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A CONSTRUCTION OF HARMONIC FORMS ON $U(p+1,q)/U(p,q)\times U(1)$

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The point of this paper is to give a method for constructing bundle valued harmonic forms on the indefinite Kähler symmetric space $U(p+1,q)/U(p,q)\times U(1)$. Such a space of harmonic forms has been studied by Rawnsley, Schmid and Wolf in order to unitarize certain representations acting on Dolbeault cohomology spaces. They primiarily study a space of "special" harmonic $(0,s)$-forms representing Dolbeault cohomology, and s is the dimension of a maximal compact subvariety. Here, harmonic forms are constructed in arbitrary degree $(0,s)$. We construct harmonic forms corresponding to "lowest K-types" and we determine the other possible K-types in the representation spanned by these. We also determine when these are L_2 (in an appropriate sense).

The methods used here are similar to those used to construct the discrete series of a semisimple symmetric space (see **[2]**, **[14]**). First we study the space of bundle valued harmonic forms of the Riemannian dual, $U(p+q,1)/U(p+q)\times U(1)$, (hyperbolic space) of $U(p+1,q)/U(p,q)\times U(1)$. This is an easy extension of the work of P. Y. Gaillard (**[5]**). Intertwining operators (called Poisson transforms) of certain principal series representations of $G=U(p+q,1)$ into the space of harmonic forms on hyperbolic space are obtained. In §2 we give a vector bundle version of the Flensted-Jensen (F.-J.) duality. To apply this we need to determine some H-finite vectors in (the hyperfunction realization of) the above principal series representations. A fairly detailed account of such H-finite vectors which are supported in a closed orbit is given in §3. In §4 we determine which of the harmonic forms constructed as above (i.e. F.-J. duals of Poisson transforms of the H-finite vectors) are L_2.

This project is motivated by the following very difficult problems. Let Y be any indefinite Kähler symmetric space and let $\mathscr{L}_\chi$ be a homogeneous holomorphic line bundle over Y.

(a) Under some negativity condition on $\mathscr{L}_\chi$, unitarize the Dolbeault cohomology space $H^s(Y,\mathscr{L}_\chi)$ by constructing L_2 harmonic forms. This is done in **[15]** when a "holomorphic fibration" condition holds. One would like to drop this condition to include many more Y.

(b) Determine the full space of L_2 harmonic $\mathscr{L}_\chi$-valued forms on Y, along with its natural invariant indefinite hermitian form.

We do not solve either (a) or (b) completely. The contribution here is to give an explicit construction of L_2 harmonic forms. For (a) one must study the natural invariant indefinite hermitian form on the image of the Poisson transform (see the remark in §4.1), we do not do this here. For (b) one would like to construct the *full* space of L_2 harmonic forms. The obstacles are the fact that the Poisson transform constructed in §1 is in general not easy to study (for example when is it onto, when does an H-finite distribution lie in its kernel?), also there does not seem to be any way of working with H-finite distributions which are not supported in closed orbits. Some results similar to the results of this paper have been obtained for $\mathrm{SO}(4,1)/\mathrm{SO}(2)\times\mathrm{SO}(2,1)$.

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1. In this section we will describe the relevant work of P. Y. Gaillard ([**5**]) on the space of harmonic forms on hyperbolic space. We will show how to extend this to the case of line bundle valued forms.

1.1. The main idea is to use the following version of Fröbenius reciprocity to construct intertwining maps from principal series representations into the space of harmonic forms. Let G be any connected linear reductive Lie group and K a maximal compact subgroup of G. Let $\mathfrak{g}_0$, $\mathfrak{k}_0$ be the Lie algebras and $\mathfrak{g}$, $\mathfrak{k}$ their complexifications. Let $P = MAN$ be the Langlands decomposition of a minimal parabolic subgroup of G. Let Σ be the roots of $\mathfrak{a}$ in $\mathfrak{g}$ ($\mathfrak{a} = \mathrm{Lie}(A)_{\mathbb{C}}$) and Σ^+ the positive system defined by $\mathfrak{n}$. For any finite dimensional representation W of P we define an induced representation as follows. $I(W)$ is the strong continuous dual of $\mathscr{A}(G/P, W^*) \equiv \{\mathscr{A}(G)\otimes W^*\otimes\mathbb{C}_\rho\}^P$ where $\mathscr{P}(\cdot)$ denotes real analytic functions (sections), $\rho = \frac{1}{2}\sum_{\alpha>0}\alpha$, $\mathbb{C}_\rho$ is the one dimensional representation with A acting by e^ρ and the superscript P means P-invariants. $I(W)$ is the space of W-valued hyperfunctions on G/P. The version of Fröbenius reciprocity that we will use is

$$(*)\qquad \mathrm{Hom}_P(W\otimes\mathbb{C}_{-\rho}, V) \simeq \mathrm{Hom}_G(I(W), V)$$

for any admissible V which is a "maximal" globalization. This follows from work of Schmid ([**17**]), see [**5**] for a discussion of this.

The case of interest to us is when (i) N acts trivially on W (ii) A acts by some character e^ν, $\nu\in\mathfrak{a}^*$ and (iii) M acts irreducibly. In this case $I(W)$ is the maximal globalization (in the sense of [**17**]) of a minimal principal series representation of G. V will be the space of harmonic forms on hyperbolic space and W will be the N-invariants in V.

We will need some facts about hyperbolic space. Let $G = U(n, 1)$, $K = U(n) \times U(1)$, then $X \equiv G/K$ is hyperbolic space. The subgroup $T \subset G$ of diagonal matrices is a compact Cartan subgroup of G. The root system $\Delta = \Delta(\mathfrak{t}, \mathfrak{g})$ is $\{\varepsilon_j - \varepsilon_k | 1 \leq j \neq k \leq n+1\}$ where

$$\varepsilon_j \begin{bmatrix} t_1 & & \\ & \ddots & \\ & & t_n \end{bmatrix} = t_j,$$

and we take $\Delta^+ = \{\varepsilon_j - \varepsilon_k | 1 \leq j < k \leq n+1\}$ as a positive system. Let $\mathfrak{g} = \mathfrak{k} + \mathfrak{p}$ be the Cartan decomposition. Set $\Delta_c \equiv \{\alpha \in \Delta | \mathfrak{g}^{(\alpha)} \subset \mathfrak{k}\}$ (the compact roots), $\Delta_c^+ \equiv \Delta_c \cap \Delta^+$ and $\Delta_{nc} = \Delta - \Delta_c$ (the noncompact roots). X has an invariant complex structure defined by $J = \mathrm{Ad}(\xi)$, where ξ is an appropriate element of the center of $\mathfrak{k}$. The holomorphic and antiholomorphic tangent spaces at eK are $\mathfrak{p}_\pm = \{\eta \in \mathfrak{p} | J(\eta) = \pm i\eta\}$. We choose ξ so that $\Delta_{nc} \cap \Delta^+ = \Delta(\mathfrak{p}_+)$ $(\equiv \{\alpha \in \Delta | \mathfrak{g}^{(\alpha)} \subset \mathfrak{p}_+\})$.

If (π, E) is a finite dimensional irreducible representation of K with highest weight $\chi \in \mathfrak{t}^*$ then we may form the associated homogeneous holomorphic vector bundle $\mathscr{E} \to X$. The Laplace-Beltrami operator on $\mathscr{E}$-valued differential forms is given by the following proposition.

Proposition (*see* [8]). $\square = \frac{1}{4}(r(\Omega) - \langle \chi, \chi + 2\rho \rangle)$ *where* $r(\Omega)$ *is the action of the Casimir as left invariant differential operator,* $\rho = \frac{1}{2}\sum_{\alpha \in \Delta^+} \alpha$ *and* $\square$, Ω *and* $\langle\ ,\ \rangle$ *are defined with respect to the trace form on* $\mathfrak{u}(n, 1)$.

Definition. The *Harmonic space* $\mathscr{H}^{(r,s)}$ or $\mathscr{H}^{(r,s)}(G/K, \mathscr{E})$ is defined to be the space of $C^\infty(r, s)$-forms ω satisfying $\square\omega = 0$.

It is known that $\mathscr{H}^{(r,s)}$ is an admissible G-module, and in fact a maximal glocalization (see [17] and [5]).

1.2. We will now show how to compute the N invariants in $\mathscr{H}^{(r,s)}$. First note that $G = NAK$ by the Iwasawa decomposition. We take A to be $\exp \mathfrak{a}_0$ with

$$\mathfrak{a}_0 = \mathbb{R} \begin{bmatrix} 0 & & 1 \\ & \ddots & \\ 1 & & 0 \end{bmatrix}.$$

Let $\Sigma = \Sigma(\mathfrak{a}, \mathfrak{g})$ be the restricted root system, so $\Sigma = \{\pm t, \pm 2t\}$, we set $\Sigma^+ = \{t, 2t\}$ and $\mathfrak{n}_0 = \mathfrak{g}_0^{(t)} \oplus \mathfrak{g}_0^{(2t)}$.

