ANALYTIC SMOOTHING EFFECT FOR NONLINEAR SCHRÖDINGER EQUATION IN TWO SPACE DIMENSIONS

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Abstract

We prove the global existence of analytic solutions to the Cauchy problem for nonlinear Schrödinger equations in two dimensions, where the nonlinearity behaves as a cubic power at the origin and the Cauchy data are small and decay exponentially at infinity.

1. Introduction

We consider the Cauchy problem for nonlinear Schrödinger equations of the form

$$(1.1) \qquad i\partial_t u + \frac{1}{2}\Delta u = f(u),$$

where u is a complex-valued function of $(t, x) \in \mathbb{R} \times \mathbb{R}^2$, Δ is the Laplacian in $\mathbb{R}^2$ and f is a complex-valued function on $\mathbb{C}$ satisfying a gauge condition $f(e^{i\theta} z) = e^{i\theta} f(z)$ for any $(\theta, z) \in \mathbb{R} \times \mathbb{C}$ and behaving as a cubic power at the origin. Typical examples of the nonlinearities f are:

$$(1.2) \qquad f(u) = \lambda |u|^2 u,$$

$$(1.3) \qquad f(u) = (\exp(\lambda |u|^2) - 1)u$$

with $\lambda \in \mathbb{C} \setminus \{0\}$ (see [2, 19] for instance).

There is a large literature on the Cauchy problem for the nonlinear Schrödinger equations (see for instance [3, 6, 30] and reference theorem). In this paper we study the analyticity of solutions to the Cauchy problem for (1.1). We refer the reader to [1, 2, 9, 10, 11, 12, 13, 14, 16, 17, 22, 23, 24, 25, 26, 29, 31] for the available results on the analyticity and related subjects. The purpose in this paper is to prove the global existence of analytic solutions to the Cauchy problem for (1.1) in two space dimensions for sufficiently small Cauchy data with exponential decay at infinity. In particular, we describe analytic smoothing effects for (1.1) with (1.2) in the $L^2(\mathbb{R}^2)$ setting and also

with (1.3) in the $H^1(\mathbb{R}^2)$ setting. The associated scaling critical space for (1.1) with (1.2) and (1.3) are known respectively as $L^2(\mathbb{R}^2)$ [4, 7, 15, 18, 33] and $H^1(\mathbb{R}^2)$ [20].

To state our result precisely we introduce the following notation. For any p with $1 \leq p \leq \infty$, L^p denotes the Lebesgue space of p-th integrable function on $\mathbb{R}^2$. For any $s \in \mathbb{R}$, $H_p^s = (1 - \Delta)^{-s/2} L^p$ and $\dot{H}_p^s = (-\Delta)^{-s/2} L^p$ denote the usual Sobolev space (or the space of Bessel potentials) and the homogeneous Sobolev space (or the space of Riesz potentials), respectively. For any $t \in \mathbb{R}$, $U(t) = \exp(i(t/2)\Delta)$ denotes the free propagator. For any $t \in \mathbb{R}$, $J = J(t) = x + it\nabla = U(t)xU(-t)$ denotes the generator of Galilei transforms. For any $t \in \mathbb{R}\backslash\{0\}$, $U(t)$ and $J(t)$ are represented as $U(t) = M(t)D(t)\mathcal{F}M(t)$ and $J(t) = M(t)it\nabla M(-t)$, respectively, where $M(t) = \exp(i|x|^2/(2t))$, $(D(t)\psi)(x) = (it)^{-1}\psi(t^{-1}x)$, $(\mathcal{F}\psi)(\xi) = (2\pi)^{-1} \int \exp(-ix \cdot \xi)\psi(x)\,dx$.

