |n\sigma^2|^{1+\delta/2} + |n\mu|^{2+\delta} \\
&\tilde{\leq} nE|Y_1-\mu|^{2+\delta} + n|\sigma^2|^{1+\delta/2} + |n\sigma^2|^{1+\delta/2} + |n\mu|^{2+\delta} \\
&\tilde{\leq} nE|Y_1|^{2+\delta} + n(E[Y_1^2])^{1+\delta/2} \\
&\quad + (nE[Y_1^2])^{1+\delta/2} + |nEY_1|^{2+\delta} \\
&\tilde{\leq} nE|Y_1|^{2+\delta} \\
&\quad + |nEY_1^2|^{1+\delta/2} + |nEY_1|^{2+\delta}.
\end{aligned}$$
□

LEMMA 4.2. *Let $V = \{V_{\underline{x}} \mid \underline{x} \in \mathbf{R}^d\}$ be iid random measures as in the definition of a cluster random measure. Let $B \subseteq \mathbf{R}^d$ be a rectangle in the nonnegative orthant with one corner at the origin and the opposite corner at $\underline{z}$ and let $B^{(n)} = \bigcup_{\underline{k}:\, n\leq k_i<n} B + \underline{z}(\underline{k})$, where $\underline{z}(\underline{k})$ is the vector with ith coordinate $k_i z_i$. Let $\underline{x}$ be chosen uniformly in $B^{(n)}$. Then for $\alpha \geq 1$*

$$E|V_{\underline{x}}(B-\underline{x})|^\alpha \leq 2^{-d}n^{-d}e[V_{\underline{x}}(\mathbf{R}^d)^\alpha].$$

Proof.

$$E|V_{\underline{x}}(B-\underline{x})|^{\alpha} = E\left[\frac{1}{|B^{(n)}|}\int_{B^{(n)}} V_{\underline{x}}(B-\underline{x})^{\alpha}\,d\underline{x}\right]$$

$$= E\left[\frac{1}{2^d n^d |B|}\sum_{\underline{k}:\ n\le k_i<n}\int_{B+\underline{z}(\underline{k})} V_{\underline{x}}(B-\underline{x})^{\alpha}\,d\underline{x}\right]$$

$$= E\left[2^{-d}n^{-d}|B|^{-1}\sum_{\underline{k}:\ n\le k_i<n}\int_{B} V_{\underline{x}}(B-\underline{x}-\underline{z}(\underline{k}))^{\alpha}\,d\underline{x}\right]$$

$$\le E\left[2^{-d}n^{-d}|B|^{-1}\int_{B}\left(\sum_{\underline{k}:\ n\le k_i<n} V_{\underline{x}}(B-\underline{x}-\underline{z}(\underline{k})\right)^{\alpha}d\underline{x}\right]$$

$$\le 2^{-d}n^{-d}|B|^{-1}E\int_{B} V_{\underline{x}}(\mathbf{R}^d)^{\alpha}\,d\underline{x} = 2^{-d}n^{-d}E[V_{\underline{x}}(\mathbf{R}^d)^{\alpha}]. \qquad \square$$

Proof of Theorem 3.1. Let $X = [U, V]$ as in the statement of the theorem and suppose that $\delta \le 2$ and that $E[V(\mathbf{R}^d)^{2+\delta}] < \infty$. Let R_n be equal to the number of occurrences of U in $B^{(n)}$ so that R_n is a Poisson random variable with parameter $\rho 2^d n^d |B|$. Also recall that conditioned on R_n the occurrences of U inside $B^{(n)}$ are independent and uniformly distributed on $B^{(n)}$.