Let $A^{(r,s)}(X, \mathscr{E})$ denote the space of C^∞ $\mathscr{E}$-valued forms on X of type (r, s) and let $\Lambda^{r,s}$ be the elements of type (r, s) in $\Lambda(\mathfrak{p})^*$ (the exterior algebra of $\mathfrak{p}^*$). Then $A^{(r,s)}(X, \mathscr{E}) \simeq \{C^\infty(G) \otimes \Lambda^{r,s} \otimes E\}^K$, the K-invariants (the K action on $C^\infty(G)$ being right translation). This is the space of functions $\omega: G \to \Lambda^{r,s} \otimes E$ such that $\omega(gk) = k^{-1} \cdot \omega(g)$, for all $g \in G$, $k \in K$.

LEMMA. *$\{A^{(r,s)}(X, \mathscr{E})\}^N \simeq C^\infty(\mathfrak{a}_0) \otimes \Lambda^{r,s} \otimes E$ as MA modules.*

Proof. Since $G = NAK$ we have $\{C^\infty(G) \otimes \Lambda^{r,s} \otimes E\}^K \simeq C^\infty(NA) \otimes \Lambda^{r,s} \otimes E$, as NA module. The N-invariants (left N-action on C^∞) are $C^\infty(A) \otimes \Lambda^{r,s} \otimes E$. □

REMARK. The space of N-invariants of the lemma is identified with C^∞ functions $\mathbb{R} \to \Lambda^{r,s} \otimes E$. Such a function φ corresponds to the form $\omega(ne^{tZ}k) = k^{-1}\varphi(t)$ where

$$Z = \begin{bmatrix} 0 & & 1 \\ & \ddots & \\ 1 & & 0 \end{bmatrix} \in \mathfrak{a}_0.$$

Also, the left action of Z is by $-d/dt$.

Let Ω and Ω_M be the Casimir elements for G and M, both w.r.t. the trace form.

LEMMA. *If $\varphi \in \{A^{(r,s)}(X, \mathscr{E})\}^N$ then*

$$\Omega\varphi = \frac{1}{2}\frac{d^2\varphi}{dt^2} - n\frac{d\varphi}{dt} + \Omega_M\varphi.$$

Proof (sketch). This is a routine calculation with root vectors which we will only describe briefly. Since $\mathfrak{g} = (\mathfrak{n} + \theta\mathfrak{n}) \oplus \mathfrak{a} \oplus \mathfrak{m}$ is an orthogonal decomposition of $\mathfrak{g}$ we may choose orthonormal bases of $\mathfrak{n} + \theta\mathfrak{n}$, $\mathfrak{a}$ and $\mathfrak{m}$ so that $\Omega = (\ ) + \frac{1}{2}Z^2 + \Omega_M$. The term in parentheses can be rearranged to lie in $\mathscr{U}(\mathfrak{g})\mathfrak{n} + \mathfrak{a}$, the terms with $\mathfrak{n}$ on the right kill $\mathfrak{n}$-invariants, the terms in $\mathfrak{a}$ give the $-nd/dt$ term. □

In order to apply the Fröbenius reciprocity (Eq. $(*)$) to $W = \{\mathscr{H}^{(r,s)}\}^N$ we must determine W as an MA-module. We are almost done once we decompose $(\Lambda^{r,s} \otimes E)|_M$. This can be done, but we are more concerned with the less complicated case when E is one dimensional and $r = 0$. So let $E = \mathbb{C}_\chi$ where

$$\chi\left(\begin{bmatrix} A & \\ & a \end{bmatrix}\right) = a^{-l},$$

for $l \in \mathbb{Z}$. We use χ to also denote the weight $-l\varepsilon_{n+1}$ and we denote the line bundle $\mathscr{E}$ by $\mathscr{L}_\chi$.

LEMMA. $(\Lambda^{(0,s)} \otimes \mathbb{C}_\chi)|_M \simeq W_1 \oplus W_2$ *(for* $s = 0, 1, \ldots, n$*) where the* W_i *are irreducible M-modules with highest weights:*

$$\sigma_1 = \left(\frac{-l-s+1}{2}, 1, \ldots 1, 0, 0, \ldots 0, \frac{-l-s+1}{2}\right) \quad \textit{and}$$

$$\sigma_2 = \left(\frac{-l-s}{2}, \underbrace{1, \ldots 1, 1}_{s}, 0, \ldots 0, \frac{-l-s}{2}\right)$$

(σ_1 *does not occur for* $s = 0$ *and* σ_2 *does not occur for* $s = n$*).*

Proof. This follows immediately from the branching law for the restriction of representations of $U(n)$ to $U(n-1)$ see **[18]**. □

Now we are in position to solve the Laplacian on $\{A^{(0,s)}(X, \mathscr{L}_\chi)\}^N$. Recall that Ω_M acts on W_i by $\langle \sigma_i, \sigma_i + 2\rho(\mathfrak{m})\rangle$. There are two cases $\varphi(t) \subset W_i$, $i = 1, 2$. By the preceding two lemmas we get

$$W_1 \colon \Box\varphi = \frac{1}{8}\left(\frac{d}{dt} - (2n - (s - l - 1))\right)\left(\frac{d}{dt} - (s - l - 1)\right)\varphi \quad \text{and}$$

$$W_2 \colon \Box\varphi = \frac{1}{8}\left(\frac{d}{dt} - (2n - (s - l))\right)\left(\frac{d}{dt} - (s - l)\right)\varphi.$$

The solutions are:

$$w_1 e^{(n \pm (n - (s - l - 1)))t}, \quad w_1 \in W_1, \quad \text{and}$$
$$w_2 e^{(n \pm (n - (s - l)))t}, \quad w_2 \in W_2.$$

Since the left action of Z is $-d/dt$ we conclude the following.

PROPOSITION. $W = \{\mathscr{H}^{(0,s)}(X, \mathscr{L}_\chi)\}^N \otimes \mathbb{C}_\rho$ ($\mathbb{C}_\rho$ *gives the proper* ρ*-shift for Fröbenius reciprocity) is the sum of four irreducible P-modules as follows:*

(i) *As an M-module* W *is the sum of two copies of* $W_1 \oplus W_2$,
(ii) A *acts by* e^ν *with* $\nu \in \mathfrak{a}^*$ *given by* $\nu(Z) = \pm(n - (s - l - 1))$ *on the two copies of* W_1, *and* $\nu(Z) = \pm(n - (s - l))$ *on the two copies of* W_2.
(iii) N *acts trivially.*

COROLLARY. *For each of the four cases above there is a nonzero Poisson transform of the corresponding principal series representations into* $\mathscr{H}^{(0,s)}$.

The operators $\overline{\partial}$ and $\overline{\partial}^*$ acting on N-invariants can be written in explicit form as follows. Let $X_1 = \left[\begin{array}{c|c} & 1 \\ \hline & \end{array}\right]$, then $\langle X_1, \overline{X}_1\rangle = 1$ where $\overline{X}_1$ is the conjugate of X_1 with respect to the real form $\mathfrak{u}(n,1)$.

LEMMA. *For $\varphi \in \{A^{(0,s)}(X, \mathscr{L}_\chi)\}^N$ decompose φ as $\varphi_1 + \varphi_2$ where $\varphi_i(t) \in W_i$. Let e and i denote exterior and interior multiplication. Then*

$$\overline{\partial}\varphi_1 = 0, \qquad \overline{\partial}\varphi_2 = \frac{1}{2}e(X_1)\left(\frac{d\varphi_2}{dt} - (s-l)\varphi_2\right) \quad \textit{and}$$

$$\overline{\partial}^*\varphi_1 = \frac{1}{2}i(\overline{X}_1)\left(\frac{d\varphi_1}{dt} - (2n-(s-l-1))\varphi_1\right), \quad \overline{\partial}^*\varphi_2 = 0.$$

Proof. This is a calculation similar to the one giving the formula for $\square$ on the N-invariants (or see **[5]**). $\square$

1.3. Nonzero maps of four principal series representations into $\mathscr{H}^{(0,s)}(X, \mathscr{L}_\chi)$ have been constructed. We now study these maps in a bit more detail. Let $\mathscr{P}\colon I(W) \to \mathscr{H}^{(0,s)}(X, \mathscr{L}_\chi)$ be one such map.

LEMMA. $\operatorname{Ker}\mathscr{P}$ *is the sum of all G-submodules of $I(W)$ not containing the K type $\Lambda^{0,s} \otimes \mathbb{C}_\chi$.*

Proof. If $U \subset \operatorname{Ker}\mathscr{P}$ is a G-submodule then $\Lambda^{0,s} \otimes \mathbb{C}_\chi$ is not contained in U because the image of $\mathscr{P}$ contains this K-type (and the K-types in $I(W)$ occur with multiplicity one). On the other hand if U does not contain this K-type then $U \subset \operatorname{Ker}\mathscr{P}$, otherwise there is a $u \in U$ such that $\mathscr{P}(u) \neq 0$, i.e., $(\mathscr{P}u)(g) \neq 0$ for some $g \in G$. But this means that $\mathscr{P}(g^{-1} \cdot u)(e) \neq 0$, so evaluation at e gives a nonzero map $U \to \Lambda^{0,s} \otimes \mathbb{C}_\chi$. $\square$

Thus, we may determine the kernels of the Poisson transforms by determining the composition series of the $I(W)$. This is well known (see **[11]**, also **[1]** has a good treatment). The results needed are given below, first for infinitesimal character ρ, then an application of the translation principle gives the results for the correct infinitesimal character.

