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THE DIMENSION FORMULA FOR THE RING OF CODE POLYNOMIALS IN GENUS 4

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1. Introduction

The purpose of this paper is to study the dimension formula for the invariant ring $C[f_a \text{ for } a \in F_2^4]^{H_4}$, which may be considered as the ring of code polynomials in genus 4. We also give all characteristic polynomials of elements in H_4. The main ingredient is the determination of the conjugacy classes of the symplectic group $Sp(8,2)$. Our result will be useful for the investigation of the Siegel modular forms in genus four.

We recall from [10, 11] that the finite group H_g is (up to ± 1) just the image of the modular group $\Gamma_g = Sp(2g,Z)$ under the theta representation (of index 1) and that the ring of modular forms of even weight is given by

$$A(\Gamma_g)_{(2)} = \bigoplus_{2|k} [\Gamma_g, k] = (C[f_a]^{H_g} / \{\text{relation}\})^N,$$

where N denotes the normalization in its field of fractions and "relation" are the theta relations. However, the generators and the dimension formulas for $A(\Gamma_g)$ are known only for genus $g \leq 3$.

On the other hand, the invariant ring $C[f_a]^{H_g}$ may be considered as the ring of code polynomials in genus g. In [9], there is the definition of the g-th weight polynomial for codes (codes mean the binary linear codes) and the connections among codes, lattices, the invariant rings of the finite groups, and the theory of modular forms were studied (cf. [1], [4], [5], [8], [16]). In particular, it was shown that the invariant ring of the group $\langle H_g, \zeta_8 \rangle$, which is the subring of $C[f_a]^{H_g}$, is generated by the g-th weight polynomials for self-dual doubly-even codes, where ζ_8 is the primitive 8-th root of unity. This invariant ring corresponds to the ring of the modular forms of weights divisible by 4.

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2. On the group H_g

In this section, we study the group H_g. In addition to Runge, the group H_g has been studied by several authors, for example, see [4], [7].

Let V be the g-dimensional vector space over the field of two elements, i.e., $V=\boldsymbol{F}_2^g$. For $x,y\in V$, let $x\cdot y$ denote the usual dot product. Set

$$T_g:=\left(\frac{1+i}{2}\right)^g[(-1)^{a\cdot b}]_{a,b\in V},$$

and for a symmetric $g\times g$ matrix S,

$$D_S:=\mathrm{diag}(i^{S[a]}\text{ with } a\in V),$$

where $S[a]:=aS^ta$. Let

$$H_g:=\langle T_g,\mathrm{D}_S\,|\,S\text{ runs over all symmetric matrices in }\mathrm{Mat}_{g\times g}(\boldsymbol{Z})\rangle$$

be the subgroup of $Gl(2^g,\boldsymbol{C})$ generated by the elements T_g and the D_S.

We get for the invarint ring $\boldsymbol{C}[f_a]^{H_g}$ the dimension formula

$$\Phi_{H_g}(t)=\sum_{d\geq 0}(\dim\boldsymbol{C}[f_a]_d^{H_g})t^d=\frac{1}{|H_g|}\sum_{\sigma\in H_g}\frac{1}{\det(1_{2^g}-t\sigma)},$$

where $\boldsymbol{C}[f_a]_d^{H_g}$ is the d-th homogeneous part of $\boldsymbol{C}[f_a]^{H_g}$, $\Phi_{H_g}(t)$ is called the Molien series of H_g.

The following lemma gives the simplification we use.

Lemma 2.1 (Lemma 2.1 [9]). *One has an exact sequence*

$$0\to N_g\to H_g\overset{\varphi}{\to}Sp(2g,2)\to 0,$$

where $N_g:=\langle i,D_S^2,T_g^{-1}D_S^2T_g\rangle$ *and the homomorphism* φ *is given by the conjugation of* H_g *on the* $\boldsymbol{F}_2$*-vector space* $N_g/\langle i\rangle$. □

Therefore we have the obvious formula

$$\Phi_{H_g}(t)=\frac{1}{|H_g|}\sum_i\sum_{n\in N_g}\frac{|C_i|}{\det(1-tz_in)},\tag{2.1}$$

where $\{C_i\}$ are the conjugacy classes of $Sp(2g,2)$ and z_i is some element of H_g with $\varphi(z_i)\in C_i$. Our computation is done using (2.1).

Finally we list the sizes of groups.

$$|N_g| = 2^{2+2g},$$

$$|Sp(2g,2)| = 2^{g^2}(2^{2g}-1)\cdots(2^2-1),$$

$$|H_g| = |N_g| \times |Sp(2g,2)| = 2^{2+2g+g^2}(2^{2g}-1)\cdots(2^2-1).$$

In our case $(g=4)$, $|N_4| = 1{,}024 = 2^{10}$, $|Sp(8,2)| = 47{,}377{,}612{,}800 = 2^{16}3^5 5^2 \cdot 7 \cdot 17$, $|H_4| = 48{,}514{,}675{,}507,\ 200 = 2^{26}3^5 5^2 \cdot 7 \cdot 17$.

REMARK. H_1, $\langle H_1, \zeta_8 \rangle$, H_2 are the reflection groups No.8, No.9, No.31 in [14], respectively (cf. Proposition 2.6 [10]).

3. On $Sp(8,2)$ and its conjugacy classes

In this section, we study the symplectic group $Sp(8,2)$. This is identified with the Chevalley group of type (C_4) over the field of two elements. We give all the characteristic polynomials of elements in H_4. We remark that we cannot read off representatives of the conjugacy classes of $Sp(8,2)$ from Atlas [3] although it is the good reference for the finite simple groups.

As is said, $Sp(8,2)$ is one of the Chevalley groups. The properties of such groups are known and we describe what we need.

Let $\Delta = \{\pm 2\xi_i, \pm\xi_i \pm \xi_j\ (i<j) \mid 1 \leq i, j \leq 4\}$ be the root system of type (C_4), and choose $a = \xi_1 - \xi_2$, $b = \xi_2 - \xi_3$, $c = \xi_3 - \xi_4$, $d = 2\xi_4$ for a fundamental system Π of roots. We denote by Δ^+ the set of positive roots with respect to Π. We write an element $\alpha a + \beta b + \gamma c + \delta d$ of Δ^+ as $\alpha\beta\gamma\delta$. For example, we write 1023 for $a + 2c + 3d$.

E_{ij} is the elementary matrix of size 8×8 with 1 in the (i,j)-entry. For $1 \leq i, j \leq 4$,

$$x_{\xi_i+\xi_j} := 1 + E_{i,4+j} + E_{j,4+i},$$

$$x_{\xi_i-\xi_j} := 1 + E_{i,j} + E_{4+j,4+i} \qquad (i<j),$$

$$x_{2\xi_i} := 1 + E_{i,4+i},$$

$$x_{-r} := {}^t x_r \qquad (r \in \Delta).$$

Then $Sp(8,2)$ is generated by x_r $(r \in \Delta)$ and is known to be one of the finite simple groups (cf. [3]).

Let X_r be the group generated by x_r and put $B = X_{0100}X_{0010}X_{0110}X_{0001}X_{0011}X_{0111}X_{0021}X_{0121}X_{0221}X_{1000}X_{1100}X_{1110}X_{1111}X_{1121}X_{1221}X_{2221}$. Then B is a Sylow 2-subgroup of $Sp(8,2)$ and is normal in $Sp(8,2)$.