$$E|X(B)|^{2+\delta} = \lim_{n\to\infty}\left[\left(\sum_{\underline{x}_i\in B^{(n)},U(\underline{x}_i)>0} V_{\underline{x}_i}(B-\underline{x}_i)\right)^{2+\delta}\right]$$

$$= \lim_{n\to\infty} E\left[E\left[\left(\sum V_{\underline{x}_i}(B-\underline{x}_i)\right)^{2+\delta} \mid R_n\right]\right]$$

$$\lesssim \lim_{n\to\infty} E[R_n E|V_{\underline{x}}(B-\underline{x})|^{2+\delta} + |R_n E V_{\underline{x}}(B-\underline{x})^2|^{1+\delta/2} + |R_n E V_{\underline{x}}(B-\underline{x})|^{2+\delta}]$$

$$\lesssim \lim_{n\to\infty} \rho 2^d n^d |B| 2^{-d} n^{-d} E[V(\mathbf{R}^d)^{2+\delta}] + (\rho 2^d n^d |B|)^{1+\delta/2}(2^{-d}n^{-d}E[V(\mathbf{R}^d)^2])^{1+\delta/2} + (\rho|B|E[V(\mathbf{R}^d)])^{2+\delta} \lesssim E[V(\mathbf{R}^d)^{2+\delta}].$$

Note that the factor $(\rho 2^d n^d |B|)^{1+\delta/2}$ in the second term follows from the fact that if R is Poisson with parameter α then $E[R^{1+\delta/2}] \le \text{const.}\alpha^{1+\delta/2}$ for nonnegative δ. This proves part (1). To see part

(2) which is finite δ-susceptibility we have a final computation.

$$
\begin{aligned}
&E[|X(B) - E[X(B)]|^{2+\delta}] \\
&= \lim_{n\to\infty} E\left[\left|\sum_{\underline{x}_i \in B^{(n)}, U(\underline{x}_i)>0} V_{\underline{x}_i}(B-\underline{x}_i) - \sum V_{\underline{x}_i}(B-\underline{x}_i)\right|^{2+\delta}\right] \\
&\le \lim_{n\to\infty} E\left[E\left[\left|\sum V_{\underline{x}_i}(B-\underline{x}_i) - E[V_{\underline{x}_i}(B-\underline{x}_i)]\right|^{2+\delta} \mid R_n\right]\right] \\
&\quad + \lim_{n\to\infty} E[|R_n E[V_{\underline{x}_i}(B-\underline{x}_i)] - ER_n E[V_{\underline{x}_i}(B-\underline{x}_i)]|^{2+\delta}].
\end{aligned}
$$

The second term is the limit of $E(|R_n - ER_n|^{2+\delta})(E[V_{\underline{x}_i}(B-\underline{x}_i)])^{2+\delta}$ which, by part (1) and some Poisson distribution calculations, is $\tilde{\le}\ (\rho 2^d n^d |B|)^{1+\delta/2}(2^{-d}n^{-d}E[V(\mathbf{R}^d)])^{2+\delta}$ which goes to zero. This leaves us with the first term which is by Lemma 4.1(2)

$$
\begin{aligned}
&\lim_{n\to\infty} E[R_n E|V_{\underline{x}_i}(B-\underline{x}_i) - E[V_{\underline{x}_i}(B-\underline{x}_i)]|^{2+\delta} \\
&\qquad\qquad + R_n^{1+\delta/2}|E[V_{\underline{x}_i}(B-\underline{x}_i)^2]|^{1+\delta/2}] \\
&\tilde{\le} \lim_{n\to\infty} E[R_n E|V_{\underline{x}_i}(B-\underline{x}_i)|^{2+\delta} + R_n^{1+\delta/2}|E[V_{\underline{x}_i}(B-\underline{x}_i)^2]|^{1+\delta/2}] \\
&\tilde{\le} \lim_{n\to\infty} \rho 2^d n^d |B| 2^{-d} n^{-d} E[V(\mathbf{R}^d)^{2+\delta}] \\
&\quad + \rho^{1+\delta/2}|B|^{1+\delta/2}(E[V(\mathbf{R}^d)^2])^{1+\delta/2} \\
&\tilde{\le} |B|^{1+\delta/2} E[V(\mathbf{R}^d)^{2+\delta}]. \qquad \square
\end{aligned}
$$

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