The irreducible admissible representations of $U(n,1)$ with infinitesimal character ρ are parametrized by non-negative integers a, b with $0 \le a+b \le n$ as follows:

For $0 \le a+b < n$, $J_{a,b}$ is the Langlands quotient of $I_{a,b} \equiv \operatorname{Ind}_P^G(\sigma \otimes \nu)$ where $\nu = n-a-b$ and $\sigma \in \hat{M}$ has highest weight

$$\left(\frac{a-b}{2}, 1, \ldots, 1, 0, \ldots, 0, -1, \ldots, -1, \frac{a-b}{2}\right)$$

with b ones and a minus ones.

For $a+b=n$, $J_{a,b}$ is the discrete series representation associated to the chamber

$$\Big(\frac{n}{2}, \frac{n}{2}-1, \ldots, \frac{n}{2}-b+1, \frac{n}{b}-b-1, \\ \frac{n}{2}-b-2, \ldots, -\frac{n}{2}+1, -\frac{n}{2}, \frac{n}{2}-b\Big).$$

PROPOSITION. *The socle filtration of* $I_{a,b}$ *is*

$$\begin{tikzcd}[column sep=small] & J_{a,b} \arrow[dl, dash] \arrow[dr, dash] & \\ J_{a+1,b} \arrow[dr, dash] & & J_{a,b+1} \arrow[dl, dash] \\ & J_{a+1,b+1} & \end{tikzcd}$$

for $0 \leq a+b < n$ *(and for* $a+b=n-1$ *the bottom term does not appear).*

From §1.2 it is clear that we are concerned with principal series representations with infinitesimal character $\chi+\rho$. The appropriate form of the translation principle is given in **[10]**, Theorem B.1, page 496. So, suppose we have a principal series representation $\mathrm{Ind}_P^G(\sigma\otimes\nu)$ with infinitesimal character λ and we want to translate to a representation with infinitesimal character λ'. We consider λ, λ' as elements of some abstract Cartan subalgebra, λ and λ' must lie in the same chamber and $\lambda-\lambda'$ must be integral. The translation ψ is accomplished by tensoring by the finite dimensional G-module with extreme weight $\tilde{\mu}=\lambda'-\lambda$. Let $\mu \in ((\mathfrak{t}\cap\mathfrak{m})\oplus\mathfrak{a})^*$ be the Cayley transform of $\tilde{\mu}$.

PROPOSITION (*Knapp, Zuckerman* **[10]**).

$$\psi(\mathrm{Ind}_P^G(\sigma\otimes\nu)) = \mathrm{Inf}_P^G(\sigma'\otimes\nu')$$

where σ' *is the finite dimensional* M*-module with infinitesimal character* $\sigma+\rho(\mathfrak{m})+\mu|_{\mathfrak{t}\cap\mathfrak{m}}$ *and* $\nu'=\nu+\mu|\mathfrak{a}$.

We apply this to translation from ρ to $\chi+\rho$. As we will see in §§3.2 and 4.2 the interesting case is then $\chi+\rho$ satisfies the negativity condition:

$$\langle\chi+\rho,\beta\rangle<0, \quad \text{for } \beta\in\Delta(\mathfrak{p}_+).$$

(Actually, a slightly weaker condition suffices but there are some added (well understood) complications. This negativity condition for $\chi=-l\varepsilon_{n+1}$ is $l<-n$, assume this holds.

LEMMA. (i) *For* $0 \leq a + b < n$, $\psi(I_{a,b}) = \operatorname{Ind}_P^G(\sigma' \otimes \nu')$ *where* $\nu' = n - 1 - b$ *and* σ' *has highest weight*

$$\left(\frac{n}{2} - b + 1, -l - n + 1, \underbrace{2, \dots, 2,}_{b-1} 1, \dots, 1, \underbrace{0, \dots, 0,}_{a} \frac{n}{2} - b + 1\right),$$

for $b \neq 0$.

When $b = 0$, $\nu' = -a - l - 1$ *and* σ' *has highest weight*

$$\left(\frac{-l - n + 1 + 1}{2}, 1, \dots, 1, 0, \dots, 0, \frac{-l - n + a + 1}{2}\right)$$

with a zeros.

(ii) *For* $a + b = n$, $\psi(J_{a,b})$ *is the discrete series representation with infinitesimal character* $\chi + \rho$ *associated to the same chamber as* $J_{a,b}$.

Proof. This follows from the preceding two propositions by straightforward, but tedious, calculations. □

Let $I_{a,b}(\chi)$ be $\psi(I_{a,b})$ and $J_{a,b}(\chi)$ the irreducible quotient. Then the $I_{a,b}(\chi)$ have the same socle filtrations as the $I_{a,b}$. The four principal series representations mapping into the harmonic space are $I(W) = I_{n-s,0}(\chi)$, $I_{n-s-1,0}(\chi)$ and their duals. The filtrations are:

(i) $W|_M = W_1, \nu(Z) = -(n - (s - l - 1))$.

$2 \leq s \leq n$:

$$\begin{tikzcd}[column sep=small] & J_{n-s,0}(\chi) \arrow[dr, dash] & \\ J_{n-s+1,0}(\chi) \arrow[ur, dash] \arrow[dr, dash] & & J_{n-s,1}(\chi) \\ & J_{n-s+1,1}(\chi) \arrow[ur, dash] & \end{tikzcd}$$

$s = 1$:

$$\begin{tikzcd}[column sep=small] & J_{n-1,0}(\chi) \arrow[dr, dash] & \\ J_{n,0}(\chi) \arrow[ur, dash] & & J_{n-1,1}(\chi) \end{tikzcd}$$

(ii) $W|_M = W_2, \nu(Z) = -(n - (s - l))$.

$0 \leq s \leq n - 1$:

$$\begin{tikzcd}[column sep=small] & J_{n-s-1,0}(\chi) \arrow[dr, dash] & \\ J_{n-s,0}(\chi) \arrow[ur, dash] \arrow[dr, dash] & & J_{n-s-1,1}(\chi) \\ & J_{n-s,1}(\chi) \arrow[ur, dash] & \end{tikzcd}$$

(iii) Reverse the diagrams for the duals.

LEMMA. $\Lambda^{0,s} \otimes \mathbb{C}_\chi$ *occurs as a K-type in* $J_{n-s,0}(\chi)$, *and occurs in no other* $J_{a,b}(\chi)$.

Proof. This K-type occurs in $I_{n-s,0}(\chi)$ and $I_{n-s-1,0}(\chi)$ and in no other $I_{a,b}(\chi)$ (otherwise there would be more Poisson transforms, see §1.2). By (i) above, $\Lambda^{0,s} \otimes \mathbb{C}_\chi$ must occur in $J_{n-s,0}(\chi)$. By (ii) it does not occur in $J_{n-s-1,0}(\chi)$. □

COROLLARY. *For the four Poisson transforms of* §1.2 *the kernels are as follows*:

$$W_1, \nu = n - (s - l - 1), \quad \operatorname{Ker}\mathscr{P} = 0,$$
$$W_1, \nu = -n + (s - l - 1),$$
$$\operatorname{Ker}\mathscr{P} = J_{n-s+1,0}(\chi) + J_{n-s,1}(\chi) + J_{n-s+1,1}(\chi),$$
$$W_2, \nu = n - (s - l), \quad \operatorname{Ker}\mathscr{P} = J_{n-s-1,0}(\chi) + J_{n-s-1,1}(\chi),$$
$$W_2, \nu = -n + (s - l), \quad \operatorname{Ker}\mathscr{P} = J_{n-s,1}(\chi) + J_{n-s-1,1}(\chi).$$

REMARK. Similar explicit calculations can be carried out for any homogeneous vector bundle $\mathscr{E} \to X$ (no negativity condition necessary). One finds that $\mathscr{H}^{r,s}(X, \mathscr{E})$ can have summands with different infinitesimal characters.

2. Our main concern is the construction of harmonic forms on the indefinite Kähler symmetric space $U(p+1, q)/U(p, q) \times U(1)$. This is the non-Riemannian dual X^0 of hyperbolic space X in the sense of Flensted-Jensen (and described below). There is a correspondence between harmonic forms on X^0 and harmonic forms on X.