Put $n_r = x_r x_{-r} x_r$ and $N = \langle n_r \mid r \in \Delta \rangle$. N is isomorphic to the Weyl group of type (C_4). We fix the following correspondence:

$$\xi_1 \leftrightarrow \underline{1} := [1,0,0,0,0,0,0,0],$$

$$\xi_2 \leftrightarrow \underline{2} := [0,1,0,0,0,0,0,0],$$

$$\xi_3 \leftrightarrow \underline{3} := [0,0,1,0,0,0,0,0],$$

$$\xi_4 \leftrightarrow \underline{4} := [0,0,0,1,0,0,0,0],$$

$$-\xi_1 \leftrightarrow \underline{-1} := [0,0,0,0,1,0,0,0],$$

$$-\xi_2 \leftrightarrow \underline{-2} := [0,0,0,0,0,1,0,0],$$

$$-\xi_3 \leftrightarrow \underline{-3} := [0,0,0,0,0,0,1,0],$$

$$-\xi_4 \leftrightarrow \underline{-4} := [0,0,0,0,0,0,0,1].$$

An element of N is uniquely determined by its natural action on $\underline{1}$, $\underline{2}$, $\underline{3}$, $\underline{4}$. If $n \in N$ satisfies $\underline{1}n=\underline{\alpha}$, $\underline{2}n=\underline{\beta}$, $\underline{3}n=\underline{\gamma}$, $\underline{4}n=\underline{\delta}$, then we denote n by $n(\alpha,\beta,\gamma,\delta)$. For example, we have

$$n(2,3,-1,-4) = \begin{bmatrix} 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&1&0 \\ 1&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \end{bmatrix}$$

In the following, we give the conjugacy classes of $Sp(8,2)$ and all characteristic polynomials of elements in H_4. To determine the conjugacy classes of $Sp(8,2)$, we have to show

(i) No two elements in the list are conjugate in $Sp(8,2)$,

(ii) $\Sigma_{0 \le i \le 80} |Sp(8,2)| / |C_{Sp(8,2)}(c_i)| = |Sp(8,2)|$,

where $C_{Sp(8,2)}(c_i)$ denotes the centralizer group of c_i in $Sp(8,2)$. These statements are proved using GAP [13]. Then we compute 1024×81 determinants of size 16×16. Since H_4 is a subgroup of $SU(2^4, \boldsymbol{Z}[\frac{1+i}{2}])$ (Proposition 2.6 [10]), the polynomials have the type $\Sigma_{0 \le i \le 16} a_i t^i$ with $a_0 = a_{16} = 1$, $\bar{a}_1 = a_{15}$, $\bar{a}_2 = a_{14}$, $\bar{a}_3 = a_{13}$, $\bar{a}_4 = a_{12}$, $\bar{a}_5 = a_{11}$, $\bar{a}_6 = a_{10}$, $\bar{a}_7 = a_9$, and $\bar{a}_8 = a_8$ (A bar denotes complex conjugation).

There are 81 boxes below and the i-th box ($0 \le i \le 80$) gives the characteristic polynomials of elements in $z_i N_4$. In each box, the first column gives the multiplicity of each polynomial. The next columns give the values of $a_1, a_2, \cdots, a_8$, respectively. For example, in the first box, the 2 in the first column means that there are 2 occurences of the polynomial in the form $1+(8-8i)t-64it^2+(-168-168i)t^3-644t^4+(-952+952i)t^5+2240it^6+(2136+2136)t^7+3334t^8+\cdots$.

Table.

order $= 1$, $c_0 = 1$, $\lvert C_{Sp(8,2)}(c_0)\rvert = 47377612800$								
2	$8-8i$	$-64i$	$-168-168i$	-644	$-952+952i$	$2240i$	$2136+2136i$	3334
504	0	0	0	-4	0	0	0	6
2	$8+8i$	$64i$	$-168+168i$	-644	$-952-952i$	$-2240i$	$2136-2136i$	3334
256	0	$8i$	0	-28	0	$-56i$	0	70
256	0	$-8i$	0	-28	0	$56i$	0	70
2	$-8-8i$	$64i$	$168-168i$	-644	$952+952i$	$-2240i$	$-2136+2136i$	3334
2	$-8+8i$	$-64i$	$168+168i$	-644	$952-952i$	$2240i$	$-2136-2136i$	3334

order $= 2$, $c_1 = x_{0001}$, $\lvert C_{Sp(8,2)}(c_1)\rvert = 185794560$								
2	$8-8i$	$-64i$	$-168-168i$	-644	$-952+952i$	$2240i$	$2136+2136i$	3334
504	0	0	0	-4	0	0	0	6
2	$8+8i$	$64i$	$-168+168i$	-644	$-952-952i$	$-2240i$	$2136-2136i$	3334
256	0	$8i$	0	-28	0	$-56i$	0	70
256	0	$-8i$	0	-28	0	$56i$	0	70
2	$-8-8i$	$64i$	$168-168i$	-644	$952+952i$	$-2240i$	$-2136+2136i$	3334
2	$-8+8i$	$-64i$	$168+168i$	-644	$952-952i$	$2240i$	$-2136-2136i$	3334

order $= 2$, $c_2 = x_{0100}$, $\lvert C_{Sp(8,2)}(c_2)\rvert = 8847360$								
4	8	24	24	-36	-120	-88	88	198
120	0	-8	0	28	0	-56	0	70
768	0	0	0	-4	0	0	0	6
120	0	8	0	28	0	56	0	70
4	-8	24	-24	-36	120	-88	-88	198
4	$-8i$	-24	$24i$	-36	$120i$	88	$88i$	198
4	$8i$	-24	$-24i$	-36	$-120i$	88	$-88i$	198

order $= 2$, $c_3 = x_{0100}x_{0121}$, $\lvert C_{Sp(8,2)}(c_3)\rvert = 2949120$								
4	8	32	88	188	328	480	600	646
240	0	0	0	-4	0	0	0	6
128	0	-8	0	28	0	-56	0	70
128	0	8	0	28	0	56	0	70
4	-8	32	-88	188	-328	480	-600	646
512	0	0	0	4	0	0	0	6
4	$-8i$	-32	$88i$	188	$-328i$	-480	$600i$	646
4	$8i$	-32	$-88i$	188	$328i$	-480	$-600i$	646

order $= 2$, $c_4 = x_{0100}x_{0121}x_{1110}$, $\lvert C_{Sp(8,2)}(c_4)\rvert = 49152$								
16	4	8	12	12	4	-8	-20	-26
32	0	-8	0	28	0	-56	0	70
384	0	0	0	-4	0	0	0	6
16	-4	8	-12	12	-4	-8	20	-26
512	0	0	0	4	0	0	0	6
32	0	8	0	28	0	56	0	70
16	$-4i$	-8	$12i$	12	$-4i$	8	$-20i$	-26
16	$4i$	-8	$-12i$	12	$4i$	8	$20i$	-26

order = 2, $c_5 = x_{0100}x_{1110}$, $\lvert C_{Sp(8,2)}(c_5)\rvert = 737280$								
16	4	0	−20	−20	36	64	−20	−90
960	0	0	0	−4	0	0	0	6
16	−4	0	20	−20	−36	64	20	−90
16	$-4i$	0	$-20i$	−20	$-36i$	−64	$-20i$	−90
16	$4i$	0	$20i$	−20	$36i$	−64	$20i$	−90

order = 2, $c_6 = x_{0100}x_{0001}$, $\lvert C_{Sp(8,2)}(c_6)\rvert = 147456$								
8	$4-4i$	$-16i$	$-20-20i$	−36	$-28+28i$	$48i$	$44+44i$	70
480	0	0	0	−4	0	0	0	6
8	$4+4i$	$16i$	$-20+20i$	−36	$-28-28i$	$-48i$	$44-44i$	70
64	0	$8i$	0	−28	0	$-56i$	0	70
384	0	0	0	4	0	0	0	6
64	0	$-8i$	0	−28	0	$56i$	0	70
8	$-4+4i$	$-16i$	$20+20i$	−36	$28-28i$	$48i$	$-44-44i$	70
8	$-4-4i$	$16i$	$20-20i$	−36	$28+28i$	$-48i$	$-44+44i$	70

order = 3, $c_7 = n(2,3,1,4)$, $\lvert C_{Sp(8,2)}(c_7)\rvert = 12960$								
16	4	6	8	17	24	22	28	36
480	0	−2	0	1	0	−2	0	4
480	0	2	0	1	0	2	0	4
16	−4	6	−8	17	−24	22	−28	36
16	$-4i$	−6	$8i$	17	$-24i$	−22	$28i$	36
16	$4i$	−6	$-8i$	17	$24i$	−22	$-28i$	36

order = 3, $c_8 = (x_{0011}n(-4,-1,-2,-3))^5$, $\lvert C_{Sp(8,2)}(c_8)\rvert = 77760$								
256	$-i$	0	$5i$	5	0	−10	$10i$	0
256	i	0	$-5i$	5	0	−10	$-10i$	0
256	1	0	5	5	0	10	10	0
256	−1	0	−5	5	0	10	−10	0

