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UNIFORM ALGEBRAS GENERATED BY HOLOMORPHIC AND PLURIHARMONIC FUNCTIONS ON STRICTLY PSEUDOCONVEX DOMAINS

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It is shown that if $f_1, \dots, f_n$ are pluriharmonic functions on a strictly pseudoconvex domain $\Omega \subset \mathbb{C}^n$ that are C^1 on $\overline{\Omega}$, and the $n \times n$ matrix $(\partial f_j / \partial \bar{z}_k)$ is invertible at every point of Ω, then the norm-closed algebra generated by $A(\overline{\Omega})$ and $f_1, \dots, f_n$ is equal to $C(\overline{\Omega})$.

Introduction.

For K a compact set in $\mathbb{C}^n$, let $A(K)$ denote the subalgebra of $C(K)$ consisting of those continuous functions on K that are holomorphic on the interior of K. Let D denote the open unit disc in the plane. If f is in $C(\overline{D})$ and f is harmonic but nonholomorphic on D, then the norm-closed subalgebra of $C(\overline{D})$ generated by the disc algebra $A(\overline{D})$ and f is equal to $C(\overline{D})$. (See [**I**] for a brief discussion of the history of this result, and [**Č**] and [**A–S**] for two different proofs.) In [**I**] a partial generalization of this result to the ball algebra is obtained: If $f_1, \dots, f_n$ are pluriharmonic on B_n (the open unit ball in $\mathbb{C}^n$) and C^1 on $\overline{B}_n$ (i.e., extend to be continuously differentiable on a neighborhood of $\overline{B}_n$), and the $n \times n$ matrix $(\partial f_j / \partial \bar{z}_k)$ is invertible at every point of B_n, then the norm-closed subalgebra of $C(\overline{B}_n)$ generated by the ball algebra $A(\overline{B}_n)$ and $f_1, \dots, f_n$ is equal to $C(\overline{B}_n)$. Extensions of this result to more general strictly pseudoconvex domains are presented there as well. It is shown that if the functions $f_1, \dots, f_n$ are assumed to be complex conjugates of holomorphic functions, then the ball can be replaced by an arbitrary strictly pseudoconvex domain. Thus for general functions $f_1, \dots, f_n$ the ball can be replaced by any simply connected strictly pseudoconvex domain. In addition, an argument due to Barnet Weinstock is presented showing that if the functions $f_1, \dots, f_n$ are assumed to be C^2, then the ball can be replaced by any strictly pseudoconvex domain with polynomially convex closure. The main purpose of the present paper is to show that the ball can be replaced by an arbitrary strictly pseudoconvex domain without any extra hypotheses on the functions $f_1, \dots, f_n$.

Given complex-valued continuous functions $f_1, \dots, f_k$ on a compact space X, we will write $[f_1, \dots, f_k]$ to denote the norm-closed subalgebra of $C(X)$

that they generate. If, in addition, A is a uniform algebra on X, then $A[f_1, \ldots, f_k]$ will denote the norm-closed subalgebra of $C(X)$ generated by A and the functions $f_1, \ldots, f_k$. As usual, $z_1, \ldots, z_n$ will denote the complex coordinate functions on $\mathbb{C}^n$. If f is a complex-valued function, and E is a subset of its domain, then by definition $\|f\|_E = \sup_{x \in E} |f(x)|$.

I would like to thank Barnet Weinstock for showing me the argument of his mentioned above which first got me thinking along the lines of the present paper. I would also like to thank David Barrett for making me aware of [**F**], and John Wermer for directing my attention to [**Ga1**].