2.1. As in §1, let $\mathfrak{g}_0 = \mathfrak{u}(n, 1)$, $\mathfrak{k}_0 = \mathfrak{u}(n) \times \mathfrak{u}(1)$, etc. The Cartan involution is $\theta = \operatorname{Ad}\left[\begin{smallmatrix} I_n & 0 \\ 0 & -1 \end{smallmatrix}\right]$. Consider the involution

$$\sigma = \operatorname{Ad}\begin{pmatrix} I_p & & \\ & -I_q & \\ & & 1 \end{pmatrix}, \qquad \text{where } p + q = n.$$

The two involutions commute so we may decompose $\mathfrak{g}_0$ (and $\mathfrak{g}$) in several ways.

$$\mathfrak{g}_0 = \mathfrak{k}_0 + \mathfrak{p}_0, \pm 1 \text{ eigenspaces of } \theta.$$
$$\mathfrak{g}_0 = \mathfrak{h}_0 + \mathfrak{q}_0, \pm 1 \text{ eigenspaces of } \sigma.$$
$$\mathfrak{g}_0 = \mathfrak{h}_0 \cap \mathfrak{k}_0 + \mathfrak{k}_0 \cap \mathfrak{q}_0 + \mathfrak{h}_0 \cap \mathfrak{p}_0 + \mathfrak{p}_0 \cap \mathfrak{q}_0.$$

Define:

$$\mathfrak{g}_0^0 = \mathfrak{h}_0 \cap \mathfrak{k}_0 + i\mathfrak{k}_0 \cap \mathfrak{q}_0 + i\mathfrak{h}_0 \cap \mathfrak{p}_0 + \mathfrak{p}_0 \cap \mathfrak{q}_0.$$
$$\mathfrak{k}_0^0 = \mathfrak{h}_0 \cap \mathfrak{k}_0 + i\mathfrak{k}_0 \cap \mathfrak{q}_0.$$
$$\mathfrak{h}_0^0 = \mathfrak{h}_0 \cap \mathfrak{k}_0 + i\mathfrak{h}_0 \cap \mathfrak{p}_0.$$

G^0 is defined to be the connected subgroup of $G_{\mathbb{C}} = \mathrm{GL}(n+1, \mathbb{C})$ with Lie algebra $\mathfrak{g}_0^0$. Thus $G^0 = U(p+1, q)$, defined by the hermitian form

$$\begin{bmatrix} I_p & & \\ & -I_q & \\ & & 1 \end{bmatrix}.$$

Similarly, $K^0 = (G^0)^\theta \simeq U(p,q)\times U(1)$, $H^0 = (G^0)^\sigma \cong U(p+1)\times U(q)$, $K = G^\theta = U(n) \times U(1)$ and $U = G^\sigma \cong U(p,1) \times U(q)$. Note that K^0 is noncompact, H^0 is a maximal compact subgroup of G^0 and σ is a Cartan involution of G^0.

DEFINITION. The *non-Riemannian dual* of $X = G/K$ is $X^0 = G^0/K^0$.

There is a G^0-invariant complex structure on X^0 so that the holomorphic and antiholomorphic tangent spaces are $\mathfrak{p}^0_\pm = \mathfrak{p}_\pm$.

2.2. Here we extend the duality of Flensted-Jensen ([2]),

$$C^\infty(X)_{H\text{-finite}} \simeq C^\infty(X^0)_{H^0\text{-finite}},$$

to a vector bundle setting. So, let (π, E) be an irreducible finite dimensional representation of K with highest weight χ and let $\mathscr{E} \to X$ be the corresponding homogeneous holomorphic vector bundle. Since K and K^0 have the same complexifications there is an irreducible finite dimensional representation (π^0, E) of K^0 with highest weight χ. There is a one-to-one correspondence between the finite dimensional irreducible representations of K and those of K^0 (for more general G, H, $K\ldots$ integrality conditions must be considered and this is not quite the case). The same holds for H and H^0.

PROPOSITION. *There is a left $\mathscr{U}(\mathfrak{g})$-isomorphism*

$$A^{(r,s)}(X, \mathscr{E})_{H\text{-finite}} \simeq A^{(r,s)}(X^0, \mathscr{E}^0)_{H^0\text{-finite}}$$

preserving the action by the invariant differential operators.

Proof. We state this a little differently as follows. Since $(T^\pm X)^* \simeq \mathscr{E}$ and $(T^\pm X^0)^* \simeq \mathscr{E}^0$ for $E \simeq \mathfrak{p}_\mp$, we may absorb the exterior algebra terms in E (i.e., replace E by $\Lambda^{r,s} \otimes E$) and consider sections instead of forms. Also, if (δ, F) is an irreducible finite dimensional H-module we need only show an isomorphism

$$C^\infty(X, \mathscr{E})_\delta \simeq C^\infty(X^0, \mathscr{E}^0)_{\delta^0},$$

where the subscript means the H (H^0) finite vectors of type δ (δ^0). Thus we show

$$(*) \qquad \{C^\infty(G) \otimes F \otimes E\}^{H,K} \simeq \{C^\infty(G^0) x F x E\}^{H^0,K^0}$$

where the action of H (resp. K) on $C^\infty(G)$ is by left (respectively right) translation and similarly for the actions of H^0 and K^0 on $C^\infty(G^0)$.

The decomposition $G = \exp(\mathfrak{h}_0 \cap \mathfrak{p}_0)\exp(\mathfrak{p}_0 \cap \mathfrak{q}_0)K$ (see **[13]**, Theorem 5, page 31) will be used. For φ in the right hand side of $(*)$, define φ^0 in the left hand side by

$$\varphi^0(\exp X \exp Yk) = (\delta(\exp X)^{-1} \otimes \pi(k)^{-1})\varphi(\exp Y)$$

where $X \in \mathfrak{h}_0 \cap \mathfrak{p}_0$, $Y \in \mathfrak{p}_0 \cap \mathfrak{q}_0$, $k \in K$ and note that $\exp(\mathfrak{p}_0 \cap \mathfrak{q}_0) \subset G \cap G^0$. It is easy to see that $\varphi \to \varphi^0$ is a linear isomorphism. The proof that it is a $\mathscr{U}(\mathfrak{g})$ isomorphism (on the sum over δ, δ^0 of each side of $(*)$) is exactly as in the case of functions ($E = \mathbb{C}$), see **[2]** or **[16]** Theorem 8.2.1.

As for the action of any invariant differential operator, note that an invariant differential operator $\mathscr{E}_1 \to \mathscr{E}_2$ may be identified with an element of $\{\mathscr{U}(\mathfrak{g}) \otimes \mathrm{Hom}_{\mathbb{C}}(E_1, E_2)\}^K$ where the action of K is by Ad on $\mathscr{U}(\mathfrak{g})$ and $k \cdot A = \pi_2(k) \cdot A \cdot \pi_1(k^{-1})$ for $A \in \mathrm{Hom}_{\mathbb{C}}(E_1, E_2)$. An element $D = \sum u_i \otimes A_i$ acts on $\{C^\infty(G) \otimes E\}^K$ by $D(\varphi) = \sum_i A_i(r(u_i)\varphi)$ where $r(u_i)$ is the right action on $C^\infty(G)$. An invariant differential operator on X^0 is given by the same formula (and the same D, note that the K-invariants coincide with the K^0-invariants), call it the dual operator. We check that $D\varphi^0 = (D\varphi)^0$. It is enough to check $(D\varphi^0)(y) = (D\varphi)^0(y)$ for $y = \exp Y \in \exp(\mathfrak{p}_0 \cap \mathfrak{q}_0)$, by the invariance of D. We show $((r(Z) \otimes A)\varphi^0)(y) = ((r(Z) \otimes A)\varphi)^0(y)$ for $Z \in \mathfrak{g}$, $A \in \mathrm{Hom}_{\mathbb{C}}(E_1, E_2)$. Since functions corresponding by $(\cdot)^0$ agree on $\exp(\mathfrak{p}_0 \cap \mathfrak{q}_0)$ we may drop the 0 on the right hand side. Also, A plays no role, so we show

$$(**) \qquad (r(Z)\varphi^0)(y) = (r(Z)\varphi)(y), \quad Z \in \mathfrak{g}, y \in \exp(\mathfrak{p}_0 \cap \mathfrak{q}_0).$$

There is a decomposition $\mathfrak{g} = \mathfrak{k} + \mathfrak{p} \cap \mathfrak{q} + \mathrm{Ad}(y^{-1})\mathfrak{h} \cap \mathfrak{p}$ for any $y \in \exp(\mathfrak{p}_0 \cap \mathfrak{q}_0)$ (see **[16]**, page 153). We check $(**)$ separately for Z in each summand.

$$\begin{aligned} Z \in \mathfrak{k}, (r(Z)\varphi^0)(y) &= \frac{d}{ds}\varphi^0(y \exp sZ)|_{s=0} \\ &= \frac{d}{ds}\pi(\exp(-sZ))\varphi(y)|_{s=0} \\ &= \frac{d}{ds}\varphi(y \exp sZ)|_{s=0} = (r(Z)\varphi)(y). \end{aligned}$$

$$\begin{aligned} Z \in \mathfrak{p} \cap \mathfrak{q}, (r(Z)\varphi^0)(y) &= \frac{d}{ds}\varphi^0(y \exp sZ)|_{s=0} \\ &= \frac{d}{ds}\varphi(y \exp sZ)|_{s=0} = (r(Z)\varphi)(y) \end{aligned}$$

$$
\begin{aligned}
Z &= \mathrm{Ad}(y^{-1}Z', Z' \in \mathfrak{h} \in \mathfrak{p}, \\
(r(Z)\varphi^0)(y) &= \frac{d}{ds}\varphi^0(y \exp(\mathrm{Ad}(y^{-1})sZ'))|_{s=0} \\
&= \frac{d}{ds}\varphi^0(\exp(sZ')y)|_{s=0} \\
&= \frac{d}{ds}\delta(\exp(-sZ'))\varphi(y)|_{s=0} \\
&= \frac{d}{ds}\varphi(\exp(sZ')y)|_{s=0} \\
&= \frac{d}{ds}\varphi(y \exp(\mathrm{Ad}(y^{-1})sZ'))|_{s=0} = (r(Z)\varphi)(y).
\end{aligned}
$$

The important cases of this are $\overline{\partial}$, $\overline{\partial}^*$ and $\square = \overline{\partial}\overline{\partial}^* + \overline{\partial}^*\overline{\partial}$. $\overline{\partial}$ is expressed by choosing a basis $\{X_j\}$ of $\mathfrak{p}_+$ so that $\langle X_j, \overline{X}_k\rangle = \delta_{jk}$, then $\overline{\partial} = \sum r(\overline{X}_j) \otimes e(X_j)$. The dual operator $\overline{\partial}^0$ on X^0 is given by the same formula, we check that this is the $\overline{\partial}$ operator on X^0.