order = 3, $c_9 = x_{0001}n(1,2,3,-4)$, $\lvert C_{Sp(8,2)}(c_9)\rvert = 4354560$								
4	8	36	112	266	504	784	1016	1107
504	0	4	0	10	0	16	0	19
4	$-8i$	−36	$112i$	266	$-504i$	−784	$1016i$	1107
504	0	−4	0	10	0	−16	0	19
4	−8	36	−112	266	−504	784	−1016	1107
4	$8i$	−36	$-112i$	266	$504i$	−784	$-1016i$	1107

order = 3, $c_{10} = x_{0001}n(-2,3,-1,-4)$, $\lvert C_{Sp(8,2)}(c_{10})\rvert = 3888$								
64	$2i$	−3	$2i$	−7	$-12i$	2	$-8i$	18
384	0	−1	0	1	0	2	0	−2
384	0	1	0	1	0	−2	0	−2
64	2	3	−2	−7	−12	−2	8	18
64	$-2i$	−3	$-2i$	−7	$12i$	2	$8i$	18
64	−2	3	2	−7	12	−2	−8	18

order = 4, $c_{11} = x_{0001}x_{1110}$, $\lvert C_{Sp(8,2)}(c_{11})\rvert = 92160$								
8	$4-4i$	$-12i$	$-4-4i$	24	$36-36i$	$-28i$	$28+28i$	78
240	0	$4i$	0	-8	0	$-12i$	0	14
8	$4+4i$	$12i$	$-4+4i$	24	$36+36i$	$28i$	$28-28i$	78
240	0	$-4i$	0	-8	0	$12i$	0	14
8	$-4+4i$	$-12i$	$4+4i$	24	$-36+36i$	$-28i$	$-28-28i$	78
8	$-4-4i$	$12i$	$4-4i$	24	$-36-36i$	$28i$	$-28+28i$	78
512	0	0	0	4	0	0	0	6

order = 4, $c_{12} = x_{0001}x_{1000}x_{1110}x_{1111}$, $\lvert C_{Sp(8,2)}(c_{12})\rvert = 92160$								
8	$4-4i$	$-20i$	$-36-36i$	-104	$-124+124i$	$252i$	$220+220i$	334
240	0	$4i$	0	-8	0	$-12i$	0	14
240	0	$-4i$	0	-8	0	$12i$	0	14
8	$4+4i$	$20i$	$-36+36i$	-104	$-124-124i$	$-252i$	$220-220i$	334
512	0	0	0	-4	0	0	0	6
8	$-4-4i$	$20i$	$36-36i$	-104	$124+124i$	$-252i$	$-220+220i$	334
8	$-4+4i$	$-20i$	$36+36i$	-104	$124-124i$	$252i$	$-220-220i$	334

order = 4, $c_{13} = x_{0100}x_{0121}x_{1100}x_{2221}$, $\lvert C_{Sp(8,2)}(c_{13})\rvert = 36864$								
16	4	12	28	52	84	116	140	150
16	$4i$	-12	$-28i$	52	$84i$	-116	$-140i$	150
16	$-4i$	-12	$28i$	52	$-84i$	-116	$140i$	150
96	0	4	0	4	0	-4	0	-10
96	0	-4	0	4	0	4	0	-10
16	-4	12	-28	52	-84	116	-140	150
768	0	0	0	4	0	0	0	6

order = 4, $c_{14} = x_{0100}x_{0121}x_{1100}$, $\lvert C_{Sp(8,2)}(c_{14})\rvert = 12288$								
16	4	4	-4	-12	-12	-4	12	22
16	-4	4	4	-12	12	-4	-12	22
96	0	-4	0	4	0	4	0	-10
96	0	4	0	4	0	-4	0	-10
256	0	0	0	4	0	0	0	6
512	0	0	0	-4	0	0	0	6
16	$-4i$	-4	$-4i$	-12	$12i$	4	$12i$	22
16	$4i$	-4	$4i$	-12	$-12i$	4	$-12i$	22

order = 4, $c_{15} = x_{0100}x_{0001}x_{1110}$, $\lvert C_{Sp(8,2)}(c_{15})\rvert = 6144$								
32	$2-2i$	0	$6+6i$	8	$-6+6i$	$16i$	$-2-2i$	-18
384	0	0	0	0	0	0	0	-2
32	$2+2i$	0	$6-6i$	8	$-6-6i$	$-16i$	$-2+2i$	-18
32	$-2+2i$	0	$-6-6i$	8	$6-6i$	$16i$	$2+2i$	-18
32	$-2-2i$	0	$-6+6i$	8	$6+6i$	$-16i$	$2-2i$	-18
512	0	0	0	4	0	0	0	6

order = 4, $c_{16} = x_{0100}x_{0001}x_{1110}x_{1111}$, $\lvert C_{Sp(8,2)}(c_{16})\rvert = 6144$								
32	$2-2i$	$-8i$	$-10-10i$	-24	$-22+22i$	$40i$	$30+30i$	46
384	0	0	0	0	0	0	0	-2
32	$2+2i$	$8i$	$-10+10i$	-24	$-22-22i$	$-40i$	$30-30i$	46
32	$-2-2i$	$8i$	$10-10i$	-24	$22+22i$	$-40i$	$-30+30i$	46
32	$-2+2i$	$-8i$	$10+10i$	-24	$22-22i$	$40i$	$-30-30i$	46
512	0	0	0	-4	0	0	0	6

order = 4, $c_{17} = x_{0100}x_{0121}x_{1000}$, $\lvert C_{Sp(8,2)}(c_{17})\rvert = 3072$								
16	4	8	12	16	20	24	28	30
128	0	−4	0	8	0	−12	0	14
192	0	0	0	0	0	0	0	−2
128	0	4	0	8	0	12	0	14
16	−4	8	−12	16	−20	24	−28	30
256	0	0	0	4	0	0	0	6
256	0	0	0	−4	0	0	0	6
16	$-4i$	−8	$12i$	16	$-20i$	−24	$28i$	30
16	$4i$	−8	$-12i$	16	$20i$	−24	$-28i$	30

order = 4, $c_{18} = x_{0100}x_{0001}x_{1121}$, $\lvert C_{Sp(8,2)}(c_{18})\rvert = 3072$								
32	$2-2i$	$-4i$	$-2-2i$	−4	$-6+6i$	$12i$	$6+6i$	6
32	$-2+2i$	$-4i$	$2+2i$	−4	$6-6i$	$12i$	$-6-6i$	6
32	$2+2i$	$4i$	$-2+2i$	−4	$-6-6i$	$-12i$	$6-6i$	6
32	$-2-2i$	$4i$	$2-2i$	−4	$6+6i$	$-12i$	$-6+6i$	6
64	0	$4i$	0	−4	0	$4i$	0	−10
64	0	$-4i$	0	−4	0	$-4i$	0	−10
384	0	0	0	−4	0	0	0	6
384	0	0	0	4	0	0	0	6

order = 4, $c_{19} = x_{0100}x_{0010}x_{0011}x_{2221}$, $\lvert C_{Sp(8,2)}(c_{19})\rvert = 1024$								
32	$2-2i$	$-4i$	$-2-2i$	0	$2-2i$	$-4i$	$-2-2i$	−2
32	$2+2i$	$4i$	$-2+2i$	0	$2+2i$	$4i$	$-2+2i$	−2
256	0	0	0	0	0	0	0	−2
32	$-2+2i$	$-4i$	$2+2i$	0	$-2+2i$	$-4i$	$2+2i$	−2
32	$-2-2i$	$4i$	$2-2i$	0	$-2-2i$	$4i$	$2-2i$	−2
256	0	0	0	4	0	0	0	6
256	0	0	0	−4	0	0	0	6
64	0	$4i$	0	−8	0	$-12i$	0	14
64	0	$-4i$	0	−8	0	$12i$	0	14

order = 4, $c_{20} = x_{0100}x_{0010}x_{1000}$, $\lvert C_{Sp(8,2)}(c_{20})\rvert = 768$								
64	2	0	−2	−4	−6	0	6	6
768	0	0	0	0	0	0	0	−2
64	−2	0	2	−4	6	0	−6	6
64	$-2i$	0	$-2i$	−4	$6i$	0	$6i$	6
64	$2i$	0	$2i$	−4	$-6i$	0	$-6i$	6