1. The Main Theorem.

Theorem 1.1. *Suppose Ω is a strictly pseudoconvex domain in $\mathbb{C}^n$. Suppose also that $f_1, \ldots, f_n$ in $C(\overline{\Omega})$ are pluriharmonic on Ω and C^1 on $\overline{\Omega}$, and that the matrix $(\partial f_j / \partial \overline{z}_k)$ is invertible at every point of Ω. Then $A(\overline{\Omega})[f_1, \ldots, f_n] = C(\overline{\Omega})$.*

It is clear that a necessary condition for the conclusion of this theorem to hold is that the maximal ideal space of $A(\overline{\Omega})[f_1, \ldots, f_n]$ be $\overline{\Omega}$. Our overall approach to proving the theorem will be to show that under the hypotheses of the theorem, this necessary condition is also sufficient, and then to show that the condition does in fact hold. The special case treated by Weinstock (mentioned in the introduction) was obtained as a consequence of a result of his concerning approximation on the graph of a smooth map [**W2**]. The general case will be obtained below as a consequence of that result as well. For convenience we state the needed result of Weinstock here.

Theorem 1.2. *Suppose X is a compact set in $\mathbb{C}^n$ and $f_1, \ldots, f_m$ are complex-valued C^1 functions on X. Let $E = \{ z \in X : \operatorname{rank} (\partial f_j / \partial \overline{z}_k) < n \}$ and let $Y = \{ (z, f_1(z), \ldots, f_m(z)) \in \mathbb{C}^{n+m} : z \in X \}$. If Y is polynomially convex in $\mathbb{C}^{n+m}$, then $[z_1, \ldots, z_n, f_1, \ldots, f_m]$ consists of those continuous functions on X that agree with some element of $[z_1, \ldots, z_n, f_1, \ldots, f_m]$ on E.*

In order to obtain Theorem 1.1 from Theorem 1.2 we will also need the following two results.

Theorem 1.3. *Suppose Ω is a strictly pseudoconvex domain in $\mathbb{C}^n$. Then the uniform algebra $A(\overline{\Omega})$ is finitely generated. In fact, it is generated by a finite collection of C^1 functions.*

Theorem 1.4. *Suppose Ω is a strictly pseudoconvex domain in $\mathbb{C}^n$ and $f_1, \dots, f_k$ in $C(\overline{\Omega})$ are pluriharmonic on Ω. Then the maximal ideal space of $A(\overline{\Omega})[f_1, \dots, f_k]$ is $\overline{\Omega}$.*

Before turning to the proofs of these results we use them to obtain Theorem 1.1.

Proof of Theorem 1.1. By Theorem 1.3, there are C^1 functions $g_1, \dots, g_l$ on $\overline{\Omega}$ such that $z_1, \dots, z_n, g_1, \dots, g_l$ generate $A(\overline{\Omega})$. For $j = 1, \dots, l+n$, and $k = 1, \dots, n$, define h_{jk} by

$$h_{jk} = \begin{cases} \dfrac{\partial g_j}{\partial \overline{z}_k} & \text{for } j = 1, \dots, l \\ \dfrac{\partial f_{j-l}}{\partial \overline{z}_k} & \text{for } j = l+1, \dots, l+n. \end{cases}$$

Let

$$E = \{ z \in \overline{\Omega} : \operatorname{rank}(h_{jk}) < n \},$$

and let

$$Y = \{ (z, g_1(z), \dots, g_l(z), f_1(z), \dots, f_n(z)) \in \mathbb{C}^{2n+l} : z \in \overline{\Omega} \}.$$

The map $\overline{\Omega} \to Y$ given by $z \mapsto (z, g_1(z), \dots, g_l(z), f_1(z), \dots, f_n(z))$ is a homeomorphism of $\overline{\Omega}$ onto Y, so it induces an identification of $C(\overline{\Omega})$ with $C(Y)$ in an obvious way. Under this identification $A(\overline{\Omega})[f_1, \dots, f_n]$ is identified with $P(Y)$, the uniform closure of the polynomials (in the complex coordinate functions) on Y. Since, by Theorem 1.4, $\overline{\Omega}$ is the maximal ideal space of $A(\overline{\Omega})[f_1, \dots, f_n]$, we conclude that Y is the maximal ideal space of $P(Y)$, and hence Y is polynomially convex. Since the algebra $A(\overline{\Omega})[f_1, \dots, f_n]$ coincides with $[z_1, \dots, z_n, g_1, \dots, g_l, f_1, \dots, f_n]$, Theorem 1.2 now shows that this algebra consists of those continuous functions on $\overline{\Omega}$ that agree with some element of $A(\overline{\Omega})[f_1, \dots, f_n]$ on E. Thus it suffices to show that E is an interpolation set for $A(\overline{\Omega})$.