The $\overline{\partial}$-operator on X^0 is given by choosing bases $\{X_j\}_{j=1,\dots,p}$ of $\mathfrak{p}_+ \cap \mathfrak{h}^0$ and $\{Y_k\}_{k=1,\dots,q}$ of $\mathfrak{p}_+ \cap \mathfrak{q}^0$ so that $\langle X_j, \overline{X}_k\rangle = -\delta_{jk}$ and $\langle Y_j, \overline{X}_k\rangle = \delta_{jk}$ (—is conjugation w.r.t. the real form G^0). Then

$$\overline{\partial}_{X^0} = -\sum_1^p r(\overline{X}_j) \otimes e(X_j) + \sum_1^q r(\overline{Y}_k) \otimes e(Y_k).$$

This is given by the same formula as $\overline{\partial}$ given in the preceding paragraph because there the conjugation is w.r.t. G and this differs from the conjugation for G^0 only by a minus sign on the $X_j \in \mathfrak{p}_+ \cap \mathfrak{h}^0$. A similar calculation for $\overline{\partial}^*$ gives the following lemma.

LEMMA. *The operators on X^0 dual to $\overline{\partial}$, $\overline{\partial}^*$ and $\square$ on X are the $\overline{\partial}, \overline{\partial}^*$-operators and the Laplacian on X^0.*

COROLLARY. *The proposition gives a one-to-one correspondence between the H-finite vectors in the space of $\mathscr{E}$-valued harmonic forms on X and the H^0 finite vectors in the space of $\mathscr{E}^0$-valued harmonic forms on X^0.*

3. By §1 we have nonzero maps of certain principal series representations into $\mathscr{H}^{(0,s)}(X, \mathscr{L}_\chi)$. Since our goal is to determine H^0-finite $\mathscr{L}_\chi^0$-valued harmonic forms on X^0, we must produce H-finite vectors in the appropriate principal series representations. These will be distributions supported in closed H-orbits in G/P.

3.1. As in §1 $P = MAN$ is a minimal parabolic subgroup of G with $\mathfrak{a}_0 = \mathbb{R}Z$,

$$Z = \begin{bmatrix} 0 & & 1 \\ & \ddots & \\ 1 & & 0 \end{bmatrix}.$$

H is the fixed points of

$$\sigma = \mathrm{Ad}\begin{pmatrix} I_p & & \\ & -I_q & \\ & & 1 \end{pmatrix}.$$

By the work of Matsuki ([**12**]) there are two H-orbits in G/P. One can see this directly as follows.

Give $\mathbb{C}^{n+1}$ the hermitian form $-\sum_{i=1}^{n} |z_i|^2 + |z_{n+1}|^2$. G/K can be identified with {positive lines in $\mathbb{C}^{n+1}$} $\simeq$ open unit ball in $\mathbb{C}^n$. G/P can be identified with {null lines in $\mathbb{C}^{n+1}$} $\simeq S^{2n-1}$ (the boundary of the domain G/K). There are two H-orbits in {null lines} $(p, q \geq 1)$ by Witt's theorems. They are:

$$\mathcal{O} \equiv H \cdot \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ - \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \simeq U(p,1)/U(p,1) \cap P \approx S^{2p-1}$$

and

$$\mathcal{O}' \equiv H \cdot \begin{bmatrix} 0 \\ \vdots \\ 0 \\ - \\ 1 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}.$$

$\mathcal{O}$ is closed and $\mathcal{O}'$ is open.

It is very important for the upcoming calculations that for the P that we have chosen $H \cap P$ is a minimal parabolic in H and $H = H \cap K \cdot A \cdot H \cap N$ is the Iwasawa decomposition of H. Thus, for the closed orbit we have $\mathscr{O} \approx H/H \cap P \approx H \cap K/H \cap M$.

3.2. Certain H-finite vectors in $I(W)$ are written down explicitly when W is $W_1 \otimes e^{\nu}$ or $W_2 \otimes e^{\nu}$ as in §1.2. These will be distributions given in terms of the invariant measure on the closed orbit and certain matrix coefficients. This is an extension of the construction in [2].

Let (δ, F) be a finite dimensional irreducible H-module. Let W be one of the four possible P-modules from §1. Suppose that

$$\{F^* \otimes W \otimes \mathbb{C}_{-\rho+2\rho_{\mathfrak{h}}}\}^{H\cap P} \simeq \mathrm{Hom}_{H\cap P}(F, W \otimes \mathbb{C}_{-\rho+2\rho_{\mathfrak{h}}})$$

contains a nonzero element t, ($\rho_{\mathfrak{h}}$ is half the sum of the positive $\mathfrak{a}$ roots in $\mathfrak{h}$). The corresponding matrix coefficients are $h \to t(\delta(h^{-1})v)$, $v \in F$, $h \in H$.

Definition. $T_v(\varphi) \equiv \int_{H\cap K} \langle \varphi(l), t(\delta(l^{-1})v)\rangle \, dl$, $\varphi \in C^\infty(K/M, W^*) \equiv \{C^\infty(K) \otimes W^*\}^M$.

Proposition. *T_v is H-finite of type (δ, H) in $I(W)$.*

Proof. We will use the following facts.

(i) $C^\infty(K/M, W^*) \simeq C^\infty(G/P, W^*)$ as K-modules, the identification is $\Phi(g) = e^{\langle \nu-\rho, H(g)\rangle}\varphi(\kappa(g))$. The Iwasawa decomposition KAN determines κ and H: $g \in \kappa(g)e^{H(g)}N$. Thus, for $h \in H$, $(h^{-1} \cdot \varphi)(l) = e^{\langle \nu-\rho, H(hl)\rangle}\varphi(\kappa(hl))$, any l.

(ii) $\int_K f(k)\,dk = \int_K f(\kappa(g^{-1}k))e^{-\langle 2\rho, H(g^{-1}k)\rangle}\,dk$ for any $g \in G$, this is proved in [7], page 197.

(iii) Let $l \in H \cap K$, $h \in H$, $l = hh^{-1}l = h\kappa(h^{-1}l)e^{H(h^{-1}l)}n$ implies $\kappa(h\kappa(h^{-1}l)) = l$ and $H(h\kappa(h^{-1}l)) = -H(h^{-1}l)$.

(iv) Let $l \in H \cap K$, $h \in H$, $v \in F$.

$$\begin{aligned} t(\delta(l^{-1})\delta(h)v) &= t(\delta(h^{-1}l)^{-1}v) = t(\delta(\kappa(h^{-1}l)e^{H(h^{-1}l)})^{-1}(v)) \\ &= e^{-H(h^{-1}l)}t(\delta(\kappa(h^{-1}l))^{-1}v) \\ &= e^{-\langle \nu-\rho+2\rho_{\mathfrak{h}}, H(h^{-1}l)\rangle}t(\delta(\kappa(h^{-1}l))^{-1}v). \end{aligned}$$

Now we prove the proposition. Let $h \in H$.

$$\begin{aligned}
(h \cdot T_v)(\varphi) &= T_v(h^{-1} \cdot \varphi) \\
&= \int_{H\cap K} e^{\langle \nu-\rho, H(hl)\rangle} \langle \varphi(\kappa(hl)), t(\delta(l^{-1})v)\rangle \, dl, \quad \text{by (i).} \\
&= \int_{H\cap K} e^{\langle \nu-\rho, H(h\kappa(h^{-1}l))\rangle} \langle \varphi(\kappa(h\kappa(h^{-1}l))), t(\delta(\kappa(h^{-1}l))^{-1}v)\rangle \\
&\quad \cdot e^{-\langle 2\rho_{\mathfrak{h}}, H(h^{-1}l)\rangle} \, dl, \quad \text{by (ii).} \\
&= \int_{H\cap K} e^{-\langle \nu-\rho+2\rho_{\mathfrak{h}}, H(h^{-1}l)\rangle} \langle \varphi(l), t(\delta(\kappa(h^{-1}l))^{-1}v)\rangle \, dl, \quad \text{by (iii).} \\
&= \int_{H\cap K} \langle \varphi(l), t(\delta(l^{-1})\delta(h)v)\rangle \, dl \\
&= T_{\delta(h)v}(\varphi).
\end{aligned}$$