order = 4, $c_{21} = x_{0100}x_{0010}x_{0011}x_{1110}$, $\lvert C_{Sp(8,2)}(c_{21})\rvert = 512$								
64	2	0	−2	0	2	0	−2	−2
64	−2	0	2	0	−2	0	2	−2
512	0	0	0	0	0	0	0	−2
256	0	0	0	−4	0	0	0	6
64	$-2i$	0	$-2i$	0	$-2i$	0	$-2i$	−2
64	$2i$	0	$2i$	0	$2i$	0	$2i$	−2

order = 4, $c_{22} = x_{0100}x_{0010}x_{0011}x_{1110}x_{2221}$, $\lvert C_{Sp(8,2)}(c_{22})\rvert = 512$								
64	2	4	6	8	10	12	14	14
64	$2i$	-4	$-6i$	8	$10i$	-12	$-14i$	14
64	$-2i$	-4	$6i$	8	$-10i$	-12	$14i$	14
64	-2	4	-6	8	-10	12	-14	14
512	0	0	0	0	0	0	0	-2
256	0	0	0	4	0	0	0	6

order = 5, $c_{23} = x_{0001}x_{0011}n(1,2,-4,3)$, $\lvert C_{Sp(8,2)}(c_{23})\rvert = 3600$								
16	$-4i$	-10	$20i$	35	$-52i$	-68	$80i$	85
480	0	-2	0	3	0	-4	0	5
16	4	10	20	35	52	68	80	85
480	0	2	0	3	0	4	0	5
16	$4i$	-10	$-20i$	35	$52i$	-68	$-80i$	85
16	-4	10	-20	35	-52	68	-80	85

order = 5, $c_{24} = x_{0010}x_{0111}n(-4,-3,-1,-2)$, $\lvert C_{Sp(8,2)}(c_{24})\rvert = 300$								
256	$-i$	0	0	0	$-3i$	-3	0	0
256	i	0	0	0	$3i$	-3	0	0
256	-1	0	0	0	-3	3	0	0
256	1	0	0	0	3	3	0	0

order = 6, $c_{25} = n(-4,-2,-1,3)$, $\lvert C_{Sp(8,2)}(c_{25})\rvert = 864$								
256	0	$-2i$	0	-1	0	$2i$	0	4
32	$2+2i$	$4i$	0	7	$8+8i$	$4i$	$6-6i$	16
384	0	0	0	-1	0	0	0	0
256	0	$2i$	0	-1	0	$-2i$	0	4
32	$-2-2i$	$4i$	0	7	$-8-8i$	$4i$	$-6+6i$	16
32	$-2+2i$	$-4i$	0	7	$-8+8i$	$-4i$	$-6-6i$	16
32	$2-2i$	$-4i$	0	7	$8-8i$	$-4i$	$6+6i$	16

order = 6, $c_{26} = n(-4,-1,-3,-2)$, $\lvert C_{Sp(8,2)}(c_{26})\rvert = 96$								
128	0	2	0	1	0	2	0	4
512	0	0	0	1	0	0	0	0
64	$-2i$	-2	0	-3	$4i$	2	$2i$	4
64	$2i$	-2	0	-3	$-4i$	2	$-2i$	4
64	2	2	0	-3	-4	-2	2	4
64	-2	2	0	-3	4	-2	-2	4
128	0	-2	0	1	0	-2	0	4

order = 6, $c_{27} = n(-4,2,1,3)$, $\lvert C_{Sp(8,2)}(c_{27})\rvert = 288$								
256	0	$2i$	0	-1	0	$-2i$	0	4
32	$2-2i$	$-4i$	$-4-4i$	-9	$-8+8i$	$12i$	$10+10i$	16
384	0	0	0	-1	0	0	0	0
32	$-2+2i$	$-4i$	$4+4i$	-9	$8-8i$	$12i$	$-10-10i$	16
256	0	$-2i$	0	-1	0	$2i$	0	4
32	$-2-2i$	$4i$	$4-4i$	-9	$8+8i$	$-12i$	$-10+10i$	16
32	$2+2i$	$4i$	$-4+4i$	-9	$-8-8i$	$-12i$	$10-10i$	16

order = 6, $c_{28} = x_{0011}x_{0021}n(2,-4,3,-1)$, $\lvert C_{Sp(8,2)}(c_{28})\rvert = 288$								
64	-2	2	-4	5	-4	6	-6	4
128	0	-2	0	1	0	-2	0	4
64	$2i$	-2	$-4i$	5	$4i$	-6	$-6i$	4
128	0	2	0	1	0	2	0	4
64	$-2i$	-2	$4i$	5	$-4i$	-6	$6i$	4
512	0	0	0	1	0	0	0	0
64	2	2	4	5	4	6	6	4

order = 6, $c_{29} = x_{0001}x_{0011}x_{0021}n(1,-2,-4,3)$, $\lvert C_{Sp(8,2)}(c_{29})\rvert = 4608$								
128	0	-4	0	10	0	-16	0	19
512	0	0	0	-2	0	0	0	3
16	-4	8	-8	2	4	0	-12	19
192	0	0	0	2	0	0	0	3
16	4	8	8	2	-4	0	12	19
16	$4i$	-8	$-8i$	2	$-4i$	0	$-12i$	19
128	0	4	0	10	0	16	0	19
16	$-4i$	-8	$8i$	2	$4i$	0	$12i$	19

order = 6, $c_{30} = x_{0021}n(1,4,-3,2)$, $\lvert C_{Sp(8,2)}(c_{30})\rvert = 13824$								
16	$-4i$	-12	$24i$	42	$-60i$	-80	$92i$	99
96	0	-4	0	10	0	-16	0	19
768	0	0	0	2	0	0	0	3
16	-4	12	-24	42	-60	80	-92	99
96	0	4	0	10	0	16	0	19
16	$4i$	-12	$-24i$	42	$60i$	-80	$-92i$	99
16	4	12	24	42	60	80	92	99

order = 6, $c_{31} = x_{0001}x_{0011}x_{0021}n(-4,-3,-2,1)$, $\lvert C_{Sp(8,2)}(c_{31})\rvert = 288$								
768	0	0	0	-1	0	0	0	0
64	2	0	-4	-5	0	4	2	0
64	$2i$	0	$4i$	-5	0	-4	$-2i$	0
64	-2	0	4	-5	0	4	-2	0
64	$-2i$	0	$-4i$	-5	0	-4	$2i$	0

order = 6, $c_{32} = x_{0010}x_{0001}x_{0121}n(1,4,3,-2)$, $\lvert C_{Sp(8,2)}(c_{32})\rvert = 4320$								
480	0	2	0	1	0	2	0	4
16	-4	6	0	-15	24	-10	-20	36
480	0	-2	0	1	0	-2	0	4
16	$-4i$	-6	0	-15	$24i$	10	$20i$	36
16	$4i$	-6	0	-15	$-24i$	10	$-20i$	36
16	4	6	0	-15	-24	-10	20	36

order = 6, $c_{33} = x_{0121}n(-1,-3,2,-4)$, $\lvert C_{Sp(8,2)}(c_{33})\rvert = 1152$								
384	0	0	0	2	0	0	0	3
384	0	0	0	-2	0	0	0	3
64	0	$-4i$	0	-10	0	$16i$	0	19
32	$2+2i$	$4i$	$-4+4i$	-6	$-2-2i$	0	$-2+2i$	-5
32	$-2-2i$	$4i$	$4-4i$	-6	$2+2i$	0	$2-2i$	-5
32	$-2+2i$	$-4i$	$4+4i$	-6	$2-2i$	0	$2+2i$	-5
32	$2-2i$	$-4i$	$-4-4i$	-6	$-2+2i$	0	$-2-2i$	-5
64	0	$4i$	0	-10	0	$-16i$	0	19