Note that the first l rows of the matrix (h_{jk}) consist entirely of zeros (since the functions $g_1, \dots, g_l$ are holomorphic), so the rank of the matrix (h_{jk}) is obviously the same as the rank of the matrix $(\partial f_j / \partial \overline{z}_k)$. Consequently, E coincides with the zero set of the function $\det(\partial f_j / \partial \overline{z}_k)$. Moreover, it follows from the hypotheses on the functions $f_1, \dots, f_n$ that the complex conjugate of the function $\det(\partial f_j / \partial \overline{z}_k)$ is in $A(\overline{\Omega})$. Thus E is a zero set for $A(\overline{\Omega})$ contained in $\partial\Omega$, and hence (by Theorem 1.1 in [**W1**]) E is an interpolation set for $A(\overline{\Omega})$. $\square$

It remains to prove Theorems 1.3 and 1.4. We present the proof of Theorem 1.3 here. Theorem 1.4 is proven in the next section.

Proof of Theorem 1.3. By Theorem 10 of Chapter 1 in [**F**], there exist a neighborhood Ω' of $\overline{\Omega}$, a holomorphic map $\psi : \Omega' \to \mathbb{C}^m$ for some positive integer m, and a convex bounded domain C in $\mathbb{C}^m$ such that

(i) ψ is biholomorphic onto a closed submanifold of $\mathbb{C}^m$ and

(ii) $\psi(\Omega) \subset C$ and $\psi(\Omega' \backslash \overline{\Omega}) \subset \mathbb{C}^m \backslash \overline{C}$.

We will show that the component functions of ψ generate $A(\overline{\Omega})$.

Since (by Theorem 5.10 in [**R–S**]) every function in $A(\overline{\Omega})$ can be approximated uniformly by functions holomorphic on a neighborhood of $\overline{\Omega}$, it suffices to show that if f is holomorphic on a neighborhood U of $\overline{\Omega}$, then f can be approximated uniformly on $\overline{\Omega}$ by polynomials in the component functions of ψ. Choose a convex neighborhood V of $\overline{C}$ such that $V \cap \psi(\Omega') \subset \psi(U \cap \Omega')$. Then $f \circ \psi^{-1}$ is defined and holomorphic on $V \cap \psi(\Omega')$. Therefore, $f \circ \psi^{-1}$ extends to a holomorphic function on V (by Theorem I5 in [**Gu**] vol. 3). Since $\overline{C}$ is convex, the Oka-Weil approximation theorem shows that this extension can be approximated uniformly on $\overline{C}$ by polynomials. In particular, $f \circ \psi^{-1}$ can be approximated uniformly on $\psi(\overline{\Omega})$ by polynomials. Thus f can be approximated uniformly on $\overline{\Omega}$ by polynomials in the component functions of ψ. □

2. The Proof of Theorem 1.4.

Assuming familiarity with the material in [**Ga1**] concerning subharmonicity with respect to a uniform algebra, a very short proof of Theorem 1.4 can be given. This will be discussed near the end of the paper. However, for the benefit of readers not familiar with [**Ga1**], a more direct proof will be given first. We begin with some preliminaries, the first of which is a slight generalization of Rossi's local maximum modulus principle and follows easily from that result.