Therefore $v \to T_v$ is an H-homomorphism of F into $I(W)$. □

REMARK. The motivation for the definition is as follows. If M is a manifold, $N \subset M$ a closed submanifold, $\mathscr{V} \to M$ a vector bundle and $\mathscr{V} \to N$ the pullback to N, then the space of $\mathscr{V}$-valued distributions on M is $\mathscr{D}'(M, \mathscr{V}) \equiv C_0^\infty(M, \mathscr{V}^* \otimes \Lambda^{\text{top}} T^*M)'$. This contains $\mathscr{D}'(N, \mathscr{V} \otimes \Lambda^{\text{top}}(TM/TN))$. Therefore the distributions on M contain the sections $C^\infty(N, \mathscr{V} \otimes \Lambda^{\text{top}}(TM/TN))$ (and does not contain $C^\infty(N, \mathscr{V})$). In our situation we have $\mathscr{O} = H/H \cap P \subset G/P$, $\Lambda^{\text{top}} T^* = \mathbb{C}_{2\rho}$, $\Lambda^{\text{top}}(TM/TN) = \mathbb{C}_{-2\rho+2\rho_{\mathfrak{h}}}$, $\mathscr{V} = W \otimes \mathbb{C}_\rho$ so we see that

$$C^\infty(H/H \cap P, W x \mathbb{C}_{-\rho+2\rho_{\mathfrak{h}}}) \subset \mathscr{D}'(G/P, W \otimes \mathbb{C}_\rho) \subset I(W)$$

(and the shift by $-\rho + 2\rho_{\mathfrak{h}}$ is explained). Finally,

$$C^\infty(H/H \cap P, W \otimes \mathbb{C}_{-\rho+2\rho_{\mathfrak{h}}})_{H\text{-finite}} \simeq \sum F \otimes \{F^* \otimes W \otimes \mathbb{C}_{-\rho+2\rho_{\mathfrak{h}}}\}^{H\cap P},$$

where the sum is over all finite dimensional irreducible H-modules.

Now we must determine which (δ, F) satisfy $\{F^* \otimes W \otimes \mathbb{C}_{-\rho+2\rho_{\mathfrak{h}}}\}^{H\cap P} \neq 0$, note that this is equivalent to

$$\operatorname{Hom}_{MA\cap H}(F/(\mathfrak{n} \cap \mathfrak{h})F, W \otimes \mathbb{C}_{-\rho+2\rho_{\mathfrak{h}}}) \neq 0.$$

Recall from §1 that there are four possibilities for W. We must decompose each $W \otimes \mathbb{C}_{-\rho+2\rho_{\mathfrak{h}}}$ as $MA \cap H$-module.

Case 1. $W = W_1 \otimes e^{n-(s-l-1)}$. $W_1|_{M\cap H}$ decomposes into $M \cap H$ modules with lowest weights

$$\left(\frac{-l-s+1}{2}, 0, \ldots 0, 1, \ldots 1 | 0, \ldots 0, 1, \ldots 1, \frac{-l-s+1}{2}\right)$$

with $s-1$ ones appearing (the line separates the pth and $p+1$th slots). After applying the Cayley transform, $W \otimes \mathbb{C}_{-\rho+2\rho_{\mathfrak{h}}}|_A$ has the following weight on the compact Cartan subalgebra:

$$\left(\frac{2p-(s-l-1)}{2}, 0, \dots 0, -\frac{2p-(s-l-1)}{2}\right).$$

Therefore $(p-s+1, 0\dots 0, 1, \dots 1|0, \dots 0, 1, \dots 1, -p-l+1)$ is the lowest weight of an H-module with $\{F^* \otimes W \otimes \mathbb{C}_{-\rho+2\rho_{\mathfrak{h}}}\}^{H\cap P} \neq 0$ provided this weight is antidominant for H. This is the case as follows: For $s \geq p+1$, $(p-s+1, 0, \dots 0, 1, \dots 1|0, \dots 0, 1, \dots 1, -p-l+1)$ when $l \leq -p$ (or $l \leq -p+1$ in case there are no ones in the first p places) and for $s = p$, $(1, 1, \dots 1|0, \dots 0, -p-l+1)$ when $l \leq -p$.

Case 2. $W = W_1 \otimes e^{-(n-(s-l-1))}$. The lowest weights of possible F are: For $s \leq n-p$, $(-n+p-l, 0, \dots 0, 1, \dots 1|0, \dots 0, 1, \dots 1, n-p-s+1)$ (with $s-1$ ones) when $l \geq -n+p$ (or $l \geq -n+p+1$ in case there are no zeros in the first p places) and for $s = n-p-1$, $(-n+p-l, 0, \dots 0|1, \dots 1, 0)$ when $l \geq -n+p$.

Case 3. $W = W_2 \otimes e^{n-(s-l)}$. The lowest weights of possible F are: For $s \geq p$, $(p-s, 0, \dots 0, 1, \dots 1|0, \dots 0, 1, \dots 1, -p-l)$ (with s ones) $l \leq -p-1$ (or $l \leq -p$ in case there are no ones in the first p places) and for $s = p-1$, $(1, \dots 1|0, \dots 0, -p-l)$ when $l \leq -p-1$.

Case 4. $W = W_2 \otimes e^{-(n-(s-l))}$. The lowest weights of possible F are: For $s \leq n-p-1$, $(-l-n+p, 0, \dots 0, 1, \dots 1|0, \dots 0, 1, \dots 1, n-p-s)$ (with s ones) when $l \geq -n+p$ (or $l \geq -n+p-1$ in case there are no zeros in the first p places) and for $s = n-p$, $(-l-n+p, 0, \dots 0|1, \dots 1, 0)$ when $l \geq -n+p$.

3.3. The following theorem gives information on the possible H-types in $I(W)$. Let W be one of the four cases listed at the end of §3.2 and assume that H-types listed there occur (i.e., the conditions on s and l hold).

THEOREM 3.3. *Let T be an H-finite distribution in $I(W)$ supported in the closed orbit and suppose that T is a lowest weight vector. Let $\Lambda \in (\mathfrak{t}\cap\mathfrak{m})^* \oplus \mathfrak{a}^*$ be this lowest weight. Then $\Lambda = \delta_\nu - \sum n_\beta \beta$, where the sum is over all positive $(\mathfrak{t}\cap\mathfrak{m}) \oplus \mathfrak{a}$-roots in $\mathfrak{q}$ with $\beta|\mathfrak{a} \neq 0$, $n_\beta \geq 0$ and δ_ν is one of the lowest weights listed in the four cases of* §3.2.

Proof. Let δ_ν be one of the lowest weights listed in §3.2, (δ, F) the corresponding finite dimensional irreducible H-module and let

$\{T_v\}_{v\in F}$ be the corresponding H-finite vectors in $I(W)$ of type δ. For $\varphi \in C^\infty(K/M, W^*)$

$$\begin{aligned}T_v(\varphi) &= \int_{H\cap K} \langle \varphi(l), t(\delta(l^{-1})v)\rangle \, dl \\ &= \int_{\overline{N}\cap H} \int_{M\cap H} \langle \varphi(\kappa(\overline{n})m), t(\delta(\kappa(\overline{n})m)^{-1}v)\rangle e^{-\langle 2\rho_{\mathfrak{h}}, H(\overline{n})\rangle} \, dm \, d\overline{n} \\ &= \int_{\overline{N}\cap H} \langle \varphi(\kappa(\overline{n})), t(\delta(\kappa(\overline{n}))^{-1}v)\rangle e^{-\langle 2\rho_{\mathfrak{h}}, H(\overline{n})\rangle} \, d\overline{n}.\end{aligned}$$

The second equality is an integration formula (see [7], page 198) and the last equality is because $m \in M \cap H$. Now take v to be a lowest weight vector v_- w.r.t. $N \cap H$ for (δ, F). Then

$$\begin{aligned}t(v_-) = t(\delta(\overline{n})^{-1}v_-) &= t(\delta(n^{-1}e^{-H(\overline{n})}\kappa(\overline{n})^{-1})v_-) \\ &= e^{-\langle \nu-\rho+2\rho_{\mathfrak{h}}, H(\overline{n})\rangle} t(\delta(\kappa(\overline{n})^{-1})v_-).\end{aligned}$$

Hence,

$$T_{v_-}(\varphi) = \int_{\overline{N}\cap H} \langle \varphi(\kappa(\overline{n})), t(v_-)\rangle e^{\langle \nu-\rho, H(\overline{n})\rangle} \, d\overline{n}.$$

There is an inclusion $\overline{N} \approx \overline{N} \cdot eP \hookrightarrow G/P \approx K/M$ as an open and dense submanifold, and $\overline{N} \cap H$ is open and dense in the closed orbit. We may restrict distributions T on K/M to distributions T' on $\overline{N}$. Under the identification of $C^\infty(K/M, W^*) \simeq C^\infty(G/P, W^*)$ we have $\Phi(g) = e^{\langle \nu-\rho, H(g)\rangle}\varphi(\kappa(g))$. Therefore,

$$T'_{v_-}(\Phi) = \int_{\overline{N}\cap H} \langle \Phi(\overline{n}), t(v_-)\rangle \, d\overline{n}.$$

This formula defines an $\overline{N} \cap H$-invariant distribution on $\overline{N}$ supported in $\overline{N} \cap H$. Since $C^\infty(\overline{N}, W^*) \simeq C^\infty(\overline{N}) \otimes W^*$ we conclude that all W^*-valued distributions on $\overline{N}$ which are $\overline{N} \cap H$-invariant and supported in $\overline{N} \cap H$ are normal derivatives of some T'_{v_-} ($v_- \in F$ for some (δ, F) listed in §3.2).