With the Cauchy data $u(0) = \phi$ at $t = 0$ the Cauchy problem for (1.1) is written as the integral equation

$$(1.4) \qquad u(t) = U(t)\phi - i \int_0^t U(t - t')f(u(t'))\,dt'.$$

We introduce the following basic function spaces:

$$\mathscr{X}_0 = L^4(\mathbb{R}; L^4) \cap L^\infty(\mathbb{R}; L^2),$$

$$\mathscr{X}_1 = L^4(\mathbb{R}; H_4^1) \cap L^\infty(\mathbb{R}; H_2^1),$$

with the associated norms defined by

$$\|u; \mathscr{X}_0\| = \max(\|u; L^4(\mathbb{R}; L^4)\|, \|u; L^\infty(\mathbb{R}; L^2)\|),$$

$$\|u; \mathscr{X}_1\| = \max(\|u; L^4(\mathbb{R}; H_4^1)\|, \|u; L^\infty(\mathbb{R}; H_2^1)\|).$$

We treat (1.4) in the following function spaces with $a > 0$:

$$G_0^a(J) = \left\{ u \in \mathscr{X}_0; \|u; G_0^a(J)\| \equiv \sum_{\alpha \geq 0} \frac{a^{|\alpha|}}{\alpha!} \|J^\alpha u; \mathscr{X}_0\| < \infty \right\},$$

$$G_1^a(J) = \left\{ u \in \mathscr{X}_1; \|u; G_1^a(J)\| \equiv \sum_{\alpha \geq 0} \frac{a^{|\alpha|}}{\alpha!} \|J^\alpha u; \mathscr{X}_1\| < \infty \right\},$$

where $J^\alpha = J_1^{\alpha_1} J_2^{\alpha_2} = M(it\partial)^\alpha M^{-1}$ for any multi-index $\alpha = (\alpha_1, \alpha_2.)$ For $\rho > 0$, we define

$$B_0^a(\rho) = \left\{ \phi \in G_0^a(x); \|\phi; G_0^a(x)\| \equiv \sum_{\alpha \geq 0} \frac{a^{|\alpha|}}{\alpha!} \|x^\alpha \phi; L^2\| \leq \rho \right\},$$

$$B_1^a(\rho) = \left\{ \phi \in G_1^a(x); \|\phi; G_0^a(x)\| \equiv \sum_{\alpha \geq 0} \frac{a^{|\alpha|}}{\alpha!} \|x^\alpha \phi; H_2^1\| \leq \rho \right\}.$$

We now state our main results.

Theorem 1. *There exists a constant $\rho > 0$ such that for any $a > 0$ and $\phi \in B_1^a(\rho)$ equation (1.4) with (1.3) has a unique solution $u \in G_1^a(J)$.*

Theorem 2. *There exists a constant $\rho > 0$ such that for any $a > 0$ and $\phi \in B_0^a(\rho)$ equation (1.4) with (1.2) has a unique solution $u \in G_0^a(J)$.*

REMARK 1. Theorems 1 and 2 describe analytic smoothing properties of solutions since functions in the space $G_0^a(J)$ are analytic in x for any $t \in \mathbb{R} \backslash \{0\}$ (see [12, 13]). A novelty consists in the fact that minimal regularity assumption regarding scaling invariance [3, 4, 7] is imposed on the Cauchy data (compare with [13, 17, 29] for instance.)

REMARK 2. By Proposition 2 in [24], the norm on $G_0^a(x)$ is described in terms of weights of exponential type. To be specific,

$$\left\| \left(\prod_{j=1}^2 e^{a|x_j|} \right) \phi; L^2 \right\| \leq \| \phi; G_0^a(x) \|$$

$$\leq (1 + 2 \log 2) \left\| \left(\prod_{j=1}^2 (1 + a|x_j|)^{1/2} e^{a|x_j|} \right) \phi; L^2 \right\|.$$

We prove Theorem 1 in Section 3 by a contraction argument of the Strichartz estimates and the Sobolev embedding in the critical case [20, 21, 27, 28]. Basic estimates for the proofs of the theorems are summarized in Section 2. In Section 4, we give a sketch of proof of Theorem 2.

2. Preliminaries

In this section we collect some basic estimate for the Schrödinger group $U(t) = \exp(i(t/2)\Delta)$ and Sobolev embedding.