order = 6, $c_{34} = x_{0001}x_{0011}x_{0111}x_{0021}x_{0121}n(1,2,3,-4)$, $\lvert C_{Sp(8,2)}(c_{34})\rvert = 69120$								
480	0	0	0	2	0	0	0	3
8	$4-4i$	$-16i$	$-24-24i$	-62	$-68+68i$	$128i$	$108+108i$	163
8	$4+4i$	$16i$	$-24+24i$	-62	$-68-68i$	$-128i$	$108-108i$	163
8	$-4-4i$	$16i$	$24-24i$	-62	$68+68i$	$-128i$	$-108+108i$	163
8	$-4+4i$	$-16i$	$24+24i$	-62	$68-68i$	$128i$	$-108-108i$	163
256	0	$4i$	0	-10	0	$-16i$	0	19
256	0	$-4i$	0	-10	0	$16i$	0	19

order = 6, $c_{35} = x_{0001}x_{0111}x_{0021}x_{0121}n(3,-4,-2,1)$, $\lvert C_{Sp(8,2)}(c_{35})\rvert = 864$								
768	0	0	0	-1	0	0	0	0
64	-2	0	0	3	0	-4	2	0
64	2	0	0	3	0	-4	-2	0
64	$2i$	0	0	3	0	4	$2i$	0
64	$-2i$	0	0	3	0	4	$-2i$	0

order = 6, $c_{36} = x_{0001}x_{0011}x_{0111}x_{0021}x_{0121}n(4,-2,-1,-3)$, $\lvert C_{Sp(8,2)}(c_{36})\rvert = 144$								
128	$-1-i$	i	$-1+i$	3	$-2-2i$	0	$-2+2i$	4
256	0	i	0	-1	0	$2i$	0	-2
128	$1-i$	$-i$	$1+i$	3	$2-2i$	0	$2+2i$	4
256	0	$-i$	0	-1	0	$-2i$	0	-2
128	$1+i$	i	$1-i$	3	$2+2i$	0	$2-2i$	4
128	$-1+i$	$-i$	$-1-i$	3	$-2+2i$	0	$-2-2i$	4

order = 6, $c_{37} = (x_{0001}n(-4,-2,-1,-3))^3$, $\lvert C_{Sp(8,2)}(c_{37})\rvert = 1296$								
256	0	i	0	-1	0	$2i$	0	-2
256	0	$-i$	0	-1	0	$-2i$	0	-2
128	$-1-i$	i	$3-3i$	-5	$2+2i$	$-8i$	$-6+6i$	4
128	$-1+i$	$-i$	$3+3i$	-5	$2-2i$	$8i$	$-6-6i$	4
128	$1+i$	i	$-3+3i$	-5	$-2-2i$	$-8i$	$6-6i$	4
128	$1-i$	$-i$	$-3-3i$	-5	$-2+2i$	$8i$	$6+6i$	4

order = 6, $c_{38} = (x_{0001}x_{0011}x_{0111}x_{0121}n(1,-4,2,3))^2$, $\lvert C_{Sp(8,2)}(c_{38})\rvert = 432$								
64	2	3	6	9	12	14	16	18
384	0	1	0	1	0	-2	0	-2
64	$-2i$	-3	$6i$	9	$-12i$	-14	$16i$	18
384	0	-1	0	1	0	2	0	-2
64	-2	3	-6	9	-12	14	-16	18
64	$2i$	-3	$-6i$	9	$12i$	-14	$-16i$	18

order = 6, $c_{39} = (x_{0001}n(4,1,-2,3))^2$, $\lvert C_{Sp(8,2)}(c_{39})\rvert = 288$								
256	-1	0	-1	1	0	-2	2	0
256	i	0	$-i$	1	0	2	$2i$	0
256	1	0	1	1	0	-2	-2	0
256	$-i$	0	i	1	0	2	$-2i$	0

order = 6, $c_{40} = (x_{0010}x_{0011}x_{0111}x_{0021}x_{0121}n(-2,-4,1,3))^2$, $\lvert C_{Sp(8,2)}(c_{40})\rvert = 1728$								
256	1	0	-3	-3	0	2	2	0
256	$-i$	0	$-3i$	-3	0	-2	$2i$	0
256	i	0	$3i$	-3	0	-2	$-2i$	U
256	-1	0	3	-3	0	2	-2	0

order = 7, $c_{41} = x_{0011}n(-4,2,1,-3)$, $\lvert C_{Sp(8,2)}(c_{41})\rvert = 42$								
64	-2	1	0	0	0	0	-2	4
384	0	-1	0	0	0	0	0	0
64	2	1	0	0	0	0	2	4
384	0	1	0	0	0	0	0	0
64	$-2i$	-1	0	0	0	0	$2i$	4
64	$2i$	-1	0	0	0	0	$-2i$	4

order = 8, $c_{42} = x_{0100}x_{0010}x_{0001}$, $\lvert C_{Sp(8,2)}(c_{42})\rvert = 384$								
32	$2-2i$	$-2i$	$2+2i$	8	$6-6i$	$-2i$	$6+6i$	14
192	0	$2i$	0	0	0	$2i$	0	-2
32	$2+2i$	$2i$	$2-2i$	8	$6+6i$	$2i$	$6-6i$	14
192	0	$-2i$	0	0	0	$-2i$	0	-2
32	$-2+2i$	$-2i$	$-2-2i$	8	$-6+6i$	$-2i$	$-6-6i$	14
32	$-2-2i$	$2i$	$-2+2i$	8	$-6-6i$	$2i$	$-6+6i$	14
512	0	0	0	0	0	0	0	2

order = 8, $c_{43} = x_{0100}x_{0010}x_{0001}x_{1111}$, $\lvert C_{Sp(8,2)}(c_{43})\rvert = 128$								
64	2	2	2	4	6	6	6	6
64	$-2i$	-2	$2i$	4	$-6i$	-6	$6i$	6
64	$2i$	-2	$-2i$	4	$6i$	-6	$-6i$	6
64	-2	2	-2	4	-6	6	-6	6
512	0	0	0	0	0	0	0	2
128	0	-2	0	0	0	2	0	-2
128	0	2	0	0	0	-2	0	-2

order = 8, $c_{44} = x_{0100}x_{0010}x_{0001}x_{1000}$, $\lvert C_{Sp(8,2)}(c_{44})\rvert = 32$								
128	$1-i$	0	$1+i$	2	$1-i$	0	$1+i$	2
128	$1+i$	0	$1-i$	2	$1+i$	0	$1-i$	2
128	$-1+i$	0	$-1-i$	2	$-1+i$	0	$-1-i$	2
128	$-1-i$	0	$-1+i$	2	$-1-i$	0	$-1+i$	2
512	0	0	0	0	0	0	0	-2

order = 8, $c_{45} = x_{0100}x_{0010}x_{0001}x_{0011}$, $\lvert C_{Sp(8,2)}(c_{45})\rvert = 384$								
32	$2-2i$	$-6i$	$-6-6i$	-12	$-10+10i$	$18i$	$14+14i$	22
192	0	$-2i$	0	-4	0	$6i$	0	6
32	$2+2i$	$6i$	$-6+6i$	-12	$-10-10i$	$-18i$	$14-14i$	22
192	0	$2i$	0	-4	0	$-6i$	0	6
32	$-2-2i$	$6i$	$6-6i$	-12	$10+10i$	$-18i$	$-14+14i$	22
32	$-2+2i$	$-6i$	$6+6i$	-12	$10-10i$	$18i$	$-14-14i$	22
512	0	0	0	0	0	0	0	2

order = 8, $c_{46} = x_{0100}x_{0010}x_{0001}x_{0011}x_{1000}$, $\lvert C_{Sp(8,2)}(c_{46})\rvert = 32$								
128	$1-i$	$-2i$	$-1-i$	-2	$-1+i$	$2i$	$1+i$	2
128	$1+i$	$2i$	$-1+i$	-2	$-1-i$	$-2i$	$1-i$	2
128	$-1-i$	$2i$	$1-i$	-2	$1+i$	$-2i$	$-1+i$	2
128	$-1+i$	$-2i$	$1+i$	-2	$1-i$	$2i$	$-1-i$	2
512	0	0	0	0	0	0	0	2