Let T be as in the theorem and T' its restriction to $\overline{N}$. T' is $\overline{N} \cap H$ invariant (because T is a lowest weight vector) and has support in $\overline{N} \cap H$. We conclude that $T' = u \cdot T'_{v_-}$ for some v_- and some $u \in \mathscr{U}(\overline{\mathfrak{n}} \cap \mathfrak{q})$. Here u is acting by left invariant differential operator on $\overline{N}$ and $\overline{\mathfrak{n}} \cap \mathfrak{q}$ is the direction in $\overline{N}$ normal to $\overline{N} \cap H$. We may also assume v_- is a lowest weight vector for $(\mathfrak{t} \cap \mathfrak{m}) \oplus \mathfrak{a}$ (since the same holds for T by assumption).

Let $Y \in (\mathfrak{t} \cap \mathfrak{m}) \oplus \mathfrak{a}$, $y = \exp Y$ and let $\Phi \in C_0^\infty(\overline{N}) \otimes W^*$.

Case (i), $Y \in \mathfrak{a}$. Note that $(y \cdot T)'(\Phi) = e^{\nu-\rho}(y)T'(\Phi \circ \mathrm{Ad}(y))$ because $(y^{-1} \cdot \Phi)(\overline{n}) = \Phi(y\overline{n}) = e^{\nu-\rho}(y)\Phi(\mathrm{Ad}(y)\overline{n})$.

$$\begin{aligned} e^{\Lambda}(y)T'(\Phi) &= (y \cdot T)'(\Phi) = e^{\nu-\rho}(y)T'(\Phi \circ \mathrm{Ad}(y)) \\ &= e^{\nu-\rho}(y)u \cdot T'_{v_-}(\Phi \circ \mathrm{Ad}(y)) \\ &= e^{\nu-\rho}(y) \int_{\overline{N}\cap H} \langle (\mathrm{Ad}(y)(u) \cdot \Phi)(\mathrm{Ad}(y)\overline{n}), t(v_-)\rangle \, d \\ &= e^{\nu-\rho}(y) \int_{\overline{N}\cap H} \langle (\mathrm{Ad}(y)u \cdot \Phi)(\overline{n}), t(v_-)\rangle e^{2\rho_{\mathfrak{h}}}(y)\, d\overline{n} \\ &= e^{\nu-\rho+2\rho_{\mathfrak{h}}}(y)(\mathrm{Ad}(y)u \cdot T'_{v_-})(\Phi). \end{aligned}$$

Case (ii), $Y \in \mathfrak{t} \cap \mathfrak{m}$. Let σ denote the representation of M on W. As above we have $(y^{-1} \cdot \Phi)(\overline{n}) = \sigma(y)^{-1}\Phi(\mathrm{Ad}(y)\overline{n})$. Therefore,

$$\begin{aligned} e^{\Lambda}(y)T'(\Phi) &= \int_{\overline{N}\cap H} \langle (\mathrm{Ad}(y)u \cdot \Phi)(\mathrm{Ad}(y)\overline{n}), \sigma(y)t(v_-)\rangle \, d\overline{n} \\ &= \int_{\overline{N}\cap H} \langle (\mathrm{Ad}(y)u \cdot \Phi)(\overline{n}), t(y \cdot v_-)\rangle \, d\overline{n} \\ &= e^{\delta_\nu}(y)(\mathrm{Ad}(y)u \cdot T'_{v_-})(\Phi). \end{aligned}$$

Since $u \in \mathscr{U}(\overline{\mathfrak{n}} \cap \mathfrak{q})$, u is a sum of terms $X^{n_1}_{-\beta_1} \cdots X^{n_j}_{-\beta_j}$ with β_j as in the theorem. Thus for $y \in (\mathfrak{t} \cap \mathfrak{m}) \oplus \mathfrak{a}$, $\mathrm{Ad}(y)u = e^{-\sum n_\beta \beta}(y)u$ and we conclude $\Lambda = \delta_\nu - \sum n_\beta \beta$. □

REMARK. The proof gives a bound on the number of times an H-type can occur in $I(W)$ with support in the closed orbit. This bound is the number of ways to write Λ as $\delta_\nu - \sum n_\beta \beta$ as in the theorem. To see this note that if S, T are as in the theorem and $S' = u \cdot T'_{v_-} = T'$ then $(S - T)' = 0$ on an open subset of the closed orbit. Since the support of an H-finite distribution is a union of H-orbits, $\mathrm{supp}(S - T) = \varnothing$ (since contained in $\mathscr{O} - \overline{N} \cap H$ and $\mathscr{O}$ is minimal).

REMARK. Theorem 3.3 holds for general semisimple symmetric spaces and any W (under conditions similar to those listed in cases 1–4 of §3.2), this is given in the appendix. In the case of $W|_M \approx \mathbb{C}$ this was done by Flensted-Jensen and Okamoto (**[4]**).

3.4. An explicit integral formula for H-finite harmonic forms on $X = G/K$ is given here. First we will explicitly write down the Poisson transform discussed in §1. Then we apply it to the H-finite distributions constructed in §3.2.

Consider the general situation of §1.1. The Fröbenius reciprocity stated there is

$$\mathrm{Hom}_P(W \otimes \mathbb{C}_\rho, V) \simeq \mathrm{Hom}_G(I(W), V)$$

and we assume that V is an admissible subrepresentation of $C^\infty(G/K,\mathscr{E})$. Given a G-homomorphism $I(W) \to V$ we obtain a P-homomorphism by restricting to $W \otimes \mathbb{C}_\rho \subset I(W)$. It takes a bit of care to go the other way. Suppose we are given a nonzero P-homomorphism $W \otimes \mathbb{C}_\rho \to V$. If this map is followed by evaluation at e we get an M-homomorphism $W|_M \to E|_M$. Let us make a few assumptions. (i) Assume that this map $W|_M \to E|_M$ is an isomorphism (there is no loss of generality), (ii) A acts on W by e^ν for some $\nu \in \mathfrak{a}^*$, and (ii) N acts trivially on W. Let $\{w_j\}$ be a basis of $E(W)$ and $\{w_j^*\}$ the dual basis of $E^*(W^*)$. It makes sense to define

$$(\mathscr{P}T)(x) = \sum_j T(e^{-\langle \nu+\rho, H(x^{-1}k)\rangle}\pi(\kappa(x^{-1})^{-1})w_j^*)w_j$$

for $T \in I(W) = \mathscr{A}(K/M, W^*)'$, $x \in G$.

PROPOSITION. *$\mathscr{P}$ is a G-homomorphism of $I(W)$ into V and the restriction of $\mathscr{P}$ to W is the original map $W \otimes \mathbb{C}_\rho \to V$.*

Proof (sketch). One must show that

$$e^{-\langle \nu+\rho, H(x^{-1}k)\rangle}\pi(\kappa(x^{-1}k)^{-1})w_j^* \in \mathscr{A}(K/M, W^*)$$

and that $\mathscr{P}T \in V \subset C^\infty(G/K,\mathscr{E})$ so that the definition makes sense. Also one must show that $\mathscr{P}$ is a G-homomorphism. These are routine (but tedious) calculations using simple properties of the Iwasawa decomposition. □

REMARK. $\mathscr{P}$ is called the *Poisson transform* of the P-homomorphism $W \to V$. In case E (and $W|_M$) are trivial this reduces to the usual Poisson transform

$$(\mathscr{P}T)(x) = \int_K e^{-\langle \nu+\rho, H(x^{-1}k)\rangle}T(k)\,dk,$$

see **[6]**.

REMARK. In case T is given by a function $f \in G^\infty(K/M, W)$ we get

$$(\mathscr{P}f)(x) = \int_K e^{-\langle \nu+\rho, H(x^{-1}k)\rangle}\pi(\kappa(x^{-1}k))f(k)\,dk.$$

We now apply $\mathscr{P}$ to the H-finite distributions constructed in §3.2 to obtain explicit formulas for H-finite harmonic forms on G/K. So let $T_v \in I(W)$ belong to the H-type (δ, F). Then

$$(\mathscr{P}T_v)(x) = \int_{H\cap K} e^{-\langle \nu+\rho, H(x^{-1}l)\rangle}\pi(\kappa(x^{-1}l))(t(\delta(l^{-1})v))\,dl.$$

This will be used in §4 to determine when these harmonic forms are L_2.