Lemma 1 ([3, 6, 28, 30, 34]). *$U(t)$ satisfies the following estimates*:
(1) *For any (q, r) with $0 \leq 2/q = 1 - 2/r < 1$*

$$\| U(\cdot)\phi; L^q(\mathbb{R}; L^r) \| \leq C \| \phi; L^2 \|.$$

(2) *For any (q_j, r_j) with $0 \leq 2/q_j = 1 - 2/r_j < 1$, $j = 1, 2$, the operator Ξ defined by*

$$(\Xi f)(t) = \int_0^t U(t - t') f(t') \, dt'$$

satisfies the estimate

$$\|\varXi f; L^{q_1}(\mathbb{R}; L^{r_1})\| \leq C \|f; L^{q_2'}(\mathbb{R}; L^{r_2'})\|,$$

where p' is dual exponent to p defined by $1/p + 1/p' = 1$.

The following lemma is crucial in the proof of convergence of the series of exponential type.

Lemma 2. *For any r with $2 \leq r < \infty$ there exists a constant $C_r > 0$ such that for any q with $r \leq q < \infty$ the following estimate holds.*

$$\|u; L^q\| \leq C_r q^{1/r'} \|u; \dot{H}_r^1\|^{2/r-2/q} \|u; L^r\|^{1-2/r+2/q}.$$

Proof. We recall the following estimate [20, Inequality (2.6)]: For any r with $1 < r < \infty$ there exists a constant $C_r' > 0$ such that for any q with $r \leq q < \infty$

$$\|u; L^q\| \leq C_r' q^{1/r'} \|u; \dot{H}_r^{2/r-2/q}\|.$$

By an interpolation inequality in the homogeneous Sobolev space [8], for any r with $2 \leq r < \infty$ there exists a constant $C_r'' > 0$ such that for any q with $r \leq q < \infty$,

$$\|u; \dot{H}_r^{2/r-2/q}\| \leq C_r'' \|u; \dot{H}_r^1\|^{2/r-2/q} \|u; L^r\|^{1-2/r+2/q}.$$

The lemma follows from those two inequalities.   □

3.   Proof of Theorem 1

For $\varepsilon > 0$ we define the metric space

$$X_1(\varepsilon) = \{u \in G_1^a(J); \|u; G_1^a(J)\| \leq \varepsilon\}$$

with metric

(3.1) $$d(u, v) = \|u - v; G_0^a(J)\|.$$

We see that $(X_1(\varepsilon), d)$ is a complete metric space. For $\phi \in B_1^a(\rho)$ and $u \in G_1^a(J)$ we define $\Phi(u)$ by

$$(\Phi(u))(t) = U(t)\phi - i(\varXi f(u))(t).$$

We prove that $\Phi: u \mapsto \Phi(u)$ is a contraction mapping on $(X_1(\varepsilon), d)$ for $\varepsilon, \rho > 0$

sufficiently small. By Lemma 2, for $u \in \mathscr{X}_1$ we estimate $f(u)$ in $L^{4/3}$ as

$$\|f(u); L^{4/3}\|$$

$$\leq \sum_{j=1}^{\infty} \frac{|\lambda|^j}{j!} \||u|^{2j}u; L^{4/3}\|$$

$$= \sum_{j=1}^{\infty} \frac{|\lambda|^j}{j!} \|u; L^{4(2j+1)/3}\|^{2j+1}$$

$$\leq \sum_{j=1}^{\infty} \frac{|\lambda|^j}{j!} \left(C_2 \left(\frac{4(2j+1)}{3} \right)^{1/2} \|u; H_2^1\| \right)^{2(j-1)} \left(C_4 \left(\frac{4(2j+1)}{3} \right)^{3/4} \|u; H_4^1\| \right)^3$$

$$= \left(\frac{C_4^3}{C_2^2} \right) \sum_{j=1}^{\infty} \frac{(C_2^2 |\lambda|)^j}{j!} \left(\frac{4}{3}(2j+1) \right)^{j+5/4} \|u; H_2^1\|^{2(j-1)} \|u; H_4^1\|^3$$

$$= F_1(\|u; H_2^1\|)\|u; H_4^1\|^3,$$

where $F_1(\rho) = (C_4^3/C_2^2) \sum_{j=1}^{\infty} ((C_2^2|\lambda|)^j / j!)((4/3)(2j+1))^{j+5/4} \rho^{2(j-1)}$ converges for any ρ with $0 \leq \rho < \sqrt{3/(8eC_2^2|\lambda|)}$ by d'Alembert's ratio test. Similarly, for $u \in \mathscr{X}_1$ we estimate $\nabla f(u)$ in $L^{4/3}$ as