order = 8, $c_{47} = x_{0100}x_{0010}x_{0001}x_{0011}x_{1110}x_{1111}x_{2221}$, $\lvert C_{Sp(8,2)}(c_{47})\rvert = 128$								
64	2	2	2	0	-2	-2	-2	-2
64	$-2i$	-2	$2i$	0	$2i$	2	$-2i$	-2
64	$2i$	-2	$-2i$	0	$-2i$	2	$2i$	-2
64	-2	2	-2	0	2	-2	2	-2
512	0	0	0	0	0	0	0	2
128	0	-2	0	4	0	-6	0	6
128	0	2	0	4	0	6	0	6

order = 9, $c_{48} = x_{0001}n(-4,-1,3,-2)$, $\lvert C_{Sp(8,2)}(c_{48})\rvert = 54$								
64	$2i$	-3	$-4i$	5	$6i$	-7	$-8i$	9
64	2	3	4	5	6	7	8	9
64	$-2i$	-3	$4i$	5	$-6i$	-7	$8i$	9
384	0	-1	0	1	0	-1	0	1
384	0	1	0	1	0	1	0	1
64	-2	3	-4	5	-6	7	-8	9

order = 9, $c_{49} = x_{1111}x_{1121}n(-4,-1,-3,-2)$, $\lvert C_{Sp(8,2)}(c_{49})\rvert = 27$								
256	$-i$	0	$-i$	-1	0	-1	i	0
256	i	0	i	-1	0	-1	$-i$	0
256	1	0	-1	-1	0	1	1	0
256	-1	0	1	-1	0	1	-1	0

order = 10, $c_{50} = x_{0011}n(-4,-2,-1,-3)$, $\lvert C_{Sp(8,2)}(c_{50})\rvert = 80$								
512	0	0	0	-1	0	0	0	1
64	$-2i$	-2	$2i$	3	$-2i$	0	0	1
128	0	2	0	3	0	4	0	5
64	$2i$	-2	$-2i$	3	$2i$	0	0	1
64	2	2	2	3	2	0	0	1
128	0	-2	0	3	0	-4	0	5
64	-2	2	-2	3	-2	0	0	1

order = 10, $c_{51} = x_{0011}n(-4,-1,-3,-2)$, $\lvert C_{Sp(8,2)}(c_{51})\rvert = 20$								
256	i	0	0	0	$-i$	1	0	0
256	$-i$	0	0	0	i	1	0	0
256	1	0	0	0	-1	-1	0	0
256	-1	0	0	0	1	-1	0	0

order = 10, $c_{52} = x_{0011}n(-4,2,-1,-3)$, $\lvert C_{Sp(8,2)}(c_{52})\rvert = 240$								
32	$2+2i$	$4i$	$-2+2i$	1	$2+2i$	0	$4-4i$	9
384	0	0	0	1	0	0	0	1
256	0	$-2i$	0	-3	0	$4i$	0	5
32	$-2-2i$	$4i$	$2-2i$	1	$-2-2i$	0	$-4+4i$	9
256	0	$2i$	0	-3	0	$-4i$	0	5
32	$-2+2i$	$-4i$	$2+2i$	1	$-2+2i$	0	$-4-4i$	9
32	$2-2i$	$-4i$	$-2-2i$	1	$2-2i$	0	$4+4i$	9

order $= 10$, $c_{53} = x_{0001}x_{0011}n(2,1,4,-3)$, $\lvert C_{Sp(8,2)}(c_{53})\rvert = 240$								
64	-2	4	-6	9	-10	12	-12	13
768	0	0	0	1	0	0	0	1
64	$-2i$	-4	$6i$	9	$-10i$	-12	$12i$	13
64	$2i$	-4	$-6i$	9	$10i$	-12	$-12i$	13
64	2	4	6	9	10	12	12	13

order $= 12$, $c_{54} = x_{0001}n(-4,-3,2,1)$, $\lvert C_{Sp(8,2)}(c_{54})\rvert = 48$								
512	0	0	0	1	0	0	0	0
128	$-1-i$	0	0	-1	0	$2i$	$1-i$	0
128	$-1+i$	0	0	-1	0	$-2i$	$1+i$	0
128	$1+i$	0	0	-1	0	$2i$	$-1+i$	0
128	$1-i$	0	0	-1	0	$-2i$	$-1-i$	0

order $= 12$, $c_{55} = x_{0001}n(-2,1,-3,-4)$, $\lvert C_{Sp(8,2)}(c_{55})\rvert = 96$								
256	0	0	0	2	0	0	0	3
128	0	-2	0	2	0	0	0	-1
256	0	0	0	-2	0	0	0	3
64	$2i$	-2	0	-2	$-2i$	0	$-2i$	3
128	0	2	0	2	0	0	0	-1
64	2	2	0	-2	-2	0	2	3
64	$-2i$	-2	0	-2	$2i$	0	$2i$	3
64	-2	2	0	-2	2	0	-2	3

order $= 12$, $c_{56} = x_{0001}n(4,1,-2,3)$, $\lvert C_{Sp(8,2)}(c_{56})\rvert = 24$								
256	$-i$	0	$-i$	-1	0	0	0	0
256	-1	0	1	-1	0	0	0	0
256	i	0	i	-1	0	0	0	0
256	1	0	-1	-1	0	0	0	0

order $= 12$, $c_{57} = x_{0011}x_{0021}n(-4,-1,3,-2)$, $\lvert C_{Sp(8,2)}(c_{57})\rvert = 48$								
128	$1-i$	$-2i$	$-2-2i$	-3	$-2+2i$	$4i$	$3+3i$	4
128	$1+i$	$2i$	$-2+2i$	-3	$-2-2i$	$-4i$	$3-3i$	4
128	$-1+i$	$-2i$	$2+2i$	-3	$2-2i$	$4i$	$-3-3i$	4
128	$-1-i$	$2i$	$2-2i$	-3	$2+2i$	$-4i$	$-3+3i$	4
512	0	0	0	-1	0	0	0	0

order $= 12$, $c_{58} = x_{0111}x_{0021}n(2,3,1,-4)$, $\lvert C_{Sp(8,2)}(c_{58})\rvert = 144$								
128	$-1-i$	$2i$	0	1	$-2-2i$	0	$-1+i$	4
128	$1+i$	$2i$	0	1	$2+2i$	0	$1-i$	4
128	$1-i$	$-2i$	0	1	$2-2i$	0	$1+i$	4
512	0	0	0	-1	0	0	0	0
128	$-1+i$	$-2i$	0	1	$-2+2i$	0	$-1-i$	4

order $= 12$, $c_{59} = x_{0001}x_{0011}x_{0111}x_{0121}n(1,-4,2,3)$, $\lvert C_{Sp(8,2)}(c_{59})\rvert = 72$								
64	$-2i$	-3	$2i$	1	0	-2	$4i$	6
384	0	-1	0	1	0	-2	0	2
64	2	3	2	1	0	2	4	6
384	0	1	0	1	0	2	0	2
64	$2i$	-3	$-2i$	1	0	-2	$-4i$	6
64	-2	3	-2	1	0	2	-4	6

order = 12, $c_{60} = x_{0001}x_{0011}x_{0121}n(-1,-2,4,-3)$, $\lvert C_{Sp(8,2)}(c_{60})\rvert = 24$								
256	0	i	0	−1	0	$-2i$	0	2
128	$-1-i$	i	$1-i$	−1	0	0	0	0
256	0	$-i$	0	−1	0	$2i$	0	2
128	$1-i$	$-i$	$-1-i$	−1	0	0	0	0
128	$1+i$	i	$-1+i$	−1	0	0	0	0
128	$-1+i$	$-i$	$1+i$	−1	0	0	0	0

order = 12, $c_{61} = x_{0011}x_{0111}x_{0021}x_{0121}n(1,3,-2,-4)$, $\lvert C_{Sp(8,2)}(c_{61})\rvert = 576$								
512	0	0	0	−2	0	0	0	3
32	$-2+2i$	$-6i$	$8+8i$	−18	$18-18i$	$32i$	$-26-26i$	39
192	0	$-2i$	0	−2	0	0	0	−1
32	$-2-2i$	$6i$	$8-8i$	−18	$18+18i$	$-32i$	$-26+26i$	39
192	0	$2i$	0	−2	0	0	0	−1
32	$2+2i$	$6i$	$-8+8i$	−18	$-18-18i$	$-32i$	$26-26i$	39
32	$2-2i$	$-6i$	$-8-8i$	−18	$-18+18i$	$32i$	$26+26i$	39