Theorem 2.1. *Let A be a uniform algebra with maximal ideal space $\mathfrak{M}_A$ and Shilov boundary ∂_A. Let x be a point in $\mathfrak{M}_A$, and let U be a neighborhood of x. Suppose f is a continuous, complex-valued function on $\overline{U}$ such that for every Jensen measure μ on $\overline{U}$ for x, we have $f(x) = \int f\, d\mu$. Then $|f(x)| \leq \|f\|_{(\partial_A \cap U) \cup \partial U}$.*

Proof. By Rossi's local maximum modulus principle we have for every g in A that

$$\|\hat{g}\|_{\overline{U}} = \|\hat{g}\|_{(\partial_A \cap U) \cup \partial U}$$

where $\hat{g}$ denotes the Gelfand transform of g. Hence $(\partial_A \cap U) \cup \partial U$ is a boundary for the uniform algebra $\overline{\hat{A}|\overline{U}}$. Consequently, there is a Jensen measure μ supported on $(\partial_A \cap U) \cup \partial U$ for x. Now by hypothesis

$$f(x) = \int_{(\partial_A \cap U) \cup \partial U} f\, d\mu.$$

Thus $|f(x)| \leq \|f\|_{(\partial_A \cap U) \cup \partial U}$. □

Lemma 2.2. *Suppose A and B are uniform algebras on a compact space X and $A \subset B$. If x is a peak point for A, and ϕ is a multiplicative linear functional on B that coincides with point evaluation at x when restricted to A, then ϕ is point evaluation at x on all of B.*

Proof. Let μ be a representing measure for ϕ (as a functional on B). Then obviously μ is a representing measure for the restriction of ϕ to A, i.e., for point evaluation at x on A. Therefore, since x is a peak point for A, μ is the point mass at x (Theorem II.11.3 in [**Ga2**]). Recalling that μ is a representing measure for ϕ, we conclude that ϕ is point evaluation at x on all of B. □

Theorem 2.3. *Suppose Ω is a strictly pseudoconvex domain in $\mathbb{C}^n$, and x is a point in Ω. Suppose also that K is a compact subset of Ω, and that μ is a Jensen measure supported on K for the functional evaluation at x on the algebra $A(\overline{\Omega})$. If f is a pluriharmonic function on Ω, then $f(x) = \int f\, d\mu$.*

Proof. First note that if g is a function of the form $\sup_{1\leq j\leq k}\{c_j \log|f_j|\}$ with the f_j in $A(\overline{\Omega})$ and $c_j \geq 0$, then

$$\begin{aligned} g(x) &= \sup_j c_j \log|f_j(x)| \\ &\leq \sup_j c_j \int \log|f_j|\, d\mu \\ &\leq \int \sup_j c_j \log|f_j|\, d\mu \\ &= \int g\, d\mu. \end{aligned}$$

By the remarks following the proof of Theorem Q9 in [**Gu**] vol. 1, every continuous plurisubharmonic function on Ω can be approximated uniformly on K by functions of the form $\sup_j\{c_j \log|f_j|\}$ with the f_j holomorphic on Ω and $c_j \geq 0$. Since Ω is strictly pseudoconvex, there is a smooth strictly plurisubharmonic defining function for Ω, i.e., a smooth strictly plurisubharmonic function ρ on a neighborhood of $\overline{\Omega}$ such that $\Omega = \{z : \rho(z) < 0\}$ and $\operatorname{grad}\rho \neq 0$ on $\partial\Omega$. Now for small $\varepsilon > 0$, the set $\Omega_\varepsilon = \{z : \rho(z) < \varepsilon\}$ is (strictly) pseudoconvex and the plurisubharmonically convex hull $\hat{K}^p_{\Omega_\varepsilon}$ of

K in Ω_ε is contained in Ω. Hence by Theorem 4.3.2 in [**H**], every function holomorphic on Ω can be approximated uniformly on $\hat{K}^p_{\Omega_\varepsilon}$ (and hence certainly on K) by functions holomorphic on Ω_ε. Thus the functions f_j can be approximated uniformly on K by elements of $A(\overline{\Omega})$ and it follows that we may assume without loss of generality that they are already in $A(\overline{\Omega})$. Hence we conclude from the first sentence of the proof that $u(x) \le \int u\, d\mu$ for every continuous plurisubharmonic function u on Ω.

