Smaoui, K.
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ERRATUM TO THE ARTICLE "BEURLING'S THEOREM FOR NILPOTENT LIE GROUPS" OSAKA J. MATH. 48 (2011), 127–147

KAIS SMAOUI

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The discussion below represents a correction of an error in the paper "Beurling's theorem for nilpotent Lie groups" Osaka J. Math. **48** (2011), 127–147.

1. Description of the error

The main result of the paper [1] was stated as follows (Theorem 1.3 in [1]):

Theorem 1.1. *Let* $G = \exp\mathfrak{g}$ *be a connected simply connected nilpotent Lie group. Let* f *be a function on* $L^2(G)$ *such that*:

$$\int_{\mathcal{W}}\int_G |f(x)| \|\pi_l(f)\|_{HS} e^{2\pi\|x\|\|l\|} |Pf(l)|\, dx\, dl < +\infty. \tag{1.1}$$

Then, $f = 0$ *almost everywhere.*

Here $\mathcal{W}$ is a suitable cross-section for the generic coadjoint orbits in $\mathfrak{g}^*$, the vector space dual of $\mathfrak{g}$.

The condition (1.1) of this theorem depends on the choice of the bases for which the norm of x in G is defined. We must define the norm of x in G before stating Theorem 1.3. For this we must fix a bases of $\mathfrak{g}$, and then define the norm of x using this bases. In addition, we shouldn't modify this bases throughout the proof of Theorem 1.3. This implies that, Remark 2.5.1 in the paper is not correct.

2. Correction of the error

First of all, Remark 2.5.1 must be deleted. This remark has no consequence for the proof of Theorem 1.3. Secondly, recall that we stated Theorem 1.3 before fixing a strong Malcev bases of $\mathfrak{g}$. Moreover, in the proof of this theorem (Sections 3 and 4 in the paper), we treated two cases, using two different strong Malcev bases of $\mathfrak{g}$. In

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the first case we used a strong Malcev bases passing through $[\mathfrak{g},\mathfrak{g}]$. In case two, we took another strong Malcev bases passing through $\mathfrak{z}(\mathfrak{g})+[\mathfrak{g},\mathfrak{g}]$, where $\mathfrak{z}(\mathfrak{g})$ is the center of $\mathfrak{g}$. To correct these deficiencies, we shall choose a fixed strong Malcev bases of $\mathfrak{g}$ before stating the main result and we will use this to proof Theorem 1.3. We start by the following definition:

DEFINITION 2.1. Let $\mathcal{B}=\{X_1,\dots,X_n\}$ be a bases of $\mathfrak{g}$. Let $\mathfrak{c}$ be an ideal of $\mathfrak{g}$. We say that $\mathcal{B}$ is a $\mathfrak{c}$-adapted bases of $\mathfrak{g}$ if $\mathcal{B}$ is a strong Malcev bases of $\mathfrak{g}$ passing through the ideal $\mathfrak{c}$.

Let $\mathcal{B}=\{X_1,\dots,X_n\}$ be a $\mathfrak{z}(\mathfrak{g})$ and $(\mathfrak{z}(\mathfrak{g})+[\mathfrak{g},\mathfrak{g}])$-adapted bases of $\mathfrak{g}$. Using this bases, we consider the Euclidean norm of $\mathfrak{g}^*$ with respect to the bases $\mathcal{B}^*$ that is,

$$\left\|\sum_{j=1}^{n} l_j X_j^*\right\| = \sqrt{(l_1^2+\cdots+l_n^2)} = \|l\|.$$

We introduce a norm function on G by setting, for $x=\exp(x_1X_1+\cdots+x_nX_n)\in G$,

$$\|x\| = \sqrt{(x_1^2+\cdots+x_n^2)}$$

(for more details see Section 2.5 in [1]). Now, we can state our main result. In fact, Theorem 1.3 must been stated as:

Theorem 2.2. *Let G be a connected simply connected nilpotent Lie group and $\mathfrak{g}$ its Lie algebra. Let $\mathfrak{z}(\mathfrak{g})$ be the center of $\mathfrak{g}$ and $\mathcal{B}$ be a $\mathfrak{z}(\mathfrak{g})$ and $\big(\mathfrak{z}(\mathfrak{g})+[\mathfrak{g},\mathfrak{g}]\big)$-adapted bases of $\mathfrak{g}$. Let f be a function on $L^2(G)$. With respect to the basis $\mathcal{B}$, suppose that:*

$$\int_{\mathcal{W}}\int_G |f(x)|\|\pi_l(f)\|_{HS}e^{2\pi\|x\|\|l\|}|Pf(l)|\,dx\,dl < +\infty. \tag{2.1}$$

Then, $f=0$ almost everywhere.

With these modifications our original proof still works. We distinguished between the cases 1 and 2. We do not need to do any changes in the proof of case 2. In case 1 (Section 3 and Section 4 in the paper) we supposed that the stabilizer of l in $\mathfrak{g}$ is included in $[\mathfrak{g},\mathfrak{g}]$ for all l in the set of generic elements in $\mathfrak{g}^*$. This implies that the center of $\mathfrak{g}$ is included in $[\mathfrak{g},\mathfrak{g}]$ and then $\mathfrak{z}(\mathfrak{g})+[\mathfrak{g},\mathfrak{g}]=[\mathfrak{g},\mathfrak{g}]$ and $\mathcal{B}$ is $[\mathfrak{g},\mathfrak{g}]$ adapted.

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References

[1] K. Smaoui: *Beurling's theorem for nilpotent Lie groups*, Osaka J. Math. **48** (2011), 127–147.

University of Sfax
higher institute of information technology and Multimedia
Pôle technologique de Sfax Route de Tunis Km 10 B.P. 242
Sfax Tunisia
e-mail: Kais.Smaoui@isimsf.rnu.tn