order = 12, $c_{62} = x_{0111}x_{0121}n(1,4,3,-2)$, $\lvert C_{Sp(8,2)}(c_{62})\rvert = 576$								
32	$-2-2i$	$2i$	0	−2	$2+2i$	0	$-2+2i$	7
192	0	$-2i$	0	−2	0	0	0	−1
32	$2+2i$	$2i$	0	−2	$-2-2i$	0	$2-2i$	7
512	0	0	0	2	0	0	0	3
192	0	$2i$	0	−2	0	0	0	−1
32	$2-2i$	$-2i$	0	−2	$-2+2i$	0	$2+2i$	7
32	$-2+2i$	$-2i$	0	−2	$2-2i$	0	$-2-2i$	7

order = 12, $c_{63} = x_{0001}x_{0011}x_{0111}x_{0121}n(4,-3,1,-2)$, $\lvert C_{Sp(8,2)}(c_{63})\rvert = 144$								
128	$-1+i$	0	$-2-2i$	3	0	$2i$	$-1-i$	0
128	$1+i$	0	$2-2i$	3	0	$-2i$	$1-i$	0
512	0	0	0	1	0	0	0	0
128	$1-i$	0	$2+2i$	3	0	$2i$	$1+i$	0
128	$-1-i$	0	$-2+2i$	3	0	$-2i$	$-1+i$	0

order = 12, $c_{64} = (x_{0001}x_{0011}x_{0021}n(-4,-1,-2,-3))^2$, $\lvert C_{Sp(8,2)}(c_{64})\rvert = 152$								
64	$2i$	0	$4i$	−2	$6i$	−8	$2i$	−9
768	0	0	0	−2	0	0	0	3
64	−2	0	4	−2	−6	8	2	−9
64	2	0	−4	−2	6	8	−2	−9
64	$-2i$	0	$-4i$	−2	$-6i$	−8	$-2i$	−9

order = 12, $c_{65} = (x_{0111}x_{0021}n(-4,-2,-3,-1))^2$, $\lvert C_{Sp(8,2)}(c_{65})\rvert = 384$								
64	$2i$	−4	$-4i$	6	$6i$	−8	$-6i$	7
512	0	0	0	2	0	0	0	3
64	−2	4	−4	6	−6	8	−6	7
64	$-2i$	−4	$4i$	6	$-6i$	−8	$6i$	7
64	2	4	4	6	6	8	6	7
256	0	0	0	−2	0	0	0	3

order $= 12$, $c_{66} = x_{0010}x_{0011}x_{0111}x_{0021}x_{0121}n(-2,-4,1,3)$, $\lvert C_{Sp(8,2)}(c_{66})\rvert = 144$								
256	-1	0	-1	1	0	2	-2	0
256	$-i$	0	i	1	0	-2	$2i$	0
256	i	0	$-i$	1	0	-2	$-2i$	0
256	1	0	1	1	0	2	2	0

order $= 14$, $c_{67} = x_{0011}n(-4,-2,1,-3)$, $\lvert C_{Sp(8,2)}(c_{67})\rvert = 14$								
256	0	$-i$	0	0	0	0	0	0
128	$1+i$	i	0	0	0	0	$1-i$	2
128	$-1-i$	i	0	0	0	0	$-1+i$	2
128	$-1+i$	$-i$	0	0	0	0	$-1-i$	2
256	0	i	0	0	0	0	0	0
128	$1-i$	$-i$	0	0	0	0	$1+i$	2

order $= 15$, $c_{68} = x_{0011}n(-4,2,-3,1)$, $\lvert C_{Sp(8,2)}(c_{68})\rvert = 90$								
64	2	1	-2	-4	-4	-1	4	7
64	-2	1	2	-4	4	-1	-4	7
384	0	-1	0	0	0	1	0	-1
384	0	1	0	0	0	-1	0	-1
64	$-2i$	-1	$-2i$	-4	$4i$	1	$4i$	7
64	$2i$	-1	$2i$	-4	$-4i$	1	$-4i$	7

order $= 15$, $c_{69} = (x_{0011}x_{0021}n(2,-4,-1,-3))^2$, $\lvert C_{Sp(8,2)}(c_{69})\rvert = 90$								
256	-1	1	-2	2	-1	2	-2	1
256	1	1	2	2	1	2	2	1
256	i	-1	$-2i$	2	i	-2	$-2i$	1
256	$-i$	-1	$2i$	2	$-i$	-2	$2i$	1

order $= 15$, $c_{70} = x_{0011}n(-4,-1,-2,-3)$, $\lvert C_{Sp(8,2)}(c_{70})\rvert = 15$								
256	$-i$	0	0	0	0	0	0	0
256	i	0	0	0	0	0	0	0
256	1	0	0	0	0	0	0	0
256	-1	0	0	0	0	0	0	0

order $= 17$, $c_{71} = x_{0001}x_{0011}n(-4,-1,-2,3)$, $\lvert C_{Sp(8,2)}(c_{71})\rvert = 17$								
256	1	1	1	1	1	1	1	1
256	i	-1	$-i$	1	i	-1	$-i$	1
256	-1	1	-1	1	-1	1	-1	1
256	$-i$	-1	i	1	$-i$	-1	i	1

order $= 17$, $c_{72} = (x_{0001}x_{0011}n(-4,-1,-2,3))^3$, $\lvert C_{Sp(8,2)}(c_{72})\rvert = 17$								
256	1	1	1	1	1	1	1	1
256	i	-1	$-i$	1	i	-1	$-i$	1
256	-1	1	-1	1	-1	1	-1	1
256	$-i$	-1	i	1	$-i$	-1	i	1

order = 18, $c_{73} = x_{0001}n(-4,-2,-1,-3)$, $\lvert C_{Sp(8,2)}(c_{73})\rvert = 18$								
256	0	$-i$	0	-1	0	i	0	1
256	0	i	0	-1	0	$-i$	0	1
128	$1-i$	$-i$	0	1	$1-i$	$-i$	0	1
128	$-1-i$	i	0	1	$-1-i$	i	0	1
128	$-1+i$	$-i$	0	1	$-1+i$	$-i$	0	1
128	$1+i$	i	0	1	$1+i$	i	0	1

order = 20, $c_{74} = x_{0001}x_{0011}n(-2,1,-4,3)$, $\lvert C_{Sp(8,2)}(c_{74})\rvert = 40$								
512	0	0	0	-1	0	0	0	1
128	$-1+i$	$-2i$	$1+i$	-1	$1-i$	$2i$	$-2-2i$	3
128	$-1-i$	$2i$	$1-i$	-1	$1+i$	$-2i$	$-2+2i$	3
128	$1-i$	$-2i$	$-1-i$	-1	$-1+i$	$2i$	$2+2i$	3
128	$1+i$	$2i$	$-1+i$	-1	$-1-i$	$-2i$	$2-2i$	3

order = 20, $c_{75} = x_{1111}x_{1121}n(-4,-1,3,-2)$, $\lvert C_{Sp(8,2)}(c_{75})\rvert = 40$								
128	$1+i$	0	$1-i$	1	$-1-i$	$-2i$	0	-1
128	$-1-i$	0	$-1+i$	1	$1+i$	$-2i$	0	-1
512	0	0	0	1	0	0	0	1
128	$-1+i$	0	$-1-i$	1	$1-i$	$2i$	0	-1
128	$1-i$	0	$1+i$	1	$-1+i$	$2i$	0	-1

order = 21, $c_{76} = x_{0001}x_{0011}n(-4,-1,-3,-2)$, $\lvert C_{Sp(8,2)}(c_{76})\rvert = 21$								
256	1	1	0	0	0	0	1	1
256	-1	1	0	0	0	0	-1	1
256	$-i$	-1	0	0	0	0	i	1
256	i	-1	0	0	0	0	$-i$	1