There is clearly no loss of generality in assuming that the function f in the statement of the theorem is real-valued. Then both f and $-f$ are plurisubharmonic so $f(x) \le \int f\, d\mu$ and $-f(x) \le \int (-f)\, d\mu$. Thus $f(x) = \int f\, d\mu$, as desired. □

Proof of Theorem 1.4. The maximal ideal space of $A(\overline{\Omega})$ is $\overline{\Omega}$ (see [**S–W**]). Consequently, if we let $\hat{z}_1, \dots, \hat{z}_n$ denote the Gelfand transforms of $z_1, \dots, z_n$ as elements of $A(\overline{\Omega})[f_1, \dots, f_k]$, and let $\pi : \mathfrak{M}_{A(\overline{\Omega})[f_1,\dots,f_k]} \to \mathbb{C}^n$ be given by

$$\pi(x) = (\hat{z}_1(x), \dots, \hat{z}_n(x)),$$

then $\pi(x) \in \overline{\Omega}$ for each point x in the maximal ideal space of $A(\overline{\Omega})[f_1, \dots, f_k]$. Moreover, it also follows that each element of the maximal ideal space of $A(\overline{\Omega})$ is uniquely determined by its values on the functions $z_1, \dots, z_n$, and hence each element of the maximal ideal space of $A(\overline{\Omega})[f_1, \dots, f_k]$ is uniquely determined by its values on the functions $z_1, \dots, z_n, f_1, \dots, f_k$. For each ζ in $\overline{\Omega}$, call $\pi^{-1}(\zeta)$ the fiber over ζ. To show that $\overline{\Omega}$ is the maximal ideal space of $A(\overline{\Omega})[f_1, \dots, f_k]$, we must simply show that each fiber consists of a single point. Hence it suffices to show that if ζ is in $\overline{\Omega}$, and x is in the fiber over ζ, then $\hat{f}_1(x) = f_1(\zeta), \dots, \hat{f}_k(x) = f_k(\zeta)$. In other words, it suffices to show that each of the functions $\hat{f}_1 - f_1 \circ \pi, \dots, \hat{f}_k - f_k \circ \pi$ is identically zero.

Since every point of $\partial\Omega$ is a peak point for $A(\overline{\Omega})$ (Cor. 1.4 in [**W1**]), Lemma 2.2 shows that the fiber over each point of $\partial\Omega$ consists of a single point. Thus $\hat{f}_j - f_j \circ \pi$ $(j = 1, \dots, k)$ is zero on the fibers over $\partial\Omega$.

Assume, to get a contradiction, that $\hat{f}_j - f_j \circ \pi$ is not identically zero. From the preceding paragraph, we see that there is a compact set K contained in Ω such that the set where $\hat{f}_j - f_j \circ \pi$ takes on its maximum modulus is contained in the fibers over the interior of K. Now let x_0 be a point where $\hat{f}_j - f_j \circ \pi$ assumes its maximum modulus, and suppose μ is a Jensen measure for x_0 supported on $\pi^{-1}(K)$. Then certainly $\hat{f}_j(x_0) = \int \hat{f}_j\, d\mu$. Moreover, if we let $\tilde{x}_0 = \pi(x_0)$, and let $\tilde{\mu}$ be the measure on K defined by $\tilde{\mu}(E) = \mu(\pi^{-1}(E))$ for every Borel set E, then $\tilde{\mu}$ is a Jensen measure for $\tilde{x}_0$ with respect to the algebra $A(\overline{\Omega})$. Therefore, by Theorem 2.3, $f_j(\tilde{x}_0) = \int f_j\, d\tilde{\mu}$, or equivalently,

$(f_j \circ \pi)(x_0) = \int (f_j \circ \pi)\, d\mu$. Hence

$$(\hat{f}_j - f_j \circ \pi)(x_0) = \int (\hat{f}_j - f_j \circ \pi)\, d\mu.$$

Now, letting $\mathrm{Int}(K)$ denote the interior of K, Theorem 2.1 shows that $|(\hat{f}_j - f_j \circ \pi)(x_0)|$ is less than or equal to the supremum of $|\hat{f}_j - f_j \circ \pi|$ over the set

$$\Big(\partial_{A(\overline{\Omega})[f_1,\dots,f_k]} \cap \pi^{-1}(\mathrm{Int}(K))\Big) \cup \partial\Big(\pi^{-1}(\mathrm{Int}(K))\Big).$$

