



KATSUO KAWAKUBO



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(1942–1999)

The late Professor Katsuo Kawakubo was born on June 20, 1942 in Nagano Prefecture, Japan. He entered the University of Tokyo and graduated in 1966. After studying for two years at Graduate School of Science, the University of Tokyo, he was appointed assistant at Osaka University in 1968. He received the doctoral degree from the University of Tokyo in 1973. He was promoted to lecturer in 1973, to associate professor in 1977, and to full professor in 1989. He worked as a visiting member at the Institute for Advanced Study, Princeton in 1970–1971 and at The State University of New York in 1971–1972, and as a guest professor at Bonn University in 1975–1976 and at the University of Helsinki in 1988. He also made efforts to develop the Mathematical Society of Japan. During the period 1990–1996, he was consecutively a member of the academic committee, the chairman of the committee, and a trustee of the Mathematical Society of Japan. Unfortunately he was affected by a cancer and died on April 24, 1999.

He was one of the pioneers and leaders of transformation groups in Japan, and made a great contribution to development of topology. He wrote a large number of papers and delivered lectures in various places in the world. He furthermore organized various symposia and meetings of transformation groups, and, in particular, held the International Conference of Transformation Groups at Osaka University in 1987.

He was also an excellent teacher and trained many graduate students and young mathematicians. He constantly encouraged and advised them, and gave them opportunities to present their results. He also published the textbook “The Theory of Transformation Groups” ([b], [d]) for them as well as for the wider audience. In addition he published several guides to mathematics for the general public ([a], [c], [e], [g], [h]).

His mathematical works are mainly concerned with the classification of equivariant manifolds and are roughly divided into five subjects as follows:

(1) *Differentiable structures on the product of spheres*: He first studied the inertia group of a product of homotopy spheres and showed in [2] that this group is in general not a combinatorial invariant, depending on more than tangential homotopy equivalence. This result gives a solution of a problem of W. Browder. Furthermore, he completely classified the possible differentiable structures on  $S^p \times S^q$ ,  $p + q \geq 6$  ([3], [4]), which is a generalization of the earlier work of S. P. Novikov. On the other hand, I. M. James asked whether any 3-sphere bundle over the 7-sphere corresponding to an element of  $\text{Im } S \cap \text{Ker } J$  is homeomorphic to  $S^3 \times S^7$ , where  $S : \pi_6(SO_3) \rightarrow \pi_6(SO_4)$  is the stable map and  $J : \pi_6(SO_4) \rightarrow \pi_{10}(S^4)$  is the  $J$ -homomorphism. In [4] Kawakubo gave an affirmative answer to this question.

(2) *Group actions on homotopy spheres*: He studied free or semifree actions on homotopy spheres and showed many interesting results by using techniques of differential topology. Some of the results give partial or complete answers to various problems posed by G. E. Bredon, W. Browder, and F. Hirzebruch. For example: (a) On a certain homotopy sphere there is no free  $S^1$ -action whose orbit space is PL-homeomorphic to a complex projective space ([1]). (b) Let  $\Sigma^{11}$  be a homotopy 11-sphere which generates the group of homotopy spheres. The homotopy 11-sphere  $32k\Sigma^{11}$  admits no free  $S^3$ -action if  $k \equiv \pm 6, 7, 8, 9, 13 \pmod{31}$ , whereas for any other  $k$  it admits infinitely many distinct free  $S^3$ -actions ([5], [6]). (c) For any  $n \geq 5$  and  $r < (n-1)/2$  there exist infinitely many distinct semifree  $S^1$ -actions on any Brieskorn  $(2n+1)$ -sphere with fixed point set of codimension  $r$  ([9], [14]). (d) There exist infinitely many distinct semifree  $S^1$ -actions on  $S^{p+2q}$  with knotted  $S^p$  as the fixed point set provided  $p \equiv 3 \pmod{4}$  and  $4q \leq p+3$  ([10]).

(3) *Equivariant characteristic classes*: He studied with F. Raymond and F. Uchida the Thom-Hirzebruch index for actions of the  $n$ -dimensional torus  $T$  on an oriented closed manifold  $M$  ([7], [8], [11], [16]), and proved the formula  $I(M) = I(M^T)$  for this index. He also found a similar formula for the generalized Todd genus of  $Z_p$ -actions ([17], [19]). Furthermore, using cobordism theory, he succeeded in estimating the dimension of the fixed point set  $M^{Z_p}$  ([18]). He also constructed an invariant  $\sigma(S^1, M)$  for  $S^1$ -actions, which is a generalization of the Atiyah-Singer invariant  $\sigma(\infty, (S^1, M))$ , and proved that  $\sigma(S^1, M^3) = 0$  if and only if  $M$  is a fiber bundle over  $S^1$  with finite structure group.

On the other hand, he established the equivariant Gysin sequences in equivariant cohomology theory, and formulated and proved the equivariant Riemann-Roch type theorem and the localization theorem ([21], [23], [29]). This theory had many important applications and led to algebraic-topological proofs of the following results: (a) the vanishing theorem of  $\hat{A}$ -genus for spin manifolds with nontrivial compact connected Lie group actions, (b) the vanishing theorem of  $\exp(c/2)\hat{A}$ -genus for  $S^1$ -manifolds, (c) the  $G$ -signature theorem, and (d) the Todd genus formula for torus actions.

(4) *Equivariant vector bundles and sphere bundles*: He studied this subject in order to classify  $G$ -homotopy types of  $G$ -manifolds for a compact Lie group  $G$ . He first proved the equivariant Dold theorem for equivariant vector bundles. Next he introduced the equivariant  $J$ -group  $J_G(M)$  of a  $G$ -manifold  $M$ , and proved that  $J_G(TM) = J_G(f^*TN)$  if  $f : M \rightarrow N$  is a  $G$ -homotopy equivalence, where  $J_G : KO_G(M) \rightarrow J_G(M)$  is the natural projection. By using these results, he showed various interesting results on  $G$ -homotopy equivalent manifolds ([25], [28], [31]). For example  $(Z_2)^k$ -homotopy equivalent manifolds are equivariantly cobordant ([27]). He also computed  $J_G(*)$  for some compact Lie groups and classified  $G$ -homotopy types of unit spheres of  $G$ -representations ([24], [26], [30]). Furthermore he gave an algebraic characterization of  $KO_G(X)$  and  $J_G(X)$ , and obtained the induction theorem ([33], [34], [35],

[37]).

(5) *Equivariant  $s$ -cobordism theory*: He studied the  $s$ -cobordism theorem in equivariant setting in order to classify  $G$ -manifolds. His first result on this subject is that there exists a  $G$ -representation  $V$  such that  $M \times D(V)$  and  $N \times D(V)$  are  $G$ -diffeomorphic if and only if  $M$  and  $N$  are tangential simple  $G$ -homotopy equivalent [38]. He studied with S. Araki properties of the equivariant Whitehead groups and proved in [39] that the equivariant  $s$ -cobordism theorem holds under some gap conditions, and in [40] that it also holds in general for semifree  $S^1$ -actions. When the gap conditions are not satisfied, there exist counterexamples for arbitrary compact Lie groups ([41], [42]).

With all these works Professor Kawakubo steadily improved the study of classifying  $G$ -manifolds, and his results enlightened many topologists on their researches. His death is a serious loss to the world of topology.

## List of publications of Katsuo Kawakubo

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