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Much to our embarrassment, we noted recently that there were some errors in our paper [2]. Proposition 2.5 (iii) is incorrect—only a weaker form is valid. There was a rather potentially serious gap in the proof of Theorem 1.2 (i), resulting from embarrassingly too many inaccuracies and typos. Fortunately the gap could be filled easily and we shall give here the complete proof. Barring Proposition 2.5 (iii), which is not used in the rest of the paper, all the other results of the paper remain valid. In what follows we use the notations of our paper [2].

Proposition 2.5 (iii) asserted that $\mathrm{Im}(\hat{q}^*)$ equals the subgalgebra invariant under the action of the symmetric group S_r on $H^*(P(m, \mathrm{Flag}(\omega), \theta); \mathbb{Z}_2)$. (The map $\hat{q} : P(m, \mathrm{Flag}(\omega), \theta) \to P(m, X, \sigma)$ is associated to the projection of the $\mathrm{Flag}(\mathbb{C}^r)$-bundle $\mathrm{Flag}(\omega) \to X$, where ω is a σ-conjugate complex vector bundle over X.) Only the following weaker assertion is valid: $\mathrm{Im}(\hat{q}^*)$ *is contained in the subalgebra invariant under the action of the symmetric group* S_r *on* $H^*(P(m, \mathrm{Flag}(\omega), \theta); \mathbb{Z}_2)$.

The above error resulted from our assertion on [2, p.79] that the subalgebra of $H^*(\mathrm{Flag}(\mathbb{C}^r); \mathbb{Z}_2)$ invariant under the action of S_r equals $H^0(\mathrm{Flag}(\mathbb{C}^r); \mathbb{Z}_2)$. This is false. Indeed the top dimensional mod 2 cohomology of $\mathrm{Flag}(\mathbb{C}^r)$ is isomorphic to $\mathbb{Z}_2$, which is invariant under *any* group action. Consequently, the last sentence in the first para of [2, p.80] also is false. The discussion in that para however establishes the aforementioned weaker assertion.

We now turn to Proof of Theorem 1.2 (i). The basic idea, as explained in the paper, is to view $\mathbb{C}^n$ as a module over the complex Clifford algebra C_r^c and to use this to obtain first an action of $(\mathbb{Z}_2)^r$ on $\mathbb{C}G_{n,k}$ without stationary points. We need only consider the case r even. In order to obtain such an action on $P(m, \mathbb{C}G_{n,k})$ one has to ensure that the generators of $(\mathbb{Z}_2)^r$, which are obtained from certain units in the Clifford algebra, act on $\mathbb{C}^n$ as *real matrices*. Our proof asserted that this was true based on the fact that C_r^c is the complexification of the real Clifford algebra C_r. But this is not sufficient to conclude that the action of each φ_j on the simple C_r^c-module is via a real matrix, *unless* C_r *is itself a matrix algebra over* $\mathbb{R}$. To fix this gap in the proof, we need to use real Clifford algebra associated to the positive definite form on $\mathbb{R}^r$ as well as the one with signature $(2, r-2)$, besides the negative definite one, the correct choice being dependent on the value of $r \mod 8$.

Let $r = 2p \geq 2$ be even so that $C_r^c \cong M_{2^p}(\mathbb{C})$. Then $C_r = M_{2^p}(\mathbb{R})$ when $p \equiv 3, 4 \mod 4$. When $p \equiv 1 \mod 4$ one may use $C_r' = M_{2^p}(\mathbb{R})$, the real Clifford algebra associated to the definite quadratic form $x \mapsto \|x\|^2$ on $\mathbb{R}^r$. When $p \equiv 2 \mod 4$, neither C_r, C_r' is isomorphic

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to $M_{2^p}(\mathbb{R})$. In this case, the real Clifford algebra $C_{2,r-2}$ associated to the indefinite (non-degenerate) quadratic form of signature $(2, r-2)$ is isomorphic to $M_{2^p}(\mathbb{R})$. See [3, Chapter 13]. So we set $C := C_r, C'_r$, or $C_{2,r-2}$, according as $p \equiv 3,4 \mod 4$, $p \equiv 1 \mod 4$, or $p \equiv 2 \mod 4$ respectively. Then C is a isomorphic to $M_{2^p}(\mathbb{R})$ and is generated by elements $\varphi_1, \ldots, \varphi_r$ satisfying the conditions $\varphi_i\varphi_j = -\varphi_j\varphi_i, i \neq j$, and $\varphi_i^2 = \pm 1$, $\forall i$. It follows that φ_j, regarded as elements of $C \otimes_{\mathbb{R}} \mathbb{C} \cong C_r^c \cong M_{2^p}(\mathbb{C})$, act on $\mathbb{C}^{2^p}$ as *real transformations*.

Given $n > k$ with $p := \nu_2(n) > \nu_2(k)$, write $n = 2^p m$ where m is odd. Set $r = 2p$ and view $\mathbb{C}^n$ as a direct sum of m copies of the simple C_r^c-module $\mathbb{C}^{2^p}$. The action of $\varphi_j, 1 \leq j \leq r$, on $\mathbb{C}^n$ yields involutive diffeomorphisms, again denoted $\varphi_1, \cdots, \varphi_r$ on $\mathbb{C}G_{n,k}$ which pairwise commute. Hence we obtain a smooth action of $(\mathbb{Z}_2)^r$ on $\mathbb{C}G_{n,k}$. Since the φ_j are real, we obtain involutions f_j on $P(m, \mathbb{C}G_{n,k})$ defined as $f_j([u, V]) = [u, \varphi_j(V)]$. This action yields a well-defined action of $G = (\mathbb{Z}_2)^r$ on $P(m, \mathbb{C}G_{n,k})$. Any stationary point for this action would result in a complex vector space V of dimension k such that $\varphi_j(V) = V$ for all j. This means that V is a module over C_r^c. Since any non-zero module over C_r^c must be a sum of simple C_r^c-module, it follows that $\dim_{\mathbb{C}} V = k$ must be divisible by 2^p. This is a contradiction since $\nu_2(k) < p = \nu_2(n)$ by hypothesis. Hence the G-action on $P(m, \mathbb{C}G_{n,k})$ has no stationary points and we conclude that $[P(m, \mathbb{C}G_{n,k})] = 0$ by [1, Theorem 30.1].

In Corollary 3.2 (ii), we need to assume that $m > 1$. When $m = 1$, $x^2 = 0$. So the same proof shows that $P(1, X)$ has a spin structure if and only if X admits a spin structure and $m + d$ is odd.

We correct a typographical error: On page 85, last para in the proof of Theorem 1.1, $\rho(m + 1 + \binom{n}{2})$ should be $\rho(m + 1 + 2\binom{n}{2})$.

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