$$\|\nabla(f(u)); L^{4/3}\|$$

$$\leq \sum_{j=1}^{\infty} \frac{|\lambda|^j}{j!}(2j+1)\|u^{2j}\nabla u; L^{4/3}\|$$

$$\leq \sum_{j=1}^{\infty} \frac{|\lambda|^j}{j!}(2j+1)\|u; L^{4j}\|^{2j}\|\nabla u; L^4\|$$

$$\leq \sum_{j=1}^{\infty} \frac{|\lambda|^j}{j!}(2j+1)(C_2(4j)^{1/2}\|u; H_2^1\|)^{2(j-1)}(C_4(4j)^{3/4}\|u; H_4^1\|)^2\|\nabla u; L^4\|$$

$$\leq \left(\frac{C_4^2}{C_2^2} \right) \sum_{j=1}^{\infty} \frac{(C_2^2|\lambda|^j)}{j!}(2j+1)(4j)^{j+1/2}\|u; H_2^1\|^{2(j-1)}\|u; H_4^1\|^3$$

$$= F_2(\|u; H_2^1\|)\|u; H_4^1\|^3,$$

where $F_2(\rho) = (C_4^2/C_2^2) \sum_{j=1}^{\infty} ((C_2^2|\lambda|)^j / j!)(2j+1)(4j)^{j+1/2}\rho^{2(j-1)}$ converges for any ρ with $0 \leq \rho < \sqrt{1/(4eC_2^2|\lambda|)}$ by d'Alembert's ratio test. Therefore, by Lemma 1 and the Hölder inequalities in space and time, we have

$$\|\Phi(u); \mathscr{X}_1\| \leq C\|\phi; H_2^1\|$$
$$+ C(F_1(\|u; L^{\infty}(\mathbb{R}; H_2^1)\|) + F_2(\|u; L^{\infty}(\mathbb{R}; H_2^1)\|))\|u; \mathscr{X}_1\|^3$$

for any $u \in \mathscr{X}_1$ with $\|u; L^\infty(\mathbb{R}; H_2^1)\| \leq \rho$, where $0 \leq \rho < \sqrt{1/(4eC_2^2|\lambda|)}$. For any multi-index α, we write $J^\alpha(f(u))$ as

$$
\begin{aligned}
J^\alpha(f(u)) &= M(it\partial)^\alpha f(M^{-1}u) \\
&= \sum_{j=1}^\infty \frac{\lambda^j}{j!} M(it\partial)^\alpha (|M^{-1}u|^{2j} M^{-1}u) \\
&= \sum_{j=1}^\infty \frac{\lambda^j}{j!} \sum_{\substack{\beta+\gamma+\delta=\alpha \\ \beta^{(1)}+\cdots+\beta^{(j)}=\beta \\ \gamma^{(1)}+\cdots+\gamma^{(j)}=\gamma}} \frac{\alpha!(-1)^{|\gamma|}}{\left(\prod_{k=1}^j \beta^{(k)}!\gamma^{(k)}!\right)\delta!} \prod_{k=1}^j J^{\beta^{(k)}}u \overline{J^{\gamma^{(k)}}u} J^\delta u
\end{aligned}
$$

to estimate

$$
\begin{aligned}
&\|J^\alpha(f(u)); L^{4/3}\| \\
&\leq \sum_{j=1}^\infty \frac{|\lambda|^j}{j!} \sum_{\substack{\beta+\gamma+\delta=\alpha \\ \beta^{(1)}+\cdots+\beta^{(j)}=\beta \\ \gamma^{(1)}+\cdots+\gamma^{(j)}=\gamma}} \frac{\alpha!}{\left(\prod_{k=1}^j \beta^{(k)}!\gamma^{(k)}!\right)\delta!} \\
&\quad \cdot \prod_{k=1}^j \|J^{\beta^{(k)}}u; L^{4(2j+1)/3}\| \|J^{\gamma^{(k)}}u; L^{4(2j+1)/3}\| \|J^\delta u; L^{4(2j+1)/3}\|.
\end{aligned}
$$