4. We now discuss the square integrability of the harmonic forms on X^0 which have been constructed above. A notion of L_2 is required, this comes from **[15]** (§7) and is described below.

4.1. The trace form defines a hermitian form $\langle X, \overline{Y}\rangle$ on $\mathfrak{p}^0$, thus giving a G^0-invariant indefinite hermitian metric on X^0. A noninvariant positive definite hermitian metric on X^0 is defined as follows. On $T_{eK^0}(X^0) \simeq \mathfrak{p}^0$ it is given by $-\langle X, \sigma\overline{Y}\rangle$. On other tangent spaces we define it by using the decomposition $G^0 = H^0 B K^0$, $B = \exp \mathfrak{b}$, $\mathfrak{b} \subset \mathfrak{p}_0 \cap \mathfrak{q}_0$ maximal abelian. An arbitrary point of X^0 is $x = h^0 b K^0$ and tangent vectors at x are of the form $v = \tau_h \circ \tau_b(X)$, for $X \in \mathfrak{p}^0$ (τ_g is left translation by $g \in G^0$). Set $\|v\|^2 = \langle X, \sigma\overline{Y}\rangle$. This is well defined because $\langle X, \sigma\overline{Y}\rangle$ is $H^0 \cap K^0 = H \cap K$-invariant.

Consider $A^{(r,s)}(X_0, \mathscr{L}_\chi)$. Let # be the Hodge-Kodaira orthocomplementation operator with respect to the noninvariant positive definite metric on X^0. Then

$$(\omega, \omega') = \int_{X^0} \omega \wedge \#\omega'$$

defines a hermitian form on the space of L_2 forms where we define a form ω to be L_2 if $\|\omega\|_{L_2}^2 \equiv (\omega, \omega) < \infty$.

DEFINITION.

$$\begin{aligned}\mathscr{H}_2^{(r,s)} &= \mathscr{H}_2^{(r,s)}(X^0, \mathscr{L}_\chi) \\ &= \{\omega \in A^{(r,s)}(X^0, \mathscr{L}_\chi) \mid \square\omega = 0 \text{ and } \|\omega\|_{L_2}^2 < \infty\},\end{aligned}$$

we call this the L_2 harmonic space.

4.2. Let ω be the harmonic form on $X^0 \simeq U(p+1,q)/U(p,q) \times U(1)$ corresponding to the harmonic form $\mathscr{P}T_v$ on X by the duality of §2.2.

PROPOSITION. *Corresponding to the four cases in* §3.2 *we have*: $\|\omega\|_{L_2} < \infty$ *as follows*;
Case 1. $l < -n + s - 1$ (*and* $l \leq -p$, *for* T_v *to exist*).
Case 2. $l > -n + s - 1$ (*and* $l \geq -n + p$).
Case 3. $l < -n + s$ (*and* $l < -p - 1$).
Case 4. $l > -n + s$ (*and* $l > -n + p$).

Proof. Let dx be the invariant measure on X^0. Let $\|\cdot\|_x$ be the positive definite form on $\Lambda T_x^*(X^0)$ coming from the positive definite form on $T_x(X^0)$. Note that $\|\cdot\|_{h^0 y} = \|\cdot\| =$ positive definite form on

$\Lambda\mathfrak{p}^*$ (coming from $\langle X, \sigma\overline{Y}\rangle$), $h^0y \in H^0 \exp\mathfrak{b}$.

$$\begin{aligned}
\|\omega\|_{L_2}^2 &= \int_{X^0} \omega(x) \wedge \#\omega(x) = \int_{X^0} \|\omega(x)\|_x^2\, dx \\
&= \int_{H^0}\int_{\mathfrak{b}^+} \|\omega(h^0y)\|_{h^0y}^2 \delta(y)\, dy\, dh^0 \\
&\qquad \text{(by an integration formula in [7], page 186)} \\
&= \int_{H^0}\int_{\mathfrak{b}^+} \|\omega(h^0y)\|^2 \delta(y)\, dy\, dh^0 \\
&\le C\sum_{i=1}^{0} \int_{\mathfrak{b}^+} \|\omega_i(y)\|^2\delta(y)\, dy \\
&= C\sum_{i=1}^{0} \int_{\mathfrak{b}^+} \|\mathscr{P}T_{v_i}(y)\|^2\delta(y)\, dy \\
&\le C'\int_{\mathfrak{b}^+} |\psi_\lambda(y)|^2\delta(y)\, dy.
\end{aligned}$$

The first inequality is because

$$\|\omega(h^0y)\| = \|((h^0)^{-1}\cdot\omega)(y)\| \le C\sum_i \|\omega_i(y)\|$$

where $(h^0)^{-1}\cdot\omega = \sum_{i=1}^{0} c_i(h^0)\omega_i$ ($d = \dim H^0$-type containing ω, and ω_i are a basis of this H^0-type). The $v_i \in F$ are chosen so that $\omega_i = \mathscr{P}(T_{v_i})$. The last inequality is as follows.

$$\begin{aligned}
\|\mathscr{P}T_v(y)\| &= \left\|\int_{H\cap K} e^{\langle\nu+\rho, H(l^{-1}y)\rangle}\pi(\kappa(l^{-1}y))t(\delta(l^{-1})v)\, dl\right\| \\
&\le \left|\int_{H\cap K} e^{-\langle\nu+\rho, H(l^{-1}y)\rangle}\|\pi(\kappa(l^{-1}y))t(\delta(l^{-1})v)\|dl\right| \\
&\le \text{const}\left|f\int_{H\cap K} e^{-\langle\nu+\rho, H(l^{-1}y)\rangle}\, dl\right| = \text{const}\,|\psi_\lambda(y)|
\end{aligned}$$

where ψ_λ is the function constructed by Flensted-Jensen (see **[16]** page 125) with $\lambda = -\nu$.

$$\int_{\mathfrak{b}^+} |\psi_\lambda(y)|^2\delta(y)\, dy < \infty, \quad \text{when } \langle\nu, \alpha\rangle < 0, \forall\alpha \in \textstyle\sum(\mathfrak{a}, \mathfrak{g})$$

(see **[16]** page 157). Checking this condition for the four possible ν parameters finishes the proof. □

Appendix. A version of Theorem 3.3 for general semisimple symmetric spaces is stated and proved. We let G/H be a semisimple symmetric space, i.e., G is a semisimple Lie group and H is the fixed point set of an involution σ of G. The Lie algebra $\mathfrak{g}_0$ of G decomposes as $\mathfrak{h}_0 + \mathfrak{g}_0$, the ± 1 eigenspaces of $d\sigma$. For a minimal parabolic subgroup $P = MAN$, the action of H on G/P has finitely many orbits, see **[12]**. Fix a closed orbit $\mathscr{O}$. Suppose P may be chosen so that $\mathscr{O} = H \cdot eP$ and P has the property that $H \cap P$ is a minimal parabolic subgroup of H and $K \cap H \cdot A \cap H \cdot N \cap H$ is an Iwasawa decomposition of H.

Fix a representation W of P so that N acts trivially. Consider the set $\{(\delta_i, F_i)\} \subset \hat{H}$ of irreducible finite dimensional representations of H for which $\{F_i^* \otimes W \otimes \mathbb{C}_{-\rho+2\rho_{\mathfrak{h}}}\}^{H\cap P} = \mathrm{Hom}_{H\cap P}(F_i, W \otimes \mathbb{C}_{-\rho+2\rho_{germh}})$ is nonzero. For each $t \in \mathrm{Hom}_{H\cap P}(F_i, W \otimes \mathbb{C}_{-\rho+2\rho_{\mathfrak{h}}})$ and for each $v \in F_i$ define a W-valued distribution T_v on K/M as in §3.2.

PROPOSITION. *T_v is H-finite in $I(W)$ of type (δ_i, F_i).*

The proof is exactly as in §3.2.

Let $W|_{H\cap P} = W_1 \oplus \cdots \oplus W_k$ and consider the following condition:

$(*)$ For each W_i there is a (δ_i, F_i) such that $\{F_i^* \otimes W_i \otimes \mathbb{C}_{-\rho+2\rho_{\mathfrak{h}}}\}^{H\cap P} \neq 0$.

THEOREM. *Suppose $(*)$ holds. Let T be an H-finite distribution in $I(W)$, assume T is a lowest weight vector. Assume T is supported in $\mathscr{O}$. Then the weight of T is $\lambda_i - \sum n_\beta \beta$ where λ_i is the lowest weight of some δ_i, and the sum is over all positive $((\mathfrak{t} \cap \mathfrak{m}) \oplus \mathfrak{a}) \cap \mathfrak{h}$ roots in $\mathfrak{g}$ with $\beta|_{\mathfrak{a}\cap\mathfrak{h}} \neq 0$, $n_\beta \geq 0$.*

The proof is exactly as in §3.3. Note that cases 1–4 at the end of §3.2 determine exactly when condition $(*)$ holds for the particular G/H and W under consideration there.

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