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CORRECTION TO "ASYMPTOTIC RADIAL SYMMETRY FOR SOLUTIONS OF $\Delta u + e^u = 0$ IN A PUNCTURED DISC"

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The negative case ($K < 0$) in the Theorem 3 of the above mentioned paper is incomplete. In this case, the authors considered three separate cases (page 273 Pacific J. Math., **163**, No. 2, 1994). After handling the first two, the authors thought a similar argument would work for the third which turns out to be incorrect. Therefore, we need to reconsider the third case, namely, the function f may take the form

$$f(z) = \frac{e^{i\alpha}(1 + g(z) + \alpha \log z)}{1 - g(z) - \alpha \log z}$$

for some $\alpha \in \mathbb{R}$ and some single-valued analytic function g on the punctured disc $D^* = \{z \in \mathbb{C} \mid 0 < |z| < 1\}$. As in this paper, we may assume that $K = -4$ and $|f| < 1$. Then we conclude that

$$\operatorname{Re} g(z) + \alpha \log r < 0, \quad \text{where } r = |z|,$$

and hence

$$r^\alpha \left| e^{g(z)} \right| < 1.$$

Therefore, 0 is not an essential singularity of $e^{g(z)}$. It implies that $g(z)$ analytically extends across 0. So, in the negative case, we have

Theorem 1. *Real smooth solutions of $\Delta u - 8e^u = 0$ in D^* are of the form*

$$u = \log \frac{|f'|^2}{(1 - |f|^2)^2}$$

with f a multi-valued locally univalent meromorphic function of the form

$$f(z) = h(z) z^\beta, \quad \beta \geq 0$$

or

$$f(z) = \frac{1 + h(z) + \alpha \log z}{1 - h(z) - \alpha \log z}, \quad \alpha \in \mathbb{R} \tag{1}$$

for some single-valued analytic function $h(z)$ *on the whole disc* $D = \{z \in \mathbb{C} \mid |z| < 1\}$.

To find the asymptotic formula, observe that (1) gives

$$u = \log \frac{|zh'(z) + \alpha|^2}{4r^2 \left(\operatorname{Re} h(z) + \alpha \log r\right)^2},$$

which implies that

$$u = -2 \log \left(r \log \frac{1}{r} \right) + O(1) \quad \text{as} \quad r \to 0.$$

Therefore, we have

Theorem 2. *Let* u *be a smooth real solution of* $\Delta u + 2Ke^u = 0$ *for* $K < 0$, *then*

$$u(z) = \alpha \log |z| + O(1), \quad \alpha > -2,$$

or

$$u(z) = -2 \log \left(|z| \log \frac{1}{|z|} \right) + O(1)$$

as $|z| \to 0$.

Finally, it is well-known that all such solution u are bounded by the Poincaré metric (the unique complete constant curvature K conformal metric) on D^* which has finite area near the origin. Therefore, all solution u satisfies $\int e^u < +\infty$ in any small region containing the origin.

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