order = 24, $c_{77} = x_{0001}x_{0011}x_{0021}n(-4,-1,-2,-3)$, $\lvert C_{Sp(8,2)}(c_{77})\rvert = 48$								
128	$-1-i$	0	0	0	$-1-i$	0	$-1+i$	1
512	0	0	0	0	0	0	0	-1
128	$-1+i$	0	0	0	$-1+i$	0	$-1-i$	1
128	$1-i$	0	0	0	$1-i$	0	$1+i$	1
128	$1+i$	0	0	0	$1+i$	0	$1-i$	1

order = 24, $c_{78} = x_{0111}x_{0021}n(-4,-2,-3,-1)$, $\lvert C_{Sp(8,2)}(c_{78})\rvert = 48$								
128	$1-i$	$-2i$	$-2-2i$	-4	$-3+3i$	$4i$	$3+3i$	5
512	0	0	0	0	0	0	0	-1
128	$-1-i$	$2i$	$2-2i$	-4	$3+3i$	$-4i$	$-3+3i$	5
128	$-1+i$	$-2i$	$2+2i$	-4	$3-3i$	$4i$	$-3-3i$	5
128	$1+i$	$2i$	$-2+2i$	-4	$-3-3i$	$-4i$	$3-3i$	5

order = 30, $c_{79} = x_{0011}x_{0021}n(2,-4,-1,-3)$, $\lvert C_{Sp(8,2)}(c_{79})\rvert = 30$								
256	1	1	0	0	-1	0	0	1
256	i	-1	0	0	$-i$	0	0	1
256	$-i$	-1	0	0	i	0	0	1
256	-1	1	0	0	1	0	0	1

order = 30, $c_{80} = x_{0011}n(-4,-2,-3,1)$, $\lvert C_{Sp(8,2)}(c_{80})\rvert = 30$								
256	0	$-i$	0	0	0	$-i$	0	-1
128	$-1-i$	i	$1-i$	-2	$2+2i$	$-3i$	$-2+2i$	3
128	$1+i$	i	$-1+i$	-2	$-2-2i$	$-3i$	$2-2i$	3
128	$-1+i$	$-i$	$1+i$	-2	$2-2i$	$3i$	$-2-2i$	3
256	0	i	0	0	0	i	0	-1
128	$1-i$	$-i$	$-1-i$	-2	$-2+2i$	$3i$	$2+2i$	3

REMARK. The determination of the conjugacy classes of Chevalley groups were studied by several authors. For example, see [2], [6], [15].

4. Main result

We have obtained the surjective homomorphism

$$\varphi : H_4 \to Sp(8,2)$$

with $\operatorname{Ker}\varphi = N_4$ and a set $\{c_i\}_{0 \le i \le 80}$ of representatives of conjugacy classes of $Sp(8,2)$ in the preceding sections. Our main result can be stated as follows:

Theorem 4.1. *The Molien series of H_4 is given by*

$$\begin{aligned}\Phi_{H_4}(t) &= N/D \\ &= 1+t^8+t^{12}+2t^{16}+3t^{20}+7t^{24}+7t^{28}+19t^{32}+27t^{36} \\ &\quad +52t^{40}+87t^{44}+172t^{48}+279t^{52}+550t^{56}+960t^{60}+1782t^{64} \\ &\quad +3183t^{68}+5485t^{72}+10288t^{76}+\cdots,\end{aligned}$$

where

$$\begin{aligned}D &= (1-t^{12})(1-t^{20})(1-t^{24})^4(1-t^{28})(1-t^{36})(1-t^{48})(1-t^{56})(1-t^{60})(1-t^{68}) \\ &\quad \times (1-t^{72})(1-t^{80})(1-t^{84})(1-t^{120}), \\ N &= (1+t^{12})g(t),\end{aligned}$$

$$\begin{aligned}g(t) &= 1+t^8-t^{12}+2t^{16}+3t^{24}+t^{28}+12t^{32}+13t^{36}+34t^{40}+43t^{44}+107t^{48} \\ &\quad +157t^{52}+335t^{56}+549t^{60}+1094t^{64}+1861t^{68}+3501t^{72}+5965t^{76} \\ &\quad +10728t^{80}+18041t^{84}+31051t^{88}+51025t^{92}+84427t^{96}+134865t^{100} \\ &\quad +215008t^{104}+333369t^{108}+513542t^{112}+773052t^{116}+1153627t^{120} \\ &\quad +1688292t^{124}+2447124t^{128}+3487706t^{132}+4922301t^{136}+6845055t^{140} \\ &\quad +9427941t^{144}+12816307t^{148}+17262549t^{152}+22980000t^{156}+30324507t^{160} \\ &\quad +39594318t^{164}+51272203t^{168}+65756890t^{172}+83679250t^{176}\end{aligned}$$

$$+105549085t^{180}+132161437t^{184}+164140047t^{188}+202451163t^{192}$$
$$+247823660t^{196}+301389903t^{200}+363960630t^{204}+436814071t^{208}$$
$$+520802553t^{212}+617312656t^{216}+727180701t^{220}+851846951t^{224}$$
$$+992056493t^{228}+1149232929t^{232}+1323941514t^{236}+1517506330t^{240}$$
$$+1730214602t^{244}+1963201767t^{248}+2216376776t^{252}+2490597453t^{256}$$
$$+2785299743t^{260}+3100983710t^{264}+3436532034t^{268}+3792023955t^{272}$$
$$+4165731123t^{276}+4557273329t^{280}+4964284431t^{284}+5385917115t^{288}$$
$$+5819175192t^{292}+6262771875t^{296}+6713128315t^{300}+7168581264t^{304}$$
$$+7625054128t^{308}+8080605429t^{312}+8530781308t^{316}+8973489231t^{320}$$
$$+9404047193t^{324}+9820361028t^{328}+10217690359t^{332}+10594101406t^{336}$$
$$+10944974102t^{340}+11268698558t^{344}+11560953224t^{348}+11820605627t^{352}$$
$$+12043796179t^{356}+12230003475t^{360}+12375970644t^{364}+12481889419t^{368}$$
$$+12545212616t^{372}+12566910422t^{376}+12545212616t^{380}+12481889419t^{384}$$
$$+12375970644t^{388}+12230003475t^{392}+12043796179t^{396}+11820605627t^{400}$$
$$+11560953224t^{404}+11268698558t^{408}+10944974102t^{412}+10594101406t^{416}$$
$$+10217690359t^{420}+9820361028t^{424}+9404047193t^{428}+8973489231t^{432}$$
$$+8530781308t^{436}+8080605429t^{440}+7625054128t^{444}+7168581264t^{448}$$
$$+6713128315t^{452}+6262771875t^{456}+5819175192t^{460}+5385917115t^{464}$$
$$+4964284431t^{468}+4557273329t^{472}+4165731123t^{476}+3792023955t^{480}$$
$$+3436532034t^{484}+3100983710t^{488}+2785299743t^{492}+2490597453t^{496}$$
$$+2216376776t^{500}+1963201767t^{504}+1730214602t^{508}+1517506330t^{512}$$
$$+1323941514t^{516}+1149232929t^{520}+992056493t^{524}+851846951t^{528}$$
$$+727180701t^{532}+617312656t^{536}+520802553t^{540}+436814071t^{544}$$
$$+363960630t^{548}+301389903t^{552}+247823660t^{556}+202451163t^{560}$$
$$+164140047t^{564}+132161437t^{568}+105549085t^{572}+83679250t^{576}$$
$$+65756890t^{580}+51272203t^{584}+39594318t^{588}+30324507t^{592}+22980000t^{596}$$
$$+17262549t^{600}+12816307t^{604}+9427941t^{608}+6845055t^{612}+4922301t^{616}$$
$$+3487706t^{620}+2447124t^{624}+1688292t^{628}+1153627t^{632}+773052t^{636}$$
$$+513542t^{640}+333369t^{644}+215008t^{648}+134865t^{652}+84427t^{656}+51025t^{660}$$

$$+31051t^{664}+18041t^{668}+10728t^{672}+5965t^{676}+3501t^{680}+1861t^{684}$$
$$+1094t^{688}+549t^{692}+335t^{696}+157t^{700}+107t^{704}+43t^{708}+34t^{712}+13t^{716}$$
$$+12t^{720}+t^{724}+3t^{728}+2t^{736}-t^{740}+t^{744}+t^{752}. \qquad \square$$

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