We use Lemma 2 to estimate three factors in the last norms as

$$
\|J^\beta u; L^{4(2j+1)/3}\| \leq C_4 \left(\frac{4(2j+1)}{3}\right)^{3/4} \|J^\beta u; H_4^1\|
$$

with $\beta = \beta^{(1)}, \gamma^{(1)}, \delta$. We use Lemma 2 to estimate other factors as

$$
\|J^\beta u; L^{4(2j+1)/3}\| \leq C_2 \left(\frac{4(2j+1)}{3}\right)^{1/2} \|J^\beta u; H_2^1\|
$$

with $\beta = \beta^{(k)}, \gamma^{(k)}$ for $k \geq 2$. Therefore, we obtain

$$
\begin{aligned}
&\|J^\alpha(f(u)); L^{4/3}\| \\
&\leq \sum_{j=1}^\infty \frac{|\lambda|^j}{j!} \sum_{\substack{\beta+\gamma+\delta=\alpha \\ \beta^{(1)}+\cdots+\beta^{(j)}=\beta \\ \gamma^{(1)}+\cdots+\gamma^{(j)}=\gamma}} \frac{\alpha!}{\left(\prod_{k=1}^j \beta^{(k)}!\gamma^{(k)}!\right)\delta!} C_4^3 C_2^{2(j-1)} \left(\frac{4}{3}(2j+1)\right)^{j+5/4} \\
&\quad \cdot \left(\prod_{k=2}^j \|J^{\beta^{(k)}}u; H_2^1\| \|J^{\gamma^{(k)}}u; H_2^1\|\right) \|J^{\beta^{(1)}}u; H_4^1\| \|J^{\gamma^{(1)}}u; H_4^1\| \|J^\delta u; H_4^1\|.
\end{aligned}
$$

Taking $L^{4/3}$ norm in time of the last inequality and using the Hölder inequality, we have

$$\|J^\alpha(f(u)); L^{4/3}(\mathbb{R}; L^{4/3})\|$$

$$\leq \sum_{j=1}^\infty \frac{|\lambda|^j}{j!} \sum_{\substack{\beta+\gamma+\delta=\alpha \\ \beta^{(1)}+\cdots+\beta^{(j)}=\beta \\ \gamma^{(1)}+\cdots+\gamma^{(j)}=\gamma}} \frac{\alpha!}{\left(\prod_{k=1}^j \beta^{(k)}!\gamma^{(k)}!\right)\delta!} C_4^3 C_2^{2(j-1)} \left(\frac{4}{3}(2j+1)\right)^{j+5/4}$$

$$\cdot \left(\prod_{k=2}^j \|J^{\beta^{(k)}}u; L^\infty(\mathbb{R}; H_2^1)\| \|J^{\gamma^{(k)}}u; L^\infty(\mathbb{R}; H_2^1)\|\right) \|J^{\beta^{(1)}}u; \mathscr{X}_1\| \|J^{\gamma^{(1)}}u; \mathscr{X}_1\| \|J^\delta u; \mathscr{X}_1\|.$$

Multiplying the last inequality by $a^{|\alpha|}/\alpha!$ and taking the summation over all multi-indices of the resulting inequality, we obtain

$$\|\Phi(u); G_0^a(J)\|$$

$$= \sum_{\alpha \geq 0} \frac{a^{|\alpha|}}{\alpha!} \|J^\alpha(\Phi(u)); \mathscr{X}_0\|$$

$$\leq C \sum_{\alpha \geq 0} \frac{a^{|\alpha|}}{\alpha!} \|x^\alpha \phi; L^2\|$$

$$+ C \sum_{j=1}^\infty \frac{|\lambda|^j}{j!} C_4^3 C_2^{2(j-1)} \left(\frac{4}{3}(2j+1)\right)^{j+5/4} \|u; G_1^a(J)\|^{2j+1}$$

$$= C \|\phi; G_0^a(x)\| + C F_1(\|u; G_1^a(J)\|) \|u; G_1^a(J)\|^3.$$

We estimate $\nabla J^\alpha(f(u))$ in $L^{4/3}$ by using the Hölder inequality with $3/4 = 1/4 + 2j/(4j)$ to obtain terms of the form

$$\prod_{k=1}^j \|J^{\beta^{(k)}}u; L^{4j}\| \|J^{\gamma^{(k)}}u; L^{4j}\| \|J^\delta u; \dot{H}_4^1\|$$

with $\beta + \gamma + \delta = \alpha$, $\beta^{(1)} + \cdots + \beta^{(j)} = \beta$, $\gamma^{(1)} + \cdots + \gamma^{(j)} = \gamma$. We use Lemma 2 to estimate two factors

$$\|J^\beta u; L^{4j}\| \leq C_4(4j)^{3/4} \|J^\beta u; H_4^1\|$$

with $\beta = \beta^{(1)}, \gamma^{(1)}$. We use Lemma 2 to estimate two factors

$$\|J^\beta u; L^{4j}\| \leq C_2(4j)^{1/2} \|J^\beta u; H_2^1\|$$

with $\beta = \beta^{(k)}$, $\gamma^{(k)}$ for $k \geq 2$. In the same way as before, we obtain

$$\sum_{\alpha \geq 0} \frac{a^{|\alpha|}}{\alpha!} \|\nabla J^{\alpha}(\Phi(u)); \mathscr{X}_0\|$$

$$\leq C \sum_{\alpha \geq 0} \frac{a^{|\alpha|}}{\alpha!} \|x^{\alpha} \phi; \dot{H}_2^1\| + C F_2(\|u; G_1^a(J)\|)\|u; G_1^a(J)\|^3.$$

Therefore,

$$\|\Phi(u); G_1^a(J)\|$$

$$\leq C\|\phi; G_1^a(x)\| + C(F_1(\|u; G_1^a(J)\|) + F_2(\|u : G_1^a(J)\|))\|u; G_1^a(J)\|^3.$$

Similarly, we have

$$\|\Phi(u) - \Phi(v); G_0^a(J)\|$$

$$\leq C(F_1(\|u; G_1^a(J)\|)\|u; G_1^a(J)\|^2 + F_2(\|v; G_1^a(J)\|)\|v; G_1^a(J)\|^2)\|u - v; G_0^a(J)\|,$$

so that the contraction argument goes through for any $\phi \in G_1^a(x)$ with $\|\phi; G_1^a(x)\| \leq \rho$, provided that ε and ρ satisfy

$$\begin{cases} C(F_1(\varepsilon)\varepsilon^2, F_2(\varepsilon)\varepsilon^2) < 1, \\ C\rho + C(F_1(\varepsilon)\varepsilon^3 + F_2(\varepsilon)\varepsilon^3) \leq \varepsilon. \end{cases}$$

This completes the proof of Theorem 1.

4. Proof of Theorem 2

For $\varepsilon > 0$ we define the metric space

$$X_0(\varepsilon) = \{u \in G_0^a(J); \ \|u; G_0^a(J)\| \leq \varepsilon\}$$

with metric

$$d(u, v) = \|u - v; G_0^a(J)\|.$$

We see that $(X_0(\varepsilon), d)$ is a complete metric space. We prove that $\Phi: u \mapsto \Phi(u)$ is a contraction mapping on $(X_0(\varepsilon), d)$ for $\varepsilon, \rho > 0$ sufficiently small. in a way similar to, and simpler than, that of the proof of Theorem 1, for any $u, v \in X_0(\varepsilon)$ we have

$$\|\Phi(u); G_0^a(J)\| \leq C\|\phi; G_0^a(J)\| + C\varepsilon^3,$$

$$\|\Phi(u) - \Phi(v); G_0^a(J)\| \leq C\varepsilon^2 \|u - v; G_0^a(J)\|,$$

and the contraction argument goes through for any $\phi \in G_0^a(x)$ with $\|\phi; G_0^a(x)\| \leq \rho$, provided that ε and ρ satisfy

$$\begin{cases} C\varepsilon^2 < 1, \\ C\rho + C\varepsilon^3 \leq \varepsilon. \end{cases}$$

This completes the proof of Theorem 2.

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