Since the Shilov boundary of $A(\overline{\Omega})[f_1, \dots, f_k]$ is obviously contained in $\overline{\Omega}$ (where $\hat{f}_j - f_j \circ \pi$ is zero), the preceding observation is easily seen to be a contradiction. □

As mentioned earlier, a very short proof of Theorem 1.4 can be given based on material in [**Ga1**]. For A a uniform algebra with maximal ideal space $\mathfrak{M}_A$ and u an upper-semicontinuous function on $\mathfrak{M}_A$, call u subharmonic with respect to A if $u(x) \le \int u\, d\sigma$ for every $x \in \mathfrak{M}_A$ and every Jensen measure σ for x. Theorems 5.9 and 6.9 in [**Ga1**] together show that the real and imaginary parts of the functions $f_1, \dots, f_k$ in Theorem 1.4 are subharmonic with respect to $A(\overline{\Omega})$, and that the same is also true with $f_1, \dots, f_k$ replaced by their negatives. Thus we have

$$f_j(x) = \int f_j\, d\sigma \qquad (j = 1, \dots, k)$$

for every $x \in \overline{\Omega}$ and every Jensen measure σ for x. Now suppose ϕ is an arbitrary multiplicative linear functional on $A(\overline{\Omega})[f_1, \dots, f_k]$, and let z_0 be the point in $\overline{\Omega}$ corresponding to the restriction of ϕ to $A(\overline{\Omega})$. Let μ be a Jensen measure for ϕ supported on $\overline{\Omega}$. Then μ is also a Jensen measure for z_0 (regarded as an element of $\mathfrak{M}_{A(\overline{\Omega})}$) and so we have

$$f_j(z_0) = \int f_j\, d\mu \qquad (j = 1, \dots, k).$$

Since μ represents ϕ, we conclude that $\phi(f_j) = f_j(z_0)$ $(j = 1, \dots, k)$. It follows that ϕ is evaluation at z_0, and hence the maximal ideal space of $A(\overline{\Omega})[f_1, \dots, f_k]$ is $\overline{\Omega}$.

When Ω is the open unit ball in $\mathbb{C}^n$, a short and elementary proof of Theorem 1.4 can be given. Let ϕ be an arbitrary multiplicative linear functional on $A(\overline{B}_n)[f_1, \dots, f_k]$, and let μ be a representing measure for ϕ. Since the only multiplicative linear functionals on $A(\overline{B}_n)$ are the point evaluations, there must be some point z_0 in $\overline{B}_n$ such that

$$(*) \qquad g(z_0) = \int g\, d\mu \qquad \text{for all } g \text{ in } A(\overline{B}_n).$$

Now suppose f in $C(\overline{B}_n)$ is real-valued and pluriharmonic on B_n. For $0 < r < 1$, let f_r denote the dilate of f defined by $f_r(z) = f(rz)$. Then f_r is pluriharmonic on the open ball of radius $1/r$, so f_r is the real-part of a holomorphic function there. Hence, since μ is a real measure, it follows from $(*)$ that $f_r(z_0) = \int f_r \, d\mu$. Since the f_r converge uniformly to f on $\overline{B}_n$, we obtain

$$f(z_0) = \int f \, d\mu. \tag{**}$$

Since the real and imaginary parts of each f_j satisfy the hypotheses on f above, we observe that $(**)$ continues to hold with f replaced by f_j $(j = 1, \ldots, k)$. Hence $\phi(f_j) = f_j(z_0)$ $(j = 1, \ldots, k)$, and it follows that ϕ is evaluation at z_0 on $A(\overline{B}_n)[f_1, \ldots, f_k]$. Thus the maximal ideal space of $A(\overline{B}_n)[f_1, \ldots, f_k]$ is $\overline{B}